Practical Electricity

W. E. AYRTON, F.R.S

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PRACTICAL ELECTRICITY:

A

LABORATORY AND LECTURE COURSE,

For First Year Students of Electrical Engineering,

BASED ON THE

Practical Definitions of the Electrical Units.

BY

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WITH NUMEROUS ILLUSTRATIONS.

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PREFACE.

This book is intended to assist students in acquiring experimentally an *exact* working knowledge of electric current, difference of potentials, resistance, electromotive force, quantity, capacity, and power. It does not merely contain short instructions for the carrying out of experiments such as may be found in existing books on practical physics, nor, on the other hand, does it resemble certain text-books, mainly of value as electrical dictionaries, which give a little information about everything that can be comprised under the head of electricity, whether it be electric eels, the history of the invention of the telegraph, the aurora, or the earliest forms of frictional machines.

During the past few years I have been gradually developing a three years' laboratory and lecture course for students of electrical technology, and this book comprises the substance of the first year's course, together with some additional matter, mainly in small print. Experience has shown me that after a student has gone intelligently through this course, under proper direction, he has obtained clear notions of the meaning of the ampere, the volt, the ohm, the coulomb, the farad, and the watt, and feels himself familiar with their connection with one another, and with the modes of employing them in actual practice. He has, in fact, mastered the basis of the exact commercial measurement of electrical quantities. It is to be hoped, therefore, that this book may be useful to other teachers as a basis of a *practical* course of instruction; and on that account I have given, at the end, two or three samples of the actual instructions

which are attached to the sets of apparatus at the City and Guilds of London Central Institution.

The subjects of magnetism, electro-magnetism, dynamo machines, electromotors, self-induction, &c., are but very briefly referred to, because the experimental treatment of these subjects, which forms my course for second-year students, will be found in a subsequent book on "Practical Magnetism."

One of the great difficulties experienced by people in mastering the *quantitative* science of electricity, arises from the fact that we do not number an electrical sense among our other senses, and hence we have no intuitive perception of electrical phenomena. During childhood we did not have years of unconscious experimenting with electrical forces as we had with the forces connected with the sensations of heaviness and lightness, loudness and softness, heat and cold. Beyond a shock or two taken perhaps from some medical galvanic apparatus, or from a Leyden jar, our senses have never been affected by electrical action, and hence we ought to begin the study of electricity as a child begins its early education. Quite an infant has distinct ideas about hot and cold, although it may not be able to put its ideas into words, and yet many a student of electricity of mature years has but the haziest notions of the exact meaning of high and low potential, the electrical analogues of hot and cold. That it is desirable that students should learn physics, as they learn to ride the bicycle, by experimenting themselves, is now generally admitted, and this is especially true in the case of electricity, since it is by experimenting, and *only* by experimenting, that a student can obtain such a real grasp of electricity that its laws become, so to say, a part of his nature.

Hence, in the courses of electricity which I arranged at the City and Guilds of London Technical College, Finsbury, and at their Central Institution, for every hour that a student spends at lecture, he spends several in the laboratory.

Readers who have been accustomed only to the ordinary books, commencing with certain chapters on statical electricity, continuing with one or more on magnetism, and ending with some on current electricity, will be surprised at the arrrangement of the subjects in this book, and will probably be astonished at what they will condemn, at the first reading, as a total want of order. But so far from the various subjects having been thrown together hap-hazard, the order in which they have been arranged has been a matter of the most careful consideration, and has been arrived at by following what appears to me to be the *natural* as distinguished from the *scholastic* method of studying electricity. I have endeavoured to treat the subject *analytically* rather than *synthetically*, because that race of successful experimental philosophers —children—adopt this method.

For example, it is not by studying geometrical optics, much less physical optics, that an infant gradually learns to appreciate the distance of objects; and later on it is not by studying a treatise on struts, nor by listening to a course of lectures on structures, that the child finds out that the table has legs, hard legs, round legs. Feeling, looking, trying, in fact a simple course of experimental investigation, gives a child its knowledge; and this, therefore, I venture to think, is the method we should adopt when commencing the study of electricity.

The subject of current is treated first, because in almost all the industries in which electricity is practically made use of, it is the electric current that is employed; secondly, because currents can be compared with one another, and the unit of current (the ampere) defined, without any knowledge of potential difference or resistance. Potential difference is next considered, and resistance the last of the three, because the very idea of resistance implies a previous acquaintance with the ideas of current and potential difference, since the resistance of a conductor is the name given to the ratio of the potential difference (measured *electrostatically*) at

its terminals to the current passing through it. And it was Ohm's experimental proof that this ratio was constant for a given conductor at a constant temperature, that led to resistance gradually coming to be considered as a fixed definite property of a given conductor like its weight or length.*

The legal unit of potential difference, however, the volt, cannot be defined until the unit of resistance, the ohm, has been considered, arising from the fact that, whereas Ohm's law, as stated by himself, furnished us with the meaning of an electrical resistance, and with the meaning of one resistance being so many times another, the Paris Electrical Congress started in their definitions with the definition of the unit of resistance, and used Ohm's law to give us the definition of a volt, and the meaning of one potential difference being so many times another. This rather complicates the logical sequence in the mind of a beginner, and, to avoid the difficulty to a certain extent, I have, in § 44, page 89, taken a provisional electrostatic definition of a volt, almost identical in value with the legal one, and superseded it by the legal one in § 81, page 141.

That a battery has a fixed E. M. F., has been developed from the laws of energy, and therefore, while potential difference is treated before resistance, E. M. F. is treated after.

The principles underlying the action of the electrophorus, and accumulating influence machines, such as Thomson's replenisher, and the Wimshurst machine, are considered late in the book, since the student can far better understand the electrical action of these machines when he has acquired clear ideas regarding capacity and condensers.

In the tables, and generally throughout the book, the legal units recommended by the Electrical Congress of 1883 have alone been employed, since, although the

* The apparatus for proving this law experimentally, is described and illustrated on pages 134—136.

legal ohm is possibly 0·19 per cent. smaller than the true ohm, it is very much nearer than the B. A. unit, which is about 1·2 per cent. too small. Several examples, however, have been introduced to illustrate the mode of converting results obtained by using the old units into the numbers which would have been obtained had the legal units been employed. The convenience of having specific resistances, &c., expressed in legal ohms, and the E. M. Fs. of important cells in legal volts, &c., will be apparent when I quote the resolutions passed last month at the meeting of the British Association at Birmingham, and which are given immediately after this Preface.

In working out the examples, Mr. Bottomley's very useful book of logarithms has been used; the answers, therefore, only contain four significant figures, the last of which is only approximately correct.

The expression difference of potentials, or even potential difference, is a cumbersome one. The use of the capital letters P. D. as an abbreviation for potential difference, employed in the latter half of this book, corresponding with the use of the letters E. M. F. for electromotive force, may, I hope, find favour with the Committee on Electrical Nomenclature. I have also, throughout the book, used capital letters to stand for currents, and small letters for resistances, as this distinction enables the equations and formulæ to be much more easily understood.

For the use of two or three of the figures I am indebted to the kindness of Mr. Cunynghame, Mr. Gray, and the Editors of the *Electrician* and *Electrical Review*. With these exceptions, the illustrations are representations of the apparatus that has been devised by Mr. Mather and myself for the first year's students at the Finsbury Technical College and the Central Institution, at both of which colleges it is in daily use. Hence, by far the greater number of the figures have been drawn for this book, and are not time-honoured representations of historical apparatus.

It will be observed that the apparatus required for each experiment is mounted complete on a board. This is to enable it to be easily carried backwards and forwards between the laboratory and the lecture-room without disarranging it. At first sight it might appear that the student finding each set of apparatus joined up quite complete, with current laid on all ready for the carrying out of the experiment, would prevent his learning to adopt expedients for overcoming experimental difficulties, and would retard his acquiring habits of originality. For first year's students, however, I have found it a good plan to have each set of apparatus complete in position; firstly, because it is only with some such arrangement that fifty or more students can commence work almost simultaneously, and in the course of two or three hours have *all* performed some *quantitative* experiment; secondly, because when the apparatus is so arranged that even beginners can perform several experiments successfully, they are less discouraged with the difficulties they subsequently meet with when selecting and arranging the apparatus for conducting some investigation, as they have acquired faith in the possibility of success.

Here and there the apparatus is referred to as having been devised by the author. In all such cases the word author is to be taken in the plural sense, as my long association with Professor Perry, and the interchange of ideas that has taken place between us for the last eleven years, render it quite impossible to distinguish to which of us the apparatus is due.

My cordial thanks are due to two of my assistants, Mr. Mather and Mr. Raine, for correcting the proofs, and making many valuable suggestions. I am also especially indebted to the former for the very earnest, thoughtful, and painstaking way in which he has for some years assisted me in developing the course of instruction for students of electrical technology, of which the present book represents the elementary portion.

W. E. AYRTON.

October, 1886.

BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

September, 1886.

Sir,—At the Birmingham meeting of the British Association, a meeting of the Committee on Electrical Standards was held, and on the motion of Sir Wm. Thomson, F.R.S., seconded by Prof. W. G. Adams, F.R.S., it was agreed that the Committee should recommend the British Government :—

(1) To adopt for a term of ten years the Legal Ohm of the Paris Congress as a legalised standard sufficiently near to the absolute Ohm for commercial purposes.

(2) That at the end of the ten years' period the Legal Ohm should be defined to a closer approximation to the absolute Ohm.

(3) That the resolutions of the Paris Congress with respect to the Ampere, the Volt, the Coulomb, and the Farad be adopted.

(4) That the Resistance Standards belonging to the Committee of the British Association on Electrical Standards now deposited at the Cavendish Laboratory at Cambridge be accepted as the English Legal Standards conformable to the adopted definition of the Paris Congress.

I remain,

Your obedient servant,

R. T. GLAZEBROOK,

Secretary Electrical Standards Committee.

Cavendish Laboratory,
Cambridge.

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PRACTICAL ELECTRICITY.

CHAPTER I.

THE ELECTRIC CURRENT AND ITS MEASUREMENT.

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1. What is meant by an Electric Current, and by its Direction of Flow.—In the various industries in which electricity is employed, as in the telegraph, telephone, electric lighting, electrotyping, electroplating, torpedo exploding, and in the working of machinery by the aid of electromotors, it is the so-called "*electric current*" that is made use of. Hence a knowledge of the laws of this electric current, a clear conception of its so-called properties, combined with a practical acquaintance with the modes of measuring it, must be of especial importance for a right understanding of the working of the apparatus employed in the above-mentioned industries. Indeed, such knowledge is absolutely necessary if the user of electrical apparatus is desirous of employing it to the best advantage, of being able to correct faults when they

occur, as well as of effecting improvements in the instruments themselves.

It is customary to speak of an electric current as if it had an independent existence apart from the "*conductor*" through which it is said to be flowing, just as a current of water is correctly spoken of as something quite distinct from the pipe through which it flows. But in reality we are sure neither of the direction of flow of an electric current, nor whether there is any motion of anything at all. And the student must not assume that the conventional expression, — the current flows from the copper pole of a galvanic battery to the zinc pole through the external circuit, — implies any knowledge of the real direction of flow any more than the railway expressions, "up train" and "down train," mean that either train is necessarily going to a higher level than the other. In the case of a stream of water flowing along a river-bed we are quite certain that there is water in motion, and every one is agreed as to which way the water is flowing; a cork or a piece of wood thrown on the water indicates by its motion the direction in which the water is moving.

Nor, again, must an electric current be supposed to be like waves of sound travelling along, since in this latter case, although there is no actual travelling along of matter, still the direction of motion of the wave of sound is perfectly definite. Indeed, a wire along which an electric current is flowing is more like a wire at each end of which a musical instrument is being played, so that the sound is travelling in both directions along the wire at the same time. In short, the statement that an electric current is flowing along a wire is only a short way of expressing the fact that the wire and the space around the wire are in a different state from that in which they are when no electric current is said to be flowing. So that when a body and the space around the body possess certain properties that they do not usually possess, an electric current is said to be flowing through that body.

2. Properties of an Electric Current.—These properties are:

(1) A suspended magnet put in nearly any position near a body through which an electric current is said to be flowing will be deflected, also a piece of iron put near this body will become magnetised, the action in both cases being produced as if the body conveying the current had become magnetic.

(2) If the circuit through which the electric current is said to be flowing be partly solid and partly liquid, then the liquid will generally be decomposed into two parts, one part going to one side of the liquid in the direction in which the current may be said to be flowing, and the other part going to the other side of the liquid in the opposite direction to the flow of the current.

(3) The body conveying the current becomes more or less heated.

In popular language the current is said:

(1) *To deflect the magnet, and magnetise the iron.*

(2) *To decompose the liquid.*

(3) *To heat the body through which it is flowing.*

But as we have no evidence of the current apart from the conductor through which it is said to flow, it is more accurate to say, that when these effects are found to be produced, a current is said to be flowing through the conductor; than to say, that the current produces these effects. The latter expression, however, for brevity's sake, is generally adopted; and, indeed, the heat generated in a wire conveying a current has so many analogies with the heat produced in a pipe by the friction of a stream of water passing through it, that we can frequently assist ourselves by thinking of an electric current as a stream of matter passing through the wire as water would pass through a pipe filled with sponge or loosely packed with sand. But the analogy, like many other analogies, must not be pressed too far, especially as there is this very great difference between a current of water flowing in a pipe and a current of electricity in a wire, viz., that

in the former case no effects are produced external to the pipe, whereas in the latter the whole space surrounding the wire is affected.

The magnetic, chemical, and heating effects of a current are utilised practically in a number of electrical instruments; for instance:

Magnetic Property.—Needle telegraph, the Morse instrument, electric bells, arc lamps, dynamo machines, electromotors, and, in fact, all instruments using electromagnets.

Chemical Property.—Electroplating, electrotyping, the cleansing of the mercury used in the extraction of gold from sand, &c.

Heating Property. — Electric lamps, contrivances for lighting gas or oil lamps electrically, fuses for torpedoes, &c.

The heating effect of the current is, as we shall see, the effect which always occurs when a current flows; that is to say, it is impossible for a current to flow through a body without some heat being produced; and not only is heat produced by the ordinary currents flowing through telegraph wires, and which are sometimes not much more than three-thousandths of the strength of the current flowing through an incandescent lamp, but even the currents used with the Bell telephone worked without a battery produce a definite amount of heat in a given telephone circuit, even though such telephone currents are very weak compared with the currents used in telegraphy. The actual measurement of the heat, however, would be extremely difficult, if not impossible, to carry out with existing apparatus.

3. Measuring the Strength of a Current.—As, then, the production of heat always accompanies the passage of a current, it might seem that the amount of heat produced in a given time ought to be taken as a measure of the strength of the current. But, in addition to the difficulty of measuring the small amount of heat produced by weak currents, the only way we have of

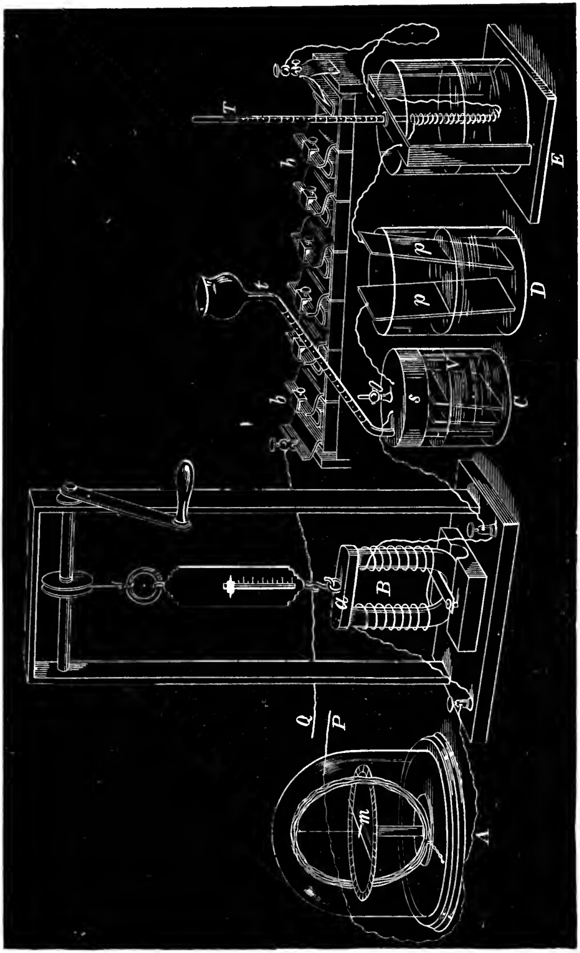

Fig. 1.

measuring the amount of heat given to a body is an indirect one, and consists in measuring its rise of temperature by means of a thermometer. But as a thermometer measures merely rise of temperature, and not the amount of heat, and as the rise of temperature of a body through which a current is passing can, without varying the current, be varied, by increasing or diminishing the facility that the body may have for cooling, various precautions have to be adopted, and further experiments have generally to be made to enable us to deduce from the observed rise of temperature the real amount of heat that was given to the body.

In order to ascertain which of the properties of a current can be best employed for measuring its strength, an experiment may be made with the following apparatus:—

A, B, C, D, E (Fig. 1) are instruments so arranged that the same electric current will be sent through them all by the battery *b b*, on joining the wires P and Q. A is a coil of cotton- or silk-covered wire, with a magnet *m* suspended so as to turn freely inside the coil, the whole arrangement forming what is called a "*galvanoscope.*" B is an "*electromagnet*" consisting of a coil of cotton- or silk-covered wire wound in opposite directions round the ends of a piece of iron of horse-shoe form. C is a "*sulphuric acid voltameter*" consisting of two platinum plates dipping into moderately dilute sulphuric acid in a vessel V, closed by an air-tight stopper *s*, through which passes a glass tube *t*, open at both ends, and with its lower end nearly touching the bottom of V. This tube is graduated in fractions of a cubic inch. D consists of two thin copper plates *p*, *p*, dipping into a solution of copper sulphate (the blue vitriol of commerce), and is called a "*copper voltameter.*" E is a coil of bare wire immersed in paraffin oil, the temperature of which can be measured by the thermometer T, the arrangement being called a "*calorimeter.*"

Connect the two wires P and Q, and allow the current

to pass for a convenient time through these five pieces of apparatus, then it will be found that:

1st. The liquid has risen a distance d_1 in the tube t of the voltameter C, indicating that the passing of the current through the liquid from one of the platinum plates to the other has caused c_1 cubic inches of gas to be generated.

2nd. One of the plates in the copper voltameter has increased in weight by W_1 grains.

3rd. The mercury in the thermometer T of the calorimeter E has risen through $D_1°$.

4th. The magnetic needle m of the galvanoscope A has all the time been kept deflected from its original position through a number of degrees $N_1°$.

5th. If at any time during the passage of the current the armature a was placed carefully on the ends of the horse-shoe electromagnet B it required a pull of w_1 lbs., as measured by the spring balance, to pull it off, when the handle at the top of the apparatus was slowly turned.

Next increase the strength of the current passing through the apparatus C, D, E, A, B, by increasing the number of cells forming the battery $b\,b$ or in any other way, such as will be described later on, then each of the effects previously observed with these instruments will be increased, and instead of the results c_1, W_1, $D_1°$, $N_1°$, w_1, we shall obtain c_2, W_2, $D_2°$, $N_2°$, w_2. But it will be found that the new values do not all bear the same ratio to the corresponding old ones. For example, if c_2 is twice c_1, then $N_2°$ may be more or less than twice $N_1°$, but will generally be less than twice, while $D_2°$ and w_2 will be found to be much greater than twice $D_1°$ and w_1 respectively. On the other hand, if the strength of the second current be so chosen as to make $D_2°$ exactly twice $D_1°$, then generally it will be found that w_2 is rather more than twice w_1, while c_2 and W_2 are much less than twice c_1 and W_1 respectively.

If, then, we *arbitrarily* define the strength of the current as being *directly* proportional to the gas evolved in the sulphuric acid voltameter, we must conclude

that if c_2 is exactly double c_1 we have doubled the current strength; but, on the other hand, if we prefer to say that strength of current is directly proportional to the angular deflection of the needle *m* in the galvanoscope A, then we must conclude that, as $N_2°$ is less than twice $N_1°$, we have not quite doubled the strength of the current; whereas if we prefer to say that current strength shall be regarded as proportional to the force required to detach the armature *a* of the electromagnet B, or, instead, proportional to the rise of temperature of the liquid in the calorimeter E in a given time, then we must conclude that the strength of the current has been much more than doubled. Which of these is right and which wrong? As long as no one of the effects varies we may be safe in concluding that the strength of the current is constant, but if the different effects to which we have been referring vary from one time to another, then which of them shall we take to represent by the magnitude of its variations the change that has taken place in the current strength?

In the case of measuring the velocity of a stream of water, or the number of gallons of water per minute discharged by a river, no two experimenters could differ. One of them, of course, by the employment of better constructed measuring instruments, or it may be from having greater experience in making such measurements, might get answers slightly different from, and more accurate than, those obtained by the other experimenter. But they could not have such totally different conceptions of what should be meant by the velocity of the water in a particular part of the channel, or of the total discharge, in gallons per minute, that the results obtained by one observer were, apart from all mere errors of experiments, twice as great as those obtained by the other. And this is because they would be dealing with the actual flow of a material substance—water.

4. Conductors and Insulators.—The various pieces of apparatus in Fig. 1 are joined by bits of copper wire,

but as long as there is even one break in the continuity, as at P Q, no current can be sent by the battery *b b* through the circuit, because the air separating the wire P from the wire Q "*insulates*" or is an "*insulator*." If P be pressed against Q, but with a thin piece of paper, or silk, or indiarubber, &c., between, still no current will flow, because all these substances are more or less good *insulators*. If, however, the ends of the wires P and Q be rubbed *clean* with *emery paper*, or be scraped clean with the back of a knife or a file, and then pressed together, the current will flow, since there is good "*conductivity*" or little "*resistance*" between the clean surfaces of metals pressed together.

5. The Strength of an Electric Current: by which of its Properties shall it be Directly Measured?—To assist us in deciding whether the amount of the magnetic action, or of the chemical action, or the amount of heat produced in a given time shall be *arbitrarily* taken to be that magnitude to which the current strength shall be defined as being *directly* proportional, we may observe that if the five pieces of apparatus A, B, C, D, E employed in the previous experiment be selected without special reference to their sizes and shapes, c_2 and W_2 will be found to be the only two out of the five quantities that bear the same ratio to their respective previous values. And in both the voltameters it was chemical decomposition that took place; in the former, this decomposition being the splitting up of the liquid into gases; in the latter, the splitting up of the copper sulphate, and the deposit of copper on one of the copper plates, together with an eating away of the other copper plate to give back to the copper sulphate solution the amount of copper taken out of it.

In A and B the effects produced are both magnetic, but it will not be found that $N_2°$ bears to $N_1°$ the same ratio that w_2 bears to w_1. Consequently, as far as we have seen at present, the amount of chemical action produced in a given time by a current appears to be a more

direct measure of its strength than the magnitude of the magnetic effect produced.

To examine this point still further, let us have *two* sulphuric acid voltameters of totally different shapes and sizes, two copper voltameters also of different shapes and sizes, the copper plates, for example, being much larger and either much nearer together or much farther apart in the one than in the other, also two galvanoscopes, two electromagnets, and two calorimeters, the two instruments in each case being selected so as to be distinctly different in size and form. Then in sending the same current through them all, the following results will be observed: In the two sulphuric acid voltameters quantities of gas equal in mass, and therefore occupying the same volume at the same pressure and temperature, will be developed in the same time, in spite of the platinum plates being of a very different size and at a very different distance apart in the two voltameters.* Similarly, in spite of the difference in size and form in the two copper voltameters, the increase in weight of the plate of the one will be exactly the same as the increase in weight of the corresponding plate of the other.† But in the case of the two galvanoscopes, the two electromagnets, and the two calorimeters, although the same current is passing through them, the effects depend on the shape, on the size, and on very many details in the arrange-

* Equality of pressure may be obtained by using for the voltameters two vessels of the same size as well as two tubes of the same bore, and filling the vessels with the same quantity of dilute sulphuric acid of the same specific gravity. In that case, if the level of the liquid in the two tubes be the same to start with, the liquids will be found to rise at exactly the same rate in them on the same current being sent through the two voltameters.

† If the plates in one of the voltameters be very small, the copper deposited may drop to the bottom of the vessel, instead of adhering to the plate. In measuring the increase of weight of the plate this copper at the bottom of the vessel must be collected and weighed. In making the experiment, however, it is better, at any rate, in the first instance, to use, in both copper voltameters, plates sufficiently large for all the copper that is deposited to adhere *firmly* to the plates. (*See* the second note on page 11.)

ment, &c. Hence, to specify the strength of a current by the magnitude of the deflection of the needle of a galvanoscope, it would be necessary to state the exact mode of constructing each part of the galvanoscope in great detail, as well as the exact position of the instrument relatively to neighbouring magnetic pieces of iron. Whereas, to specify the strength of a current by the amount of gas produced in a given time in a sulphuric acid voltameter, or by the amount of copper deposited in a given time on one of the plates of a copper voltameter, neither the shape nor size of the plates, nor the distance between them, need be taken into account within wide limits.

6. Definition of the Unit Current.—We shall, therefore, define *the strength of a current as being directly proportional to the amount of chemical decomposition produced in a given time;* and the current that *deposits* 0·00111815 *gramme, or* 0·017253 *grain, of silver per second on one of the plates of a silver voltameter, the liquid employed being a solution of silver nitrate, containing from* 15 *to* 30 *per cent. of the salt,* we shall call an "*ampere,*" and take it as our unit current.*

The same current is found to deposit 0·00032959 gramme, or 0·005084 grain, of copper per second on one of the plates of a copper voltameter,† and 0·0003392

* The silver is usually deposited on the inside of a *light* platinum bowl, and Lord Rayleigh finds that if a fairly strong solution be employed, and the deposition be not continued for more than a quarter of an hour, a uniform adherent deposit of silver will be obtained if the current does not exceed about one ampere per six square inches; that is to say, if not more than about three-thousandths of a grain of silver be deposited per second on a square inch of the surface of the platinum bowl. The other pole should consist of a silver disc placed horizontally, and wrapped in filtering paper to prevent particles of oxide of silver which may become detached from the silver plate dropping on to the platinum, and making the weight appear to be too great. The edge of the silver disc should be about equi-distant from the side and the bottom of the bowl. (*See* § 207, page 395.)

† In order that a current may be measured accurately in amperes with a copper voltameter, Dr. Hammerl finds that the plates may be conveniently put at about half an inch from one another. If put too near, what is called "*polarisation*" will occur if the current to be measured is strong, and it will be difficult to keep it constant in

gramme, or 0·005232 grain, of zinc per second on one of the plates of a zinc voltameter,* and also to decompose 0·00009326 gramme, or 0·001439 grain, of dilute sulphuric acid per second. The acid in the sulphuric acid voltameter may be conveniently diluted with water until the specific gravity of the mixture is about 1·1, which corresponds with about 15 per cent. by weight of pure sulphuric acid at 15° C.

The volume of mixed gas (oxygen and hydrogen) that is produced per second by the decomposition corresponding with a current of one ampere equals in cubic centimetres

$$\frac{0{\cdot}1738 \times 76\,(273 + \mathrm{C.}^\circ)}{h \times 273},$$

where C.° is the temperature of the mixed gas in degrees Centigrade, and h the pressure in centimetres of mercury.

If the volume be measured in cubic inches, the temperature in degrees Fahrenheit, and the pressure in inches of mercury, the formula becomes

$$\frac{0{\cdot}01058 \times 30\,(491 + \mathrm{F.}^\circ - 32^\circ)}{h \times 491}.$$

Example 1.—How many amperes would deposit 5 grammes of copper in half an hour, the current being supposed constant?

strength. The plates should be as square as possible, and in order that the deposit of copper should adhere well to the plate, the surface of each of the two plates immersed in the copper sulphate solution, should be at least two square inches for each ampere of current to be measured, this area being reckoned only on the sides of the plates opposed to one another. The plate on which the copper is deposited, and which is the only one that need be weighed, should be made of *hard thin* copper, so as to be as light as possible for its area, in order that the weight of the film of copper deposited on it may be accurately determined.

* The chemical equivalents here employed in calculating the weights of copper and zinc deposited per second by an ampere from the weight of silver deposited by that current are: silver, 107·66; copper, 63·47; zinc, 65·33.

0·0003295 grammes are deposited in 1 second by 1 ampere.

∴ 5 grammes are deposited in 1 second by

$$\frac{5}{0\cdot0003295} \text{ amperes.}$$

5 grammes are deposited in 30×60 seconds by

$$\frac{5}{0\cdot0003295 \times 30 \times 60} \text{ amperes.}$$

Answer.—8·430 amperes.

Example 2.—How many grammes of copper would be deposited by a steady current of 40 amperes acting for 5 hours?

1 ampere acting for 1 second deposits 0·0003295 grammes.
40 amperes acting for $60 \times 60 \times 5$ seconds

deposit $0\cdot0003295 \times 40 \times 60 \times 60 \times 5$ grammes.

Answer.—237·24 grammes.

Example 3.—How many amperes would deposit 9 grammes of copper in $2\frac{1}{2}$ hours, the current being constant? *Answer.*—3·035 amperes.

Example 4.—How many grammes of copper would be deposited by a steady current of 1·5 amperes acting for 16 seconds? *Answer.*—0·007908 grammes.

Example 5.—How many grammes of sulphuric acid would be decomposed by a steady current of 12 amperes acting for one hour? *Answer.*—4·028 grammes.

Example 6.—How many amperes would deposit 18 grammes of zinc in $1\frac{3}{4}$ hour, the current being constant? *Answer.*—8·428 amperes.

Example 7.—If the mixed gas produced in a sulphuric acid voltameter be at 20° C., and the barometer stand at 77·5 centimetres, what volume of gas would be produced in half a minute by a steady current of 18 amperes?

1 ampere in 1 second produces

$$\frac{0{\cdot}1738 \times 76 \times (273 + 20)}{77{\cdot}5 \times 273} \text{ cubic centimetres of gas.}$$

18 amperes in 30 seconds produce

$$\frac{0{\cdot}1738 \times 76 \times 293 \times 18 \times 30}{77{\cdot}5 \times 273} \text{ cubic centimetres of gas.}$$

Answer.—98·77 cubic centimetres of gas.

Example 8.—If the temperature of the mixed gas in a sulphuric acid voltameter be 19°·5 C., and the height of the barometer 75 centimetres, what current would produce 50 cubic centimetres of mixed gas in one minute?

Answer.—4·418 amperes.

7. Definition of the Direction of the Current.—The next thing to define is the direction of the current, which, as already explained, can only be done in a conventional way. In the case of a sulphuric acid voltameter we have hitherto only spoken of the total quantity of gas given off at both platinum plates, but if these gases be collected in separate tubes, as can very conveniently be done in the Hoffmann's voltameter (Fig. 2), then it is found that at one of the plates P oxygen gas is given off, and at the other P hydrogen, exactly in the proportions in which these two gases have to be combined together to form water; viz., two volumes of hydrogen and one of oxygen.* So that the "*electrolytic*" action effected by sending a current from one platinum plate to another in dilute sulphuric acid, is exactly the same as if the water had simply been decomposed. That sulphuric acid must be added to distilled water in order that an electric current may flow through it and produce oxygen and hydrogen, may easily be shown experimentally, but we are not sure of the exact action of the sulphuric acid; it may be that the sulphuric acid has to be added

* That the gases are hydrogen and oxygen can be proved by the fact that on turning the stop-cocks S, S, the one H when lighted will burn with a pale blue flame, and the other O will ignite a glowing piece of wood.

merely to make the non-conducting distilled water more conducting in order that it may become possible to send a strong current through the mixture with ordinary batteries; or it may be that it is the sulphuric acid that is decomposed by the current, and that the water is decomposed by a secondary chemical action. In the latter case the action would be represented in chemical symbols as follows:

Electrical decomposition

$$H_2SO_4 = H_2 + SO_4.$$

Subsequent chemical action

$$H_2O + SO_4 = H_2SO_4 + O.$$

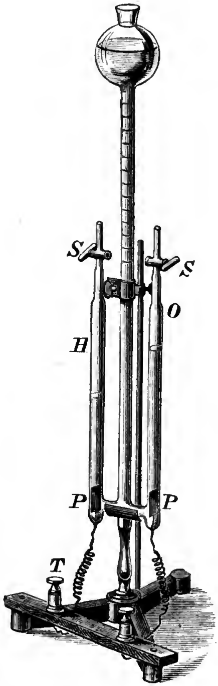

Fig. 2.

Whichever may be the true explanation, the effect of the "*electrolysis*" of dilute sulphuric acid is that two volumes of hydrogen come off at one platinum plate and one volume of oxygen at the other, and the current is said to travel through the liquid towards the plate at which the hydrogen is given off, or *the current flows through the liquid with the hydrogen*, so that in the Hoffmann's voltameter, shown in Fig. 2, the current would be said to flow through the liquid, in the short horizontal tube, from right to left.

If an acid, a copper, and a zinc voltameter be all joined together, so that the same current passes through them, then it will be found that the hydrogen in the first, the copper in the second, and the zinc in the third, all travel in the same direction, so that if through the liquid in an acid voltameter the current be said to go in the

direction in which the hydrogen travels, then through the liquids in a copper and in a zinc voltameter, it must be said to go in the direction in which the copper and the zinc travel. With this definition of direction of current

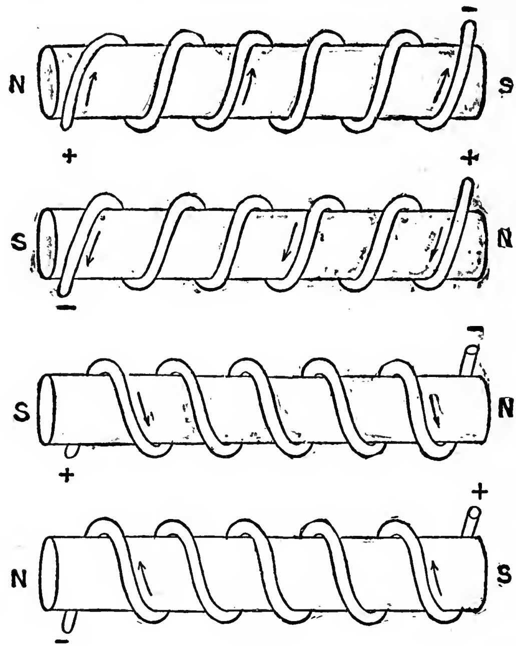

Fig. 3.

we find that if a compass needle be placed under a telegraph wire running north and south, the north-seeking *

* The "*north-seeking*" end of a magnet is the one that points towards the geographical north. The simple expression "*north*" end is confusing, since in England it refers generally to the end of a magnet that points to the north, while in France it refers to the end that points to the south, the French using that definition because that end is attracted by the earth's magnetism situated in the southern

end of the compass needle is deflected towards the east when the current is flowing along the telegraph wire from north to south.

Or, again, if a wire conveying a current be coiled round a piece of iron shown end-on to an observer, then *the end of the iron nearest him will act as the north-seeking end of a magnet when the current appears to the observer to flow round the wire in the direction opposite to that in which the hands of a clock go (or simply contra-clockwise)*. If the observer now look at the other end of the bar, he will, of course, see the south-seeking end, and in his new position the current will now appear to him to flow round the wire

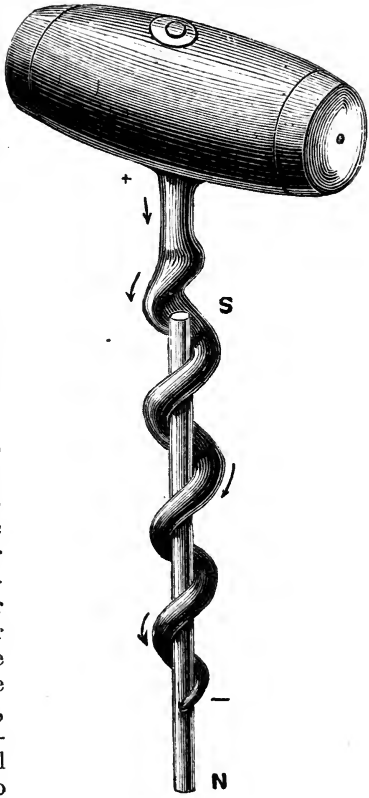

Fig. 4.

hemisphere, and the *unlike* ends *attract one another*. Calling the ends of magnets "*red*" and "*blue*" is equally confusing, as some people use one of these two colours, and others the other colour, to stand for the same end. As, however, the north-seeking end of a magnet is usually marked by instrument makers with a scratch or a cut, it would probably be best to call the north-seeking and south-seeking ends of a magnet the "*marked end*" and "*unmarked end*" respectively.

in the same direction as that in which the hands of a clock go (or clockwise). The relative magnetic polarity of the iron bar and the direction of the current, as indicated by the arrows, are shown in Fig. 3.

Perhaps the simplest method for remembering the connection between the magnetic polarity of an iron bar and the direction in which a current circulates round it is, that if a current circulates round the bar in the direction in which the iron of the thread of a corkscrew (Fig. 4) moves when the corkscrew is screwed down or up, the point of the screw will move towards the north-seeking magnetic end of the iron bar.

8. Objection to the Usual Mode of Constructing Voltameters.—The sulphuric acid voltameters, as usually pictured in books, and which are the forms obtainable at shops, are extremely unsuitable for practical use, as it is troublesome, after the tubes in which the gas is collected are full of gas, to fill them with liquid again for a new experiment. The apparatus shown in Fig. 2 is very convenient when it is required to collect the oxygen and hydrogen separately, but it has the inconvenience that, the platinum plates being small and far apart, it requires the employment of several galvanic cells to make the gas come off quickly; for although the quantity of gas produced in a given time by the same current is independent of the shape and size of the plates, the ease with which this current can be generated depends very materially on the size of the plates and their distance apart, and if we wish to produce chemical decomposition quickly, we ought to have the plates large and very near together, and the liquid employed ought to contain something like 33 per cent. of strong sulphuric acid by weight, the mixture having a specific gravity of about 1·25 at 15° C.

9. Description of a Practical Form of Sulphuric Acid Voltameter.—In Fig. 5 is shown a very convenient form of voltameter, designed by the author, consisting of a glass vessel closed at the top with an indiarubber stopper I, and containing moderately dilute sulphuric

acid. The two platinum plates P are held together by indiarubber bands, but prevented from touching one another by small pieces of glass tubing put between the plates at the top and bottom. Wires coated with gutta-percha, to prevent their being corroded by acid being spilt over them, go from the plates, one to the "*key*" K, which is raised up above the general level of the apparatus also to prevent its being corroded by drops of acid, and the other wire to one of the terminal binding screws seen in the figure. On pressing down K, the current produced by a generator attached by wires to the two binding screws, seen at the right-hand side of the figure, is allowed to pass through the apparatus. The graduated tube t, which passes air-tight through the india-rubber stopper, and reaches nearly to the bottom of the vessel, terminates at the upper end in a thistle funnel, so that if the current is by accident kept on for a longer time than is necessary to cause the liquid to rise to the top of the graduated tube, the liquid collects in the funnel instead of spilling over. This tube is also sloped so that the rise of liquid in the tube may increase the pressure of the gas in the upper part of the voltameter as little as possible.* The second tube might be simply terminated with a piece of indiarubber tubing

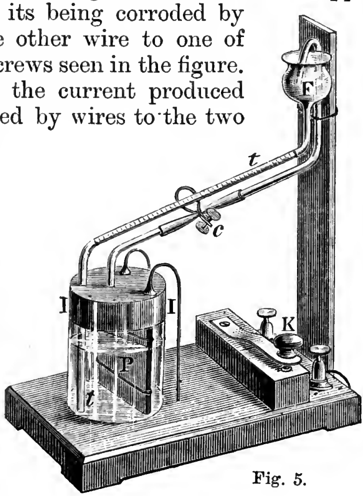

Fig. 5.

* If the vessel be full of liquid so that there is no gas between the top of the liquid and the indiarubber stopper F at the commencement of the experiment, the error arising from the compression of the gas produced by the rise of liquid in the tube t may be neglected.

closed with a pinch-cock, on opening which the gas is allowed to escape, and the liquid runs back out of the tube *t*. If this is done suddenly, however, there is a tendency for small particles of the liquid to be jerked out of the lower tube. To prevent these particles being thrown on to the stand of the apparatus, the tube is carried up, and its cord is bent over into the thistle funnel.

10. Relative Advantages of Voltameters and Galvanometers.—The disadvantage of employing a voltameter for the practical measurement of currents is, that it requires a strong current to produce any visible decomposition in a reasonable time. Even the current of one ampere, which is about that used in an ordinary Swan incandescent lamp, would require two hours, fifty-eight minutes, and forty-five seconds to decompose one gramme of dilute sulphuric acid, whereas the weak currents used in telegraphy, and, still more, the far weaker currents used in testing the insulating character of specimens of guttapercha, indiarubber, &c., might pass for many days through a sulphuric acid voltameter before their presence could be detected, much less their strength measured. Indeed, not to mention the enormous waste of time, and the difficulty of keeping the current strength which it was desired to measure constant all this time, the leakage of the gas which would take place at all parts of the apparatus that were not hermetically sealed,* would render such a mode of testing quite futile. Hence, although the voltametric method is the most direct way of measuring a current strength, and although it is constantly made use of for measuring the large currents now used industrially, still the very fact that the amount of chemical decomposition produced in a given time by a certain current is independent of the shape or size of the instrument, makes it impossible to increase its sensibility.

* A glass vessel is said to be hermetically sealed when any opening that previously existed in it has been closed, by heating the glass round the opening until it becomes soft and sticky, and pressing the edges together.

Consequently some other apparatus must be employed for practically measuring small currents, and the law of the apparatus, that is, the connection between the real strength of the current and the effect produced in the apparatus, must be experimentally ascertained by direct comparison with a voltameter.

But if we are going to compare together the indications of the two instruments produced by various currents, the second instrument cannot be much more sensitive than the voltameter, and what advantage can arise from employing such an instrument? This leads us to the fact, that whereas in a voltameter there is only one way by which the production of the gas can be more easily measured, viz., by diminishing the bore of the graduated tube *t* (Fig. 5), up which the liquid is forced by the production of the gas, there are two quite distinct ways in which the magnitude of the deflection of a "*galvanometer*"* needle can be more easily read. The first consists in using a microscope or some magnifying arrangement, or in simply lengthening the pointer, both of which methods correspond with using a tube of smaller bore in a voltameter; the second consists in winding a long fine wire, instead of a shorter thicker wire, on the bobbin of the galvanometer, and which causes the deflection of the magnet to be greater with the same current. This second mode has no analogy with any possible change in a single voltameter.

Now experiment shows *that a galvanometer of a particular shape and size, and with a definite magnetic needle, acted on by a definite controlling force, produced say by the earth's magnetism, or by some fixed permanent magnet, has a perfectly definite law connecting the magnitude of the deflection with the strength of the current producing it,*

* While a "*galvanoscope*" is the name given to an instrument used for ascertaining whether a current is flowing, or merely which of two currents is the stronger, a "*galvanometer*" is the name given to an instrument by means of which the relative strengths of currents can be compared. Any galvanoscope when calibrated becomes a more or less sensitive galvanometer.

although the absolute value of the current in amperes necessary to produce any particular deflection can be increased or diminished by using fewer turns of thick wire or more turns of fine wire to make a coil of the same dimension. If, for example, with a particular gauge of wire employed to fill up the bobbin it requires $2\frac{3}{5}$ times as many amperes to produce a deflection of 40° as it requires to produce a deflection of 20°, then if a much finer gauge of wire be employed to fill the bobbin there will still be required $2\frac{3}{5}$ times as many amperes to produce a deflection of 40° as are required to produce a deflection of 20°. But in the second case $\frac{1}{1000}$ of an ampere may be all that is required to produce the 20° deflection, whereas five amperes may be required to produce the same deflection in the first. The law of the instrument remains the same, although its sensibility may be increased 5,000 times by using finer wire to wind on the bobbin.

Thus, while we take advantage of the absolute character of the amount of chemical action to furnish us with our "*standard current meter*," we avail ourselves of the variation that can easily be made in the deflection of a galvanometer needle corresponding with the same current, to furnish us with instruments of greater and greater degrees of delicacy.

11. Meaning of the Relative and the Absolute Calibration of a Galvanometer.—Two distinct things are required to be known with reference to a particular galvanometer: first, the law connecting the various deflections with the *relative* strength of the currents required to produce them; secondly, the *absolute* values of the currents, that is, the number of amperes required for the same purpose, or, what is sufficient if the first has been ascertained, the number of amperes required to produce some one deflection. The first is sometimes called the "*relative calibration*," the second the "*absolute calibration*" of the galvanometer.

A galvanometer with its bobbin wound with thick

wire may be compared directly with a voltameter, and the relative calibration of the galvanometer determined; then if the same space on the bobbin be wound with any other gauge of wire the relative calibration of the galvanometer will be the same, and therefore known, provided that neither the length of the suspended magnet nor the magnitude of the controlling force is in any way altered. Or if a galvanometer wound with thick wire be compared with a voltameter, and its absolute calibration determined, and if, further, the law of change of sensibility with gauge of wire has also been ascertained experimentally, then the absolute calibration of the same galvanometer, when wound with any gauge of wire, filling the same space, will be known without further experiments, provided that the length of the suspended magnet and the magnitude of the controlling force remain unaltered.

If the length of the suspended magnet, or, more accurately, the distance between its "*magnetic poles*," remains unaltered, a change in the strength of its poles will neither affect the relative nor the absolute calibration of the galvanometer. For when the current is sent round the galvanometer, the suspended magnet takes up a particular position, because in that position the forces on its two ends, due to the current, balance the controlling force produced by the earth's magnetism or by some permanent magnet. And as any variation of strength of the poles of the suspended magnet will alter these two sets of forces exactly in the same ratio, they will still balance one another for the same position of the suspended magnet.

A magnet whose length is great compared with its breadth and thickness, acts as if all the magnetism were concentrated at its two ends, or the *magnetic poles* are at its ends. If the breadth or thickness be not small compared with its length, the *poles* are not quite at its ends, and the distribution of magnetism along the bar may be measured as follows.

11a. Measuring the Distribution of Magnetism in a Permanent Magnet.—This may be done with the apparatus shown in Fig. 5a, where M M is the permanent magnet placed on a board, one end of which is attached

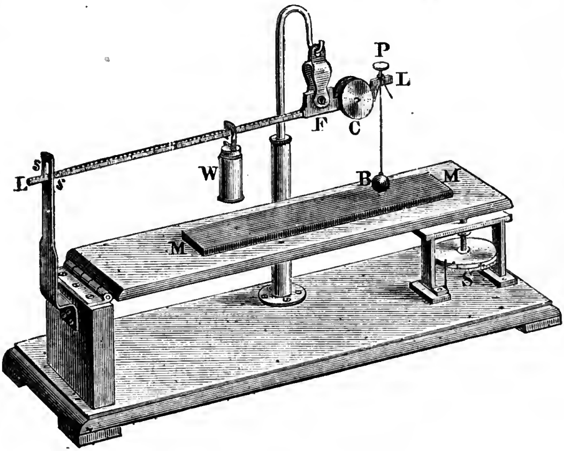

Fig. 5a.

to a hinge, while the other end can be raised or lowered by turning the "*micrometer screw*" S.* L L is a brass bar, supported on knife edges at F, like the beam of an ordinary balance, and on the upper surface of this beam there is a series of equidistant grooves, in any one of which can be placed a knife edge made like a hook, and from which hangs a brass box, W, containing leaden shot. A *soft iron ball*, B, hangs by a thread, which passes

* A micrometer screw is a screw of small pitch, accurately cut, and provided with a large head, the circumference of which is accurately subdivided. If the distance between two of the threads of the screw be, say $\frac{1}{20}$th of an inch, and the circumference be subdivided into 200 equal parts, the screw will advance $\frac{1}{4000}$th of an inch when the head is turned through a space equal to one division.

through a small vertical hole in the beam, from a brass pin P, to which the thread is attached. Before the magnet is placed on the board, the quantity of shot in this box and the counterpoise C are so adjusted that when the knife edge supporting W is placed in the groove marked nought, the beam rests horizontal. Turning P winds up, or unwinds, a little of the thread, and so slightly raises or lowers the ball. The experiment is performed by first cleaning the upper surface of the magnet and the lower surface of the ball with fine emery cloth, and wiping off the emery. The board is next levelled, the magnet put on it, and the pin P turned until the ball is just in contact with the magnet, when the left-hand end of the beam is resting at the bottom of the slot *s s*, in which position the beam is horizontal. The knife edge carrying the weight is now placed in the different grooves on the upper edge of the beam until, by trial, two are found *close to one another*, such that if the knife edge is put in the one of them nearer the fulcrum F the iron ball remains in contact with the magnet, when the micrometer screw S is turned *without shaking*, so as to lower the magnet—or in other words the left-hand end of the beam rises up as the magnet is lowered,—whereas if the knife edge carrying W be put in the next groove, the magnet cannot pull the ball down with it when it is lowered—or turning the micrometer screw S so as to lower the magnet, fails to raise the left hand end of the beam. It may then be assumed that if the knife edges were put about half-way between these two adjacent grooves, the weight W would produce a force exactly equal to that exerted by the magnet on the ball, and which, therefore, is known. Of course the experiment should be repeated several times, hanging the knife edge first in one of the grooves and then in the other, to make quite sure that the two right grooves have been found, and that the detaching of the magnet was not produced by shaking.

The magnet is now moved along the board to a new position, and the force which is exerted when the iron

ball is put in contact with another part of it ascertained in a similar way, care being taken that in all cases the thread is quite vertical. If experiments be made at points equidistant from one another all along, say, the central line of the magnet, it will be found that the force exerted by the magnet on the ball is very large towards each end, rapidly diminishes as we approach the centre, and becomes practically nought at the middle of the magnet. If similar experiments be conducted along a line parallel to the long edge of the magnet, but much nearer to one edge than the other, similar results will be obtained, but the forces at the ends of the magnet will be even greater than before. If the magnet be "*uniformly magnetised*" the attraction of the iron ball will not indicate any difference between the forces at two points similarly situated relatively to the two ends of the magnet, but if we approach our bar magnet M M to a suspended compass needle we find that the north-seeking end of the compass needle is attracted by one end of the bar magnet and repelled by the other, and so for the south-seeking end of the compass needle.

Hence, although the forces exerted on a piece of *soft iron* by points symmetrically situated relatively to the two ends of a uniformly magnetised steel bar are the same in every respect, the forces exerted by the two ends of the large magnet on one end of a compass needle are opposite in character.

Further, if we slip the bar magnet M M through a stirrup of paper suspended by a filament of unspun silk, and place it so that it is balanced and turns freely, we can find which is its north-seeking and which is its south-seeking pole, by observing the position it takes up relatively to the earth. This being done we note that it was *the north-seeking pole of our large magnet that attracted the south-seeking pole of the compass needle, and repelled the north-seeking pole.* Hence we are led to the general rule *that similar poles repel one another, dissimilar poles attract one another.*

12. Experiment for Calibrating a Galvanometer Relatively or Absolutely.—Fig. 6 shows a voltameter V, connected up with a galvanometer G, and a "*box of resistance coils*" R, ready for use for a relative or for an absolute calibration experiment. The course of the current is shown by the thick and dotted lines; the thick lines representing the wires above, and the dotted lines the wires underneath, the board on which the apparatus is placed, and by means of which it can be moved

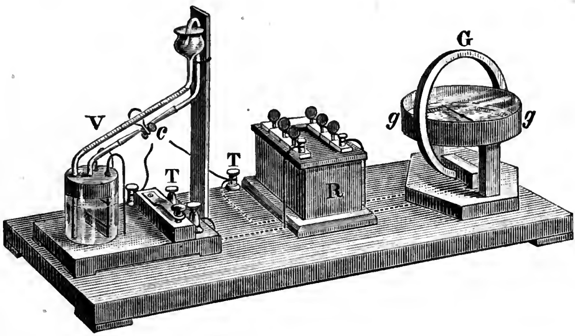

Fig. 6.

about from place to place without disconnecting the instrument. T T are the terminals, or binding screws, to which the wires coming from the battery, dynamo machine, accumulators, or other source of electricity, are attached. The galvanometer in this case consists of a vertical circular coil of wire G, at the centre of which is suspended a very short magnetic needle carrying a long pointer of aluminium or of brass wire, or, best of all, made of a thin thread of glass. *g* is a shallow circular box, with a glass lid. A scale is fixed to the bottom of the box, and from the centre of the glass lid the small magnetic needle hangs by a filament of *unspun* silk. The position of the pointer on the scale can easily be read off if

the ends of the pointer are blackened, and parallax* can be avoided by fixing the scale close under the pointer. As this, however, is liable to lead to one or other of the ends of the pointer touching the scale, if the instrument is not very well made and carefully levelled, it is better to avoid parallax by fastening the scale, which in this case takes the form of a mere circular ring, to a disc of looking-glass, and by the observer always taking care, when making a reading, to hold his head so that the pointer exactly hides its reflection in the looking-glass underneath it.

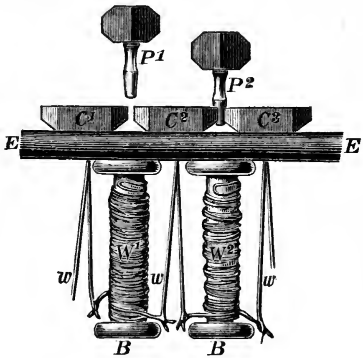

Fig. 7.

Fig. 7 shows the interior of the resistance box R, which contains coils of wire W^1, &c., wound on wooden or ebonite bobbins B, &c. The ends of these coils are soldered to stiff wires *w*, which again are fastened to the brass pieces C^1, C^2, C^3, &c., the latter being screwed to the wooden or ebonite top, E E, of the resistance box. When a plug P^2 is inserted tightly between the contact pieces, C^2 and C^3 (which can be best done by giving to the plug a downward screwing motion) the current flows along the short path, $C^2\ P^2\ C^3$, across the metal plug, and practically none through the wire wound on the bobbin W^2. If, however, a plug P^1 be withdrawn, then all the current passes through the coil W^1, and none across the space

* Parallax is the error arising from looking at the pointer rather sideways, instead of looking directly down on it, and so causing its end to appear to be over a part of the scale a little to the right, or a little to the left, of its true position.

separating c^1 and c^2. Hence, by taking out one or more plugs the path for the current may be lengthened at will,* and the strength of the current diminished. The brass pieces, c^1, c^2, c^3, are *undercut*, as seen in the figure, so that a strip of clean washleather can be inserted between them, and the ebonite cleaned. If the ebonite between the brass pieces were left dirty there would be leakage of the electricity across the film of dirt when the plug was removed, and the resistance between two of the brass pieces would be a little less than that of the coil of wire connecting them. (*See* § 140, page 266.)

For the benefit of those who may be accustomed to use resistance coils, it may be noticed that in the particular experiment shown in Fig. 6, it is quite unnecessary to know the length or gauge of the wire that has been wound on the various bobbins, nor is it at all necessary that all the coils should be made of the same wire, since whatever resistance be inserted in the box R, the current that passes through the voltameter is the same as the current that passes through the galvanometer, so that the variation in strength of the current is known from the voltameter observations, and not from the length of wire that has been introduced into the circuit. Indeed the resistance box in this experiment may be dispensed with altogether when there is any easy mode of altering the current strength by using different numbers of cells or a different kind of battery to produce the current, but in practice this result is generally most easily attained by the use of a box of resistance coils.

The calibration is performed by observing for a number of different currents the rise of the liquid in the graduated tube of the voltameter V (Fig. 6), in a given time, and the corresponding steady deflection of the needle, or of the pointer, of the galvanometer. More accurate observations can be made if, instead of observing the different lengths of the tube through which the liquid rises in the

* Further details of the construction of resistance coils will be found in § 89, page 151; § 94, page 159; § 95, page 163.

same time corresponding with the different currents, the times be noted during which the liquid rises through a fixed length of the tube, say the whole of it, and from these results a calculation be made of the distances through which the liquid would have risen in the same time. In this case two marks only are necessary, one at each end of the tube.

If the tube *t* (Fig. 5) has been graduated in cubic centimetres or cubic inches, and if the apparatus be so constructed that it can be kept during the experiment under water, so that the temperature of the gas is the same as that of the water, and therefore can be easily measured by a thermometer dipping into the water, then the actual currents in amperes producing any particular deflection on the galvanometer will, from what is given previously on page 12, be known, or the galvanometer will have been calibrated *absolutely*. If, however, the tube has been divided into portions having equal volumes, but of *unknown* value in cubic centimetres, or in cubic inches, or if, what is approximately the same in the case of a well-drawn tube, the divisions merely mark off equal lengths of the tube, then the result of the experiment will merely give the *relative* calibration of the galvanometer.

13. Graphically Recording the Results of an Experiment.—The results of this experiment, and indeed of all experiments, are best recorded graphically by points on a sheet of squared paper,* that is, paper subdivided into a number of small squares, by a large number of straight lines drawn at right angles to one another. The

* Prior to the commencement of the courses at the Finsbury Technical College, in 1879, squared paper was practically used in England only for the recording of results of original experiments. And as these results, rather than the training of the experimenter, were the most important part of the investigation, the paper was very accurately divided, and sold at a high price totally out of the reach of students. It became, therefore, necessary to have squared paper specially made, cheap, and at the same time sufficiently accurately divided for students' purposes; and such paper, machine-ruled, can now be obtained at between a farthing and a halfpenny per sheet, or at about one-twentieth of the cost of the older squared paper.

distances of the points from O Y (Fig. 8) should be taken to represent the deflections on the galvanometer G, and the distances of the same points from O X the corresponding amounts of gas produced in a given time, that is, the corresponding values of the current. In Fig. 8 the two sets of lines at right angles to one another, which divide

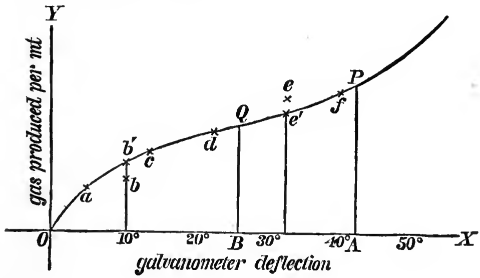

Fig. 8.

the paper into squares, have been omitted to avoid confusion. They will, however, be seen on referring to Fig. 93, page 245.

It may be asked how distances along a line can represent the angular deflections on a galvanometer, or the amount of gas produced in a given time. What is meant is this: the line O X is subdivided into a number of equal divisions by the ruling on the squared paper; one or any convenient number of these subdivisions is taken arbitrarily to stand for 1°, then any deflection is represented by this number of divisions that we have arbitrarily taken to stand for 1°, multiplied by the number of degrees on the deflection. Similarly one or any convenient number of the divisions along O Y is taken arbitrarily to stand for one cubic centimetre of gas, or the volume, it may be, contained in unit length of the tube, then any number of cubic centimetres, or the volume contained in any length of the tube, will be represented by the number of divisions along O Y that has

been taken to stand for one cubic centimetre, or for unit length of the tube, multiplied into the number of cubic centimetres, or into the length of the tube.

In selecting the scale, that is, in determining the number of divisions along O X or along O Y, that is to be taken to represent 1° deflection, or unit volume of the tube, we must remember that it is desirable that the curve, which we are about to draw, shall be as large as possible, since the larger it is the more accurately we can draw it. The scale should, therefore, be so selected that the maximum deflection of the galvanometer that has been used in the experiment should be represented by nearly the whole of O X, and the corresponding maximum quantity of gas developed in the given time by nearly the whole of O Y, since with this arrangement the curve would occupy nearly the whole of the sheet of squared paper. For example, suppose that the length O X is divided by the ruling of the paper into 170 equal divisions, and O Y into 100, and suppose that the maximum galvanometer deflection was 60°, and that when that deflection was produced the liquid ascended from the zero mark at the bottom of the tube to the top mark in twenty-two seconds, then, if one minute be the fixed time decided on, the most suitable scales for distances measured along O X and along O Y would be selected as follows:—

$$\frac{170}{60} = 2{\cdot}8 \text{ about.}$$

$$\frac{60}{22} = 2{\cdot}7 \quad \text{,,}$$

$$\frac{100}{2{\cdot}7} = 37 \quad \text{,,}$$

2·8 divisions per 1° would be a little awkward to employ when deflections of 17°, 29½°, &c., had to be represented; 2½ divisions per 1°, or 25 divisions per 10°, would therefore be better. 37 divisions along O Y, to represent the whole length of the tube would just

enable the maximum volume, corresponding with 2·7 lengths of the tube in the minute, to be represented by the whole of O Y; but 37 divisions for the whole length would be a little awkward to employ when other lengths of the tube had to be represented; probably, therefore, 30 divisions along O Y, to stand for the whole of the tube, would be more convenient.

Having obtained a sufficient number of points by experiment, a curve should be drawn connecting these points. Such a curve can be best drawn by bending an *elastic* piece of wood, and holding it so as to pass as nearly as possible through all the points that are plotted on the squared paper to record the results, and then using the bent piece of wood as a ruler, along which to draw a line. But unless the experiment has been performed with great accuracy—to attain which requires, not merely the careful attention of those engaged in making the experiment, but a certain amount of practice in experimenting—it must not be expected that a curve so drawn will pass through all the points; some of them, *b*, are sure to be a little too low, meaning that the deflection on the galvanometer has been read too high, or that the rise of liquid in the graduated tube has been read too low, from, perhaps, an error having been made in taking the time, or from the current not having been kept on for a sufficient time before the pinch-cock *c* (Fig. 5) was closed for the gas to have commenced to come off regularly. Some of the points *e* (Fig. 8), on the other hand, are sure to be too high, meaning that the deflection on the galvanometer has been read too low, or the rise of liquid in the graduated tube too high; or it may be that the experiments were fairly well made, and that *b* and *e* are merely plotted incorrectly, and so do not represent the results of the experiment.

14. Practical Value of Drawing Curves to Graphically Record the Results of Experiments.—It may be asked, But is it not possible that the points *b* and *e*, although not on the curve, may be quite correct? The

answer is, No, because experience makes us quite sure, from the fact that the connection between the deflection of the galvanometer G and the current strength must be a *continuous* one, that the points correctly representing the true connection must all lie on an *elastic curve*, or on such a curve as can be obtained by bending a thin piece of wood or steel, and, consequently, that if no mistake has been made in plotting the points *b* and *e*, some mistake must have been made in taking the observations. But what is even more important, we are also sure that the points *b′* and *e′* on the curve, obtained by drawing lines through *b* and *e* respectively parallel to O Y, give far more accurately the relative strengths of the currents producing respectively the two deflections in question, than the currents obtained directly from the experiment itself. *Drawing the curve, then, corrects the results obtained by the experiment.* But it does something more than that—*it gives, by what is called "interpolation," the results that would have been obtained from intermediate experiments correctly made*, that is to say, it tells us what would be the relative strengths of the currents that would produce deflections intermediate between the deflections that were actually observed. For example, suppose it be required to know the strength of current which will produce a deflection of 43°, for which deflection no experiment has been made, compared with that which will produce a deflection of, say 27°, for which deflection also no experiment has been made, then all that is necessary is to draw a line parallel to O Y, through the point A in O X corresponding with 43°, similarly to draw a line parallel to O Y, through the point B in O X, corresponding with 27°, and observe the lengths of the lines between O X and the points P and Q, where they cut the curve, then the strength of the current which produces the deflection 43° on this particular galvanometer bears to the strength of the current that produces the deflection 27° the ratio of the length A P to the length B Q.

If the curve is an absolute and not merely a relative

calibration curve, then the scale on which it is drawn will be known, and therefore the number of amperes corresponding with either A P or B Q.

The method of plotting the results of experiment on squared paper, and drawing a curve through them to graphically record the result, has a third important use in that *it enables us to see the nature of the law connecting the current with the deflection*, which might easily escape observation if only a few disconnected experiments had been made. For example, suppose that the results obtained in some particular case are :—

Deflection.	Relative Strength of Current.
10	24.
17·3	41·5.
22·8	54·7.
29·5	70·8.
37·4	89·7.

then plotting the results on squared paper a straight line is obtained, and from this we see at once that this particular galvanometer has, somehow or other, been so made that the angular deflection of the needle is directly proportional to the strength of the current.

CHAPTER II

GALVANOMETERS.

15. Tangent Galvanometer. — Using the particular galvanometer of the shape shown as G (Fig. 6), experiment proves that the calibration curve has the shape shown in Fig. 9, page 37, if—

(1st) The controlling force be produced by the needle moving in a "*uniform magnetic field,*" like that produced by the earth's magnetism, and in which the force acting on a given magnetic pole is uniform in magnitude and direction;

(2nd) The diameter of the bobbin round which the wire is wound be large compared with the length of the suspended magnetic needle;

(3rd) The centre of this needle be at the centre of the bobbin;

(4th) The plane of the bobbin be so placed that it contains the "*magnetic axis*" of the needle, that is, the

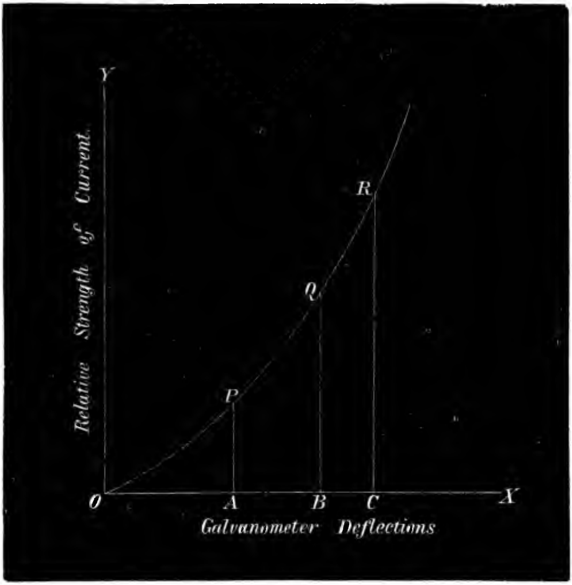

Fig. 9.

line joining its magnetic poles, when no current is passing round the coil.

And it is easy to ascertain by measurement that if any three points, P, Q, R, be taken on this curve, the lengths A P, B Q, C R, parallel to O Y, bear to one another the ratios of the tangents* of the angles

* To find the tangent of any angle A O B (Fig. 10). In *either* line O A or O B take *any* point P, and drop a perpendicular P Q on the other. Then in the triangle P O Q we have two perpendiculars: one, P Q,

represented by O A, O B, and O C respectively. Such a galvanometer is, therefore, called a "*tangent galvanometer*,"

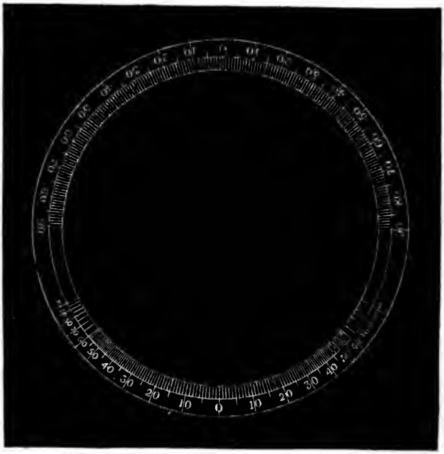

Fig. 11.

and it may be henceforth used without reference to any voltameter for the comparison of current strengths, as they will be simply proportional to the tangents

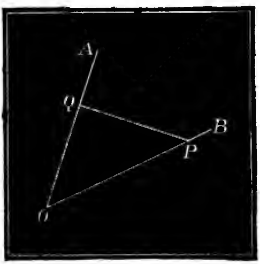

Fig. 10.

opposite to the given angle; the other, O Q, *adjacent* to it; and a third side, opposite the right angle, called the *hypotenuse*. The ratio of the *opposite* side to the *adjacent* side is called the *tangent* of the angle A O B,

$$\text{or } \frac{PQ}{OQ} = \text{tan. } AOB.$$

The ratio of the opposite side to the *hypotenuse* is called the sine of that angle,

$$\text{or } \frac{PQ}{OP} = \text{sin. } AOB.$$

of the angles through which the magnetic needle is deflected.

16. Scale for a Tangent Galvanometer.—The scales of tangent galvanometers are frequently simply divided into degrees, and a reference has constantly to be made to a table of tangents to enable the galvanometer to be used. A better plan is to divide the scale, not into equal divisions, but into divisions, the lengths of which become smaller and smaller as we depart from the zero or un-

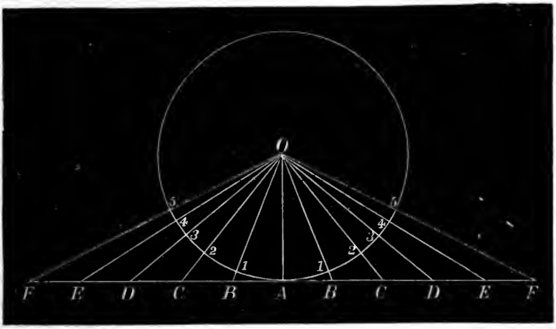

Fig. 12.

deflected position of the needle, in such a way that the number of divisions in any arc is proportional, but not necessarily equal, to the tangent of the angle corresponding with that arc. Or the scale may, as shown in Fig. 11, be divided into degrees on one side, and on the tangent principle on the other.

17. Mode of Making a Tangent Scale. —Fig. 12 shows the method of constructing such a tangent scale. The lengths A B, B C, C D, &c., along the line A F, which is a tangent to the circle at the point A, are all made equal to one another; hence if from the centre, O, of the circle straight lines, O A, O B, O C, &c., be drawn, cutting the circumference of the circle in the points A, 1, 2, 3, &c., the

numbers 1, 2, 3, 4, &c., will be respectively proportional to the tangents of the angles A O 1, A O 2, A O 3, &c.

$$\text{For tan. } AO1 = \frac{AB}{OA}$$

$$\text{tan. } AO2 = \frac{AC}{OA} = \frac{2\,AB}{OA}$$

$$\text{tan. } AO3 = \frac{AD}{OA} = \frac{3\,AB}{OA}$$

and so on.

Beginners are apt to think that, because the divisions on such a tangent scale are very much crowded together in the higher part of the scale, the value of a current can be more accurately ascertained by taking a reading on the degree side, and then finding the value of the tangent in a table of tangents, than by reading it off on the tangent scale. But this seeming greater accuracy is quite delusive, since what has to be ascertained in either case is the tangent of the angle, not merely the angle, and although on the degree side of the scale the angle can be read much more accurately than can be its tangent, or a number proportional to its tangent, on the other side, this only indicates that the error of a tenth of a degree in a large angle, although a much smaller proportional error than a tenth of a degree in a smaller angle, produces a far greater proportional error in the tangent. For example, if 20°·1 be read instead of 20°, the error is $\frac{1}{200}$, whereas if 85°·1 be read instead of 85°, the error is only $\frac{1}{850}$, or less than a quarter of the preceding error. But the tangents are in the first case 0·3659, and 0·3640, the error in the tangent, therefore, is $\frac{19}{3640}$, or about $\frac{1}{192}$, whereas the tangents in the second case are 11·66 and 11·43, so that

the proportional error is $\frac{23}{1143}$, or about $\frac{1}{50}$, which is nearly four times as great as before. Hence in this case, when the proportional angular error is diminished to one-quarter, the corresponding proportional error in the tangents is increased four times. The crowding together of the divisions on the tangent scale at the higher readings is, therefore, a correct indication of the inaccuracy likely to occur in taking readings in that part of the scale.

18. Best Deflection to use with a Tangent Galvanometer.—It can be shown that if one current strength has to be measured by a tangent galvanometer, the result, other things being the same, will be most accurate when the deflection produced is 45°; or if two currents are to be measured, the measurements will be most accurate when the deflections are as nearly as possible at equal distances on the two sides of 45°.

19. When the Tangent Law is True.—Any galvanometer may now be calibrated either relatively or absolutely, by comparison with a tangent galvanometer; and if the galvanometer to be calibrated be a very sensitive one, a tangent galvanometer with a bobbin wound with fine wire should be selected. Before, however, entering into the calibration of other galvanometers in this way, it may be well to consider under what circumstances a galvanometer will be a tangent galvanometer, especially as beginners are too apt to think that if the law of some galvanometer is unknown to them, then it must be the tangent law.

The apparatus shown in Fig. 13 enables us to decide under what conditions a force acting on a body turning on a pivot is proportional to the tangent of the angle through which the body is deflected from the position it had before the force acted on it. A *short* piece of wood, N N′, turning on a pivot, O, is acted on by a weight, W, which produces a force constant both in magnitude and direction. Variable weights, W′, are put into the scale-pan hanging at the end of a long cord, which passes over a distant pulley, *p*, and which is attached at its other end to the piece of wood

at N. The height of the pulley, *p*, is such that the long portion of the cord is *horizontal* when N N′ is vertical, that is, when there is no weight in the scale-pan, which in the figure is shown holding a weight, W′. And owing to the pulley being *distant* from N N′, the long portion of the cord remains nearly horizontal, even when the piece

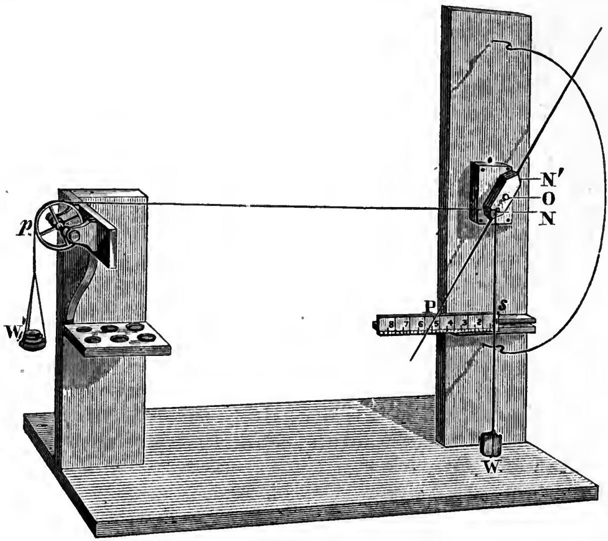

Fig. 13.

of wood N N′ is deflected through an angle. Under these circumstances experiment shows that the weights W′ put successively into the scale-pan are proportional to the distance *s* P, intercepted between the position *s* on the scale where the cord supporting W cuts the scale when N N′ is vertical, and the point P where the pointer P N cuts the scale when N N′ is deflected by the weight put into the scale-pan. Now this length *s* P divided by *s* O is the tangent of the angle through which N N′ is deflected, and,

therefore, since s O is a constant length, s P is proportional to the tangent of the angle through which N N′ is deflected. Hence with the apparatus the tangent law holds. What are the conditions of the apparatus? They are :—

1st. *The controlling force is unaltered in magnitude and direction by the motion of* N N′.

2nd. *The deflecting force always acts in the same direction, and at right angles to the controlling force.*

Hence, whenever these two conditions are fulfilled the deflecting force will be measured by the tangent of the angle of deflection.

20. Preceding Conditions are Fulfilled in the Tangent Galvanometer.—The first condition, constancy in magnitude and direction of the controlling force, is practically fulfilled in all galvanometers where the controlling force is produced by a *distant* magnet, since such a magnet produces a practically uniform magnetic field throughout the space in which the galvanometer needle can move, for, as the length of the needle is small compared with its distance from the poles of the controlling magnet, the controlling force exerted on the needle cannot be materially altered in magnitude and direction when it is deflected. In all galvanometers, therefore, in which the controlling force is due to the attraction produced by the earth's magnetism, condition (1) is absolutely fulfilled. Next with reference to condition (2)—with all flat coils the magnetic force due to a current passing round them is perpendicular to the plane of the coil for all points in the plane of the coil. But the direction of this force rapidly alters as we proceed outside the coil, unless we are near the axis, in which case the direction of the force remains practically perpendicular to the plane of the coil. And, indeed, for all points on the axis itself the magnetic force is strictly perpendicular to the plane of the coil, that is, acts along the axis. In Fig. 14 are seen a number of lines, called "*lines of force.*" These lines tell us the paths along which a magnetic pole would be pulled, or pushed, by the action of a current passing round a circular

wire or coil* perpendicular to the paper, and cutting it in the two small circles *c c*. It will be seen that at any point *p* on the axis A A of the coil the direction is everywhere perpendicular to the plane of the coil, also that near the axis the direction is nearly perpendicular to this plane for

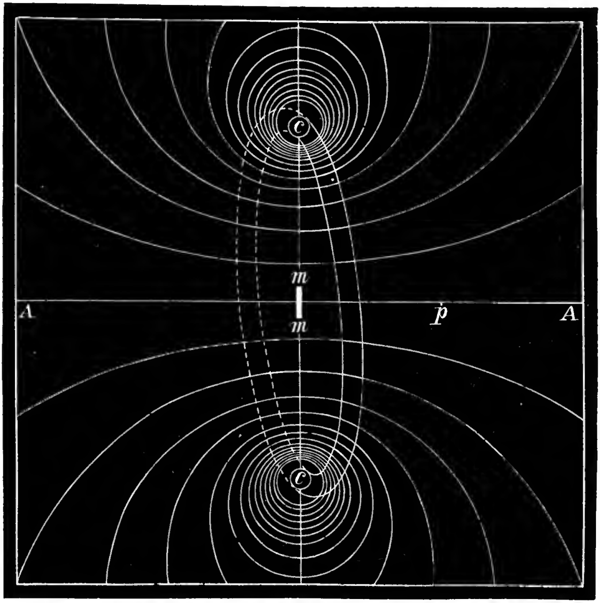

Fig. 14.

a considerable distance, while near the coil itself the direction of the force changes rapidly. Hence, if we suspend at the centre of a coil a very short magnetic needle, *m m*, having a length not greater than one-tenth or one-

* This wire or coil, the plane of which is in reality perpendicular to that of the paper, is represented in the figure in a kind of oblique perspective by a double line.

twelfth the diameter of the coil, the deflecting force due to a current passing round the coil will be perpendicular to the plane of the coil, even after the needle is deflected, and will be also perpendicular to the controlling force, if the controlling force acts in the plane of the coil, that is, if the coil is so placed that its plane contains the magnetic axis of the suspended needle when no current is passing through the coil.

In fact, if the coil occupies the position of the semicircular wire seen in Fig. 13, and if this wire is in the "*plane of the magnetic meridian*,"* the conditions necessary for the deflecting force being proportional to the tangent of the deflection will be fulfilled.

We have seen, from the experiment described in § 15, page 36, that the tangent of the deflection of the needle of a tangent galvanometer is directly proportional to the current strength, or simply to the current; hence, we may conclude that the force acting on a magnetic pole at a fixed point on, or near, the axis of a circular coil is directly proportional to the current flowing round that coil. Later on we shall see that this law is true for a fixed magnetic pole in any position relatively to the coil acted on by a current flowing round a coil of any shape.

It is not necessary that the coil of a tangent galvanometer should be circular, but in order to obtain the straightness of the lines of force in the neighbourhood of the axis, as seen in Fig. 14, and not merely for points actually on the axis, of which we could only avail ourselves by using an infinitely short magnet, the diameter of all parts of the coil must be large. Hence, if an elliptic, or other non-circular coil, were used, its smallest diameter would have to be large, and consequently its largest diameter unnecessarily so.

From what has been said, and from an examination of Fig. 14, it will be seen that for *very small* deflections of the needle any galvanometer, no matter what be the

* The "*plane of the magnetic meridian*" at any place is that vertical plane in which lies the axis of a compass needle.

size of the needle and of the coil, or how near be the controlling magnet, will be a tangent galvanometer. And further, since the tangents of very small angles are simply proportional to the angles, the deflections of the needle, as long as they are *very small*, in any galvanometer are directly proportional to the strengths of the currents producing them.

21. Adjusting the Coil of a Tangent Galvanometer.—Returning now to ordinary tangent galvanometers to be used for large deflections, how can we adjust the coil so as to be sure that its plane contains the axis of the needle? Owing to the coil having a certain breadth, it is impossible to see the needle when looking down on to the coil; indeed, it is for this reason that the long light pointer attached to the needle is placed at right angles to the needle. It would not be right to assume that because the instrument has been so turned that the pointer points to the zero on the scale, therefore the plane of the coil contains the magnetic axis of the needle, for even if the scale has been attached to the instrument so that the line of zeros is at right angles to the plane of the coil, it does not follow that the pointer itself is at right angles to the needle. The two may even have been placed at right angles to one another by the maker, and yet the pointer may have been bent subsequently, so that they are not at right angles at present; or no experiment may have been made by the maker to test this, as he is aware that the user will probably make a test and adjust the pointer for himself. This test may most simply be made as follows:—Turn the instrument until the pointer points to 0°, send any convenient current through it, and observe the deflection, then reverse the direction of the current without altering its strength, and observe the deflection on the other side. If these deflections are exactly equal, then the plane of the coil contains the axis of the needle when the pointer points to 0°, and the instrument is properly adjusted. But if, on the other hand, one deflection is, say, 47° to the left, and the other, say, 44° to the right, the

pointer is not at right angles to the magnetic axis of the needle, supposing, of course, that the scale has been so fixed that the line of zeros is exactly at right angles to the plane of the coil. Next, turn the instrument a little about its centre in the direction opposite to that in which the needle moved when the greater deflection was obtained. The pointer will now, of course, not point to zero; let it stand at 1° to the left. Again send a current, first in one direction, obtaining a deflection, say, 46° to the left, and in another direction, when it gives a deflection of, say, 45° to the right. Now remembering that the pointer started from 1° to the left, the true deflections of the needle are respectively, $46° - 1°$, or 45° to the left, and $45° + 1°$, or 46° to the right. Hence, the fault is now on the other side, or the left deflection is smaller than the right, and we have, consequently, turned the instrument too much. Turn, therefore, the coil round a very little in the opposite direction, so that when no current is passing through the instrument the pointer stands at, say, $\frac{1}{2}°$ to the left, and send as before reverse currents of equal strength, obtaining apparent deflections, $45\frac{1}{2}°$ to the left and $44\frac{1}{2}°$ to the right, which, corrected for the initial zero error, correspond with equal deflections of 45° to either side.

The instrument will now be correct when it is so placed that for no current the pointer stands at $\frac{1}{2}°$ left, and it can be so used, but not, however, with the tangent scale. To enable us to employ the side of the dial graduated in tangents, as well as to avoid having to remember the $\frac{1}{2}°$ left error, do not alter the position of the instrument, but bend the pointer until it points to 0° for the same position of the instrument in which it previously pointed to $\frac{1}{2}°$ left. The instrument will now behave as a correct tangent galvanometer when the pointer stands at 0° for no current.

We have spoken of reversing the direction of the current without altering its value. This may be done by causing the current to pass through any galvanoscope,

the law of which may be quite unknown; and taking care that the deflection of the needle after the current has been reversed is the same in amount as it was before the current was reversed; indeed, if we reverse the connections of the galvanoscope at the same time that we reverse the connections of the battery or other current generator employed in the experiment, it will not be even necessary to know that the coil and needle of this auxiliary galvanoscope are symmetrical, or that the strength of a current producing a deflection to the right is the same as that of a current producing a deflection to the left.

22. Variation of the Sensibility of a Galvanometer, with the number of Windings and with the Diameter of the Bobbin.—A tangent galvanometer, on the bobbin of which a short thick wire has been coiled, can be calibrated absolutely by direct comparison with a voltameter. To obtain a more delicate tangent galvanometer, we must replace this thick wire with many turns of fine wire, and the numbers of amperes or fractions of an ampere producing any particular deflection on this delicate galvanometer will also be known if we know the exact change in the sensibility produced by replacing the thick wire with many turns of fine. The apparatus shown in Fig. 15 is for the purpose of enabling this to be experimentally tested, as well as for testing the variation in sensibility produced by altering the diameter of the coil. *g g* is a flat cylindrical box, containing, as in Fig. 6, a scale fastened to its bottom, and a short needle carrying a long light pointer, suspended by a short piece of unspun silk, fastened to the centre of a circular piece of glass, forming the cover. C C is a bobbin of large diameter, and such that its centre is exactly the same height above the base-board B B as is the centre of the suspended magnetic needle. *c c* is a smaller bobbin, of which the diameter is exactly half that of the larger bobbin, but still large compared with the length of the suspended magnet. The centre of the smaller bobbin is also on the same level

as the suspended magnet when the base-board *b b* of the smaller bobbin is placed on that of the larger. On the larger bobbin C C are wound two distinct coils of insulated wire, one consisting of twelve convolutions, and having its ends attached to two of the binding screws, 1, 2, the other

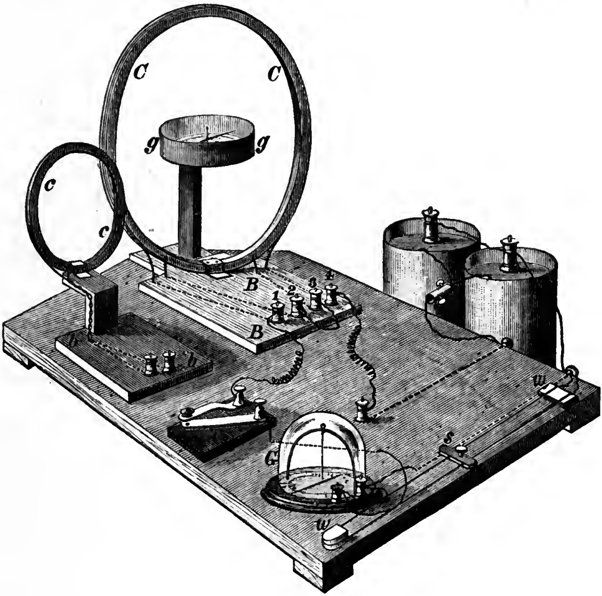

Fig. 15.

of four convolutions, and having its ends attached to the other two binding screws, 3, 4. If the binding screw 2 at the end of the first coil be joined by a piece of wire, as shown in the figure, to the binding screw 3 attached to the beginning of the second, the current will go 12 + 4, or sixteen times round the bobbin; whereas if the wire connect the end of the first coil, 2, with the end of the

second. 4. and the current enter and finally leave the bobbin by the two binding screws 1, 3, attached respectively to the beginnings of the two coils, then the current will go twelve times round the bobbin in one direction and four times in the other, or practically $12 - 4$, or eight times round the bobbin. Now, experiment shows that if the controlling magnet be untouched, and a current of constant strength be passed successively first four, then eight, then twelve, then sixteen times round the bobbin, which is kept fixed in position during the experiment, the tangents of the corresponding deflections produced will be as four to eight, to twelve to sixteen, that is, simply proportional to the number of times the current passes round the bobbin. The constancy of the current can be tested by the deflection on the auxiliary galvanoscope G, and if the insertion in the circuit of the greater or less number of coils on the bobbin C C, or any other cause, tends to make it vary in strength, its constancy can be maintained by sliding the screw clip *s* along the stretched wires *w w*,* by means of which the length of the wire in the circuit can be increased or diminished, and the current strength diminished or increased. If we next experiment with the bobbin *c c* of half the diameter, and on which a coil of four convolutions is wound, we find that if the two bobbins be placed so as to be in one plane, and if their centres coincide with that of the suspended magnet, the tangent of the deflection produced by a certain current flowing round the smaller one is twice as great as the tangent of the deflection produced by the same current flowing four times round the larger bobbin; and also if the same current pass four times round the smaller in one direction, and eight times round the larger in the opposite direction, that no deflection is produced.

* To prevent these wires being accidentally damaged, it is better to put them in a groove formed in the base-board instead of above the board as shown in Fig. 15. In that case it is convenient to shape the clip *s* so that it can slide in the groove in the base-board, the ends of the clip being guided by the sides of the groove.

From this we learn that the tangent of the deflection produced by a current, that is, *the sensibility of the instrument is directly proportional to the number of convolutions of wire, and inversely proportional to their diameter.* On the bobbin c c the sixteen convolutions of wire all occupy practically the same position relatively to the suspended magnet. If, however, many turns are to be wound on a bobbin, the bobbin will have a certain depth in the direction of the diameter of the coil, and a certain width at right angles to the plane of the coil. The error introduced by the depth of the coil is that of making the convolutions of wire have different diameters, and the effect of this we have just seen. The error introduced by the width of the coil can be seen by observing how the deflection produced by a constant current varies as the bobbin c c is moved parallel to itself along its axis. The additional error introduced by the non-centring of the coil and the needle may also be experimentally investigated by examining how the deflection produced by a constant current alters as the bobbin is slid in its own plane.

It is not necessary in this book to consider exactly how to correct these errors, nor the error arising from the diameter of the bobbin in all actual tangent galvanometers not being infinitely large compared with the length of the needle; and it will be sufficient to state that with a tangent galvanometer made with a single bobbin having a rectangular channel, within which the coils of insulated wire are to be wound, Prof. Silvanus Thompson has shown that the tangent law is most accurately fulfilled when the depth of the channel in the radial direction bears to the breadth in the axial direction the ratio of

$$\sqrt{3} \text{ to } \sqrt{2},$$

or about eleven to nine.

When an experiment is made to determine the alteration in sensibility produced by moving the coil parallel to

itself along its axis, it is found that the tangent of the deflection produced by the same current when a coil of radius r is made to occupy different positions parallel to itself at distances x, measured along the axis from the centre of the needle, is proportional to

$$\frac{r^3}{\left(\sqrt{r^2 + x^2}\right)^3}$$

that is, the sensibility of the galvanometer is proportional to this expression.

Example 9.—A tangent galvanometer is made with two coils of equal diameter, the first consisting of 500 convolutions of wire, the second of one convolution. If a current of 0·25 ampere sent through the first cause a deflection of 45°, what current sent through the second in the opposite direction, while the same current was still flowing through the first, would cause the deflection to become one of 10°?

Let x be the unknown number of amperes:

$$\text{Then } \frac{500 \times 0{\cdot}25 - x}{500 \times 0{\cdot}25} = \frac{\tan. 10^\circ}{\tan. 45^\circ}$$

Answer.—103 amperes.

Example 10.—A galvanometer is about to be constructed of two coils: the first, six inches in diameter, consists of 350 convolutions of wire; the second has two convolutions only. A current of 0·4 ampere sent through the first causes a deflection of 30°. What must be the diameter of the second coil, in order that a current of 80 amperes, in the opposite direction, sent through it, while 0·4 amperes is still flowing through the first, may cause the deflection to become 5°?

Let x be the diameter of the second coil.

Since the effect of the current is directly proportional to the number of convolutions, and inversely proportional to the diameter—

$$\frac{\dfrac{0\cdot4 \times 350}{6} - \dfrac{80 \times 2}{x}}{\dfrac{0\cdot4 \times 350}{6}} = \frac{\tan. 5^\circ}{\tan. 30^\circ}$$

Answer.—8 inches nearly.

Example 11.—A galvanometer is about to be constructed of two coils: the first, seven inches in diameter, consists of 600 convolutions of wire; the second is to be 5·5 inches in diameter. A current of 0·1656 ampere sent through the first causes a deflection of 40°. Of how many convolutions of wire must the second coil consist, in order that while 0·1656 ampere is still flowing through the first, a current of 65 amperes flowing through the second may cause the deflection to become 8°?

Answer.—One convolution.

23. Thomson's Galvanometer for Large Currents.—A tangent galvanometer, with a scale graduated in tangents, and controlled by a permanent magnet rigidly fixed to the instrument, has been arranged by Sir William Thomson, and is shown in Fig. 16. It has the peculiarity that the needle, scale, and permanent magnet M can be slid along a board P, and so withdrawn parallel to itself farther and farther from the action of the coil C; hence a wide range of sensibility can be given to the instrument, in accordance with the last formulas. To prevent the current which flows in the long wires connecting the galvanometer with the rest of the circuit acting directly on the suspended magnetic needle, these coming and going wires are twisted together into a form of cable, which is shown in the figure, and which is supplied with the instrument.

The advantage of this galvanometer is that, first, owing to its being a tangent galvanometer the ratio of two current strengths can be very accurately compared; secondly, from the method of sliding the needle away from the coil, two currents, widely differing in strength, can be compared. The disadvantage is that, on account of the

small action that the coil, even with a very strong current flowing round it, can exert on the needle, when they are at opposite ends of the baseboard, the controlling force of the permanent magnet has to be kept small; hence the instrument, as we shall see afterwards, cannot be made very "*dead beat*" (*see* § 38, page 78), and further, the indications are much disturbed by any external magnet. In fact, the instrument is rather for use in a laboratory, where the magnetic field is constant in strength, and known, than in a dynamo room or workshop, where large pieces of iron and powerful magnets are being moved about.

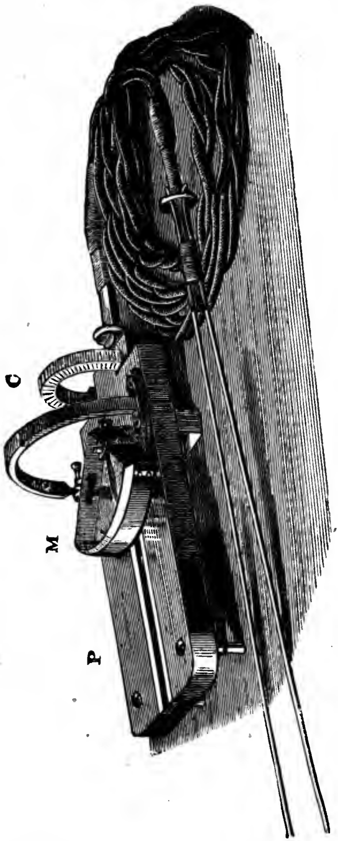

Fig. 16.

24. Values in Amperes of the Deflections of a Tangent Galvanometer controlled only by the Earth's Magnetism. — The sensibility of a tangent galvanometer depends not merely on the bobbin, but also on the strength of the controlling field. If, however, the

"*horizontal component of the earth's magnetic force*"* in London be alone employed as the controlling force, and if the instrument be used with the centre of the coil and the centre of the needle coinciding, then the connection between the current A in amperes, the deflection d in degrees, the radius r of the coil in inches, and the number of convolutions N of wire on the bobbin, is given by the following equation for 1886:

$$A = \frac{0{\cdot}73735 \times r \times \tan. d}{N};$$

the coefficient 0·73735 for 1886 becoming 0·73844 for 1887, 0·73953 for 1888, and 0·74062 for 1889. From this it follows that in the year 1887 a deflection of 45° will be given by one ampere when there are five convolutions of wire on a bobbin 6·772 inches in radius.

Example 12.—How many amperes would deflect the needle of a tangent galvanometer 60° in the year 1886, the controlling force being the horizontal component of the earth's magnetism, and the galvanometer having a bobbin five inches in radius, wound with six convolutions of wire?

The number of amperes is $\dfrac{0{\cdot}73735 \times 5 \times \sqrt{3}}{6}$.

Answer.—1·064 amperes.

Example 13.—Through what angle would 0·598 ampere deflect the needle of a galvanometer with a bobbin seven inches in radius, wound with five convolutions of wire, in the year 1888, the controlling force being the horizontal component of the earth's magnetism?

* The *horizontal component of the earth's magnetic force* is that portion of the earth's force which acts on a compass needle.

$$0{\cdot}598 = \frac{0{\cdot}73953 \times 7 \times \tan. d}{5}$$

$$\therefore \tan. d = \frac{5 \times 0{\cdot}598}{0{\cdot}73953 \times 7}$$

$$= 0{\cdot}5775$$

$$\therefore \quad d = 30°. \qquad \textit{Answer.}—30°.$$

Having tan. d, d may be found either by looking in a table of tangents or in the following way :—

Take a sheet of squared paper, and on it select two axes, or lines of reference, O X, O Y, at right angles to one another. Choose any number of the divisions on your paper to represent unity, taking care that there are more than 100 of these larger divisions along O X, and at least 58 along O Y. These numbers are chosen because the tangent of the angle required is approximately given by the ratio $\frac{57{\cdot}7}{100}$. Along O X mark off O A, equal to 100 of the divisions, then on the line through A, parallel to O Y, mark off A B as nearly as possible equal to 57·7 of the divisions. Join O B. Then B O A is the angle d.

$$\text{For } \tan. \text{B O A} = \frac{\text{B A}}{\text{O A}}$$

$$= \frac{57{\cdot}7}{100}$$

$$= \tan. d.$$

The angle d may now be found by means of a protractor.

Example 14.—If the horizontal component of the earth's magnetism in 1887 be the controlling force in a tangent galvanometer, the bobbin of which is 11 inches in diameter, how many convolutions of wire must be wound on the bobbin in order that a current of 1·015 amperes may give a deflection of 45°?

Answer.—4 convolutions.

Example 15.—If the horizontal component of the earth's magnetism in 1885 be the controlling force in a tangent galvanometer, the bobbin of which is wound with eight convolutions of wire, what must be the radius of the bobbin in order that a current of 0·384 ampere may give a deflection of 50°? *Answer*.—$3\frac{1}{2}$ inches.

Tan. 50° may be found either in a table of tangents or in the following way:—

Take a sheet of squared paper; on it take axes O X, O Y; with a protractor make the angle B O X, equal to 50°, and produce O B as far as the paper will allow. Let A B be the farthest line from O, parallel to O Y, which cuts B O. Then $\tan. 50° = \frac{AB}{OA}$.

Count the number of divisions and fractions of a division in A B and O A, and divide the one by the other.

If the angle be large, great care must be taken to lay it down accurately with the protractor, since a small error in a large angle will introduce a large error in the tangent.

Example 16.—About how many times the horizontal component of the earth's magnetism must the controlling force be in a tangent galvanometer, having a bobbin five inches in radius wound with six convolutions of wire, in order that a current of 20 amperes may make a deflection of 45°? *Answer*.—Nearly $32\frac{1}{2}$ times.

25. Galvanometers having an Invariable Absolute Calibration.—In order that the absolute calibration of any galvanometer may remain invariable, the magnetic field in which the suspended magnet moves must remain constant in strength; and if the galvanometer is to be moved about near masses of iron, or near the large powerful electromagnets of dynamo machines, probably the most satisfactory of all the methods that have been tried for securing approximate constancy of the controlling field is either to attach a powerful permanent magnet to the instrument, or still better to substitute the force of

a spring for a magnetic controlling force.* In either case this controlling force must, of course, be large compared with any magnetic forces that are likely to be exerted by outside magnets on the suspended needle, and must be very many times as large as that due to the earth's magnetism. But, in that case, unless the instrument is only to be employed to measure the most powerful currents, the coil must be near the needle, so that the condition (No. 1, page 36) for obtaining the tangent law cannot be complied with. And generally the necessity of having a coil of very large diameter compared with the length of the needle makes a tangent galvanometer unsuitable for a portable galvanometer, or else necessitates the employment of so short a needle that its oscillations are much impeded by the mass of even an extremely light pointer attached to it. Hence with all portable galvanometers, and especially in the case of those which may be used near masses of iron or dynamos without serious error, it is better to abandon any attempt to obtain the tangent law, and calibrate the galvanometer by direct comparison with a tangent galvanometer.

26. Calibrating any Galvanometer by Direct Comparison with a Tangent Galvanometer.—Fig. 17 shows the simplest way of doing this. G is the standard tangent galvanometer, D the galvanometer, which, if rough and portable, is sometimes called a "detector," requiring to be calibrated. V is a vessel containing two zinc plates dipping into a small quantity of a solution of zinc sulphate, which is used for varying the strength of the currents passing through G and D by altering the distance between the bottoms of the plates. The wires coming from the generator of electricity are attached to the terminals, one only of which, T, is seen in the figure, and a key placed between G and D enables the current to be made or broken. As the same current passes through G and D, it is quite unneces-

* For further information on *shielding galvanometers from extraneous magnetic disturbance, see* § 36, p. 73; § 53, p. 103; and § 202, p. 390.

sary to know the value of the resistance introduced by V; all that has to be done is to observe a number of corresponding deflections of the needles of G and of D, then, since the true value of the current is proportional to the tangent of the deflection in G, a calibration curve can be drawn for D, in which horizontal distances represent the observed angular deflection of the needle of D, and vertical distances the relative strengths of the currents producing these deflections. If the number of amperes

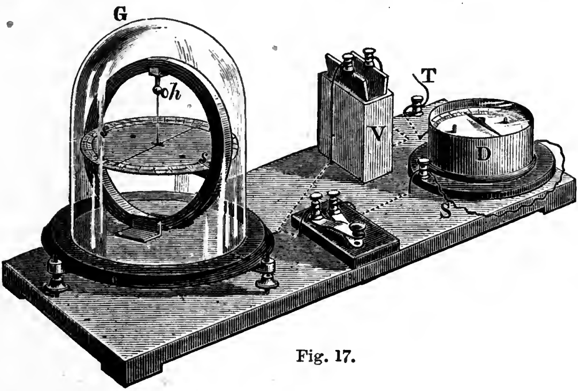

Fig. 17.

producing any particular deflection in G is also known, then D will be calibrated absolutely.

It frequently happens that, on account of the great increase in sensitiveness produced by putting the wires conveying the current close to the needle, a rough galvanometer with a few turns of wire is even more sensitive than a tangent galvanometer with many turns. Under such circumstances it would be difficult to compare them, as a large deflection on D would only correspond with a small one on G, and a smaller deflection on D would not produce deflections on G large enough to be read at all accurately. This difficulty may, however, be overcome by putting a piece of wire S (Fig. 17), a "*shunt*" as it is called,

between the terminals of D, and which allows a portion of the current to pass through it instead of through D. As, however, for the same shunt the same *fraction* of the total current is, as we shall see later on (page 178), always shunted past D, the sensibility alone of D, and not the law connecting current strength with deflection, is altered by using such a shunt. The use of a shunt, therefore, alters the absolute but not the relative calibration of a galvanometer; consequently, if D is absolutely calibrated, the same shunt must always be employed when it is desired to use the absolute calibration curve of that galvanometer.

27. **Pivot and Fibre Suspensions.**—The galvanometers G and D differ also in another particular, namely, in the way in which the magnetic needle is supported. In D the little magnet has a jewel in its centre, and rests on a sharp pivot, as in an ordinary pocket compass; whereas in G the needle is supported by a fine fibre of *unspun* silk, the upper end of which is rolled round a brass pin *h*, by turning which the needle can be lowered on to the card *s s*, on which the scale is engraved, when the instrument is being carried about, or raised again so as to be in the centre of the coil when the instrument is in use. The fibre suspension introduces far less friction to the motion of the needle than the best jewel and pivot, and, in addition, costs far less; but with a fibre suspension it is generally necessary that the instrument should have levelling screws, such as are seen attached to G, Fig. 17, and that it should be levelled before being used.

There is one form of fibre suspension, however, which is used by Sir Wm. Thomson in his "*marine galvanometer*," and which, although not employed in other instruments, has advantages that make it worthy of more general adoption in portable galvanometers. To a silk fibre stretched between a fixed support and one end of a spring, there is attached the magnetic needle and pointer, or other indicating arrangement, and when these are well balanced, the whole instrument may be tilted through

several degrees without any practical alteration of the deflection. (*See* § 53, page 103.)

28. Sine Law: Under what Conditions it is True.—When the controlling force acting on the needle of a galvanometer remains constant in magnitude and direction on the needle being deflected (a result that will always practically happen when the controlling force is produced by the attraction of a *distant* magnet), there is a very simple plan, suggested to the author by Prof. Carey Foster, for calibrating the galvanometer relatively by employing what is known as the "*sine principle*," in a particular way, and which does not require the use of any other galvanometer at all. We have already seen under what conditions a force acting on a body is proportional to the tangent of the angle through which the body is deflected, and in a similar way the apparatus shown in Fig. 18 will enable us to decide under what circumstances a force acting on a body is directly proportional to the "*sine*" of the angle of deflection. N O is a piece of wood, in this case not necessarily short, turning on a pivot at O, and having suspended from its lower end a weight W, which produces a force constant both in magnitude and direction. The same end of the piece of wood N O is also acted upon by a force produced by a cord carrying the

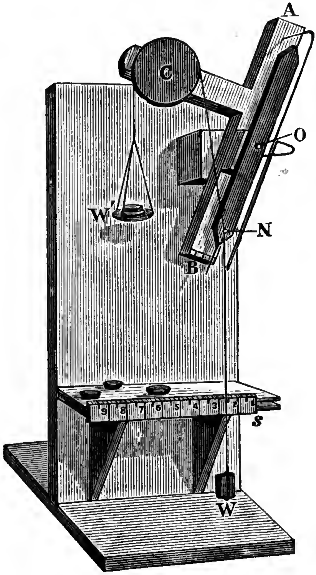

Fig. 18.

scale-pan in which is placed the weight w′, the magnitude of which can be varied. Now experiment shows that, if different weights be successively put into the scale-pan, and if in each case the framework A B carrying the pulley C be turned about the centre O, so that the piece of wood N O always occupies the same position relatively to A B, the weights are proportional to the horizontal distance (S *s*, Fig. 19), measured along the scale between the point where the cord carrying W cuts now, and where it cut it when W was nought. But S *s*, or P N, which is equal to it, divided by N O, the half-length of the deflected lever, is equal to the sine of the angle P O N, through which N O has been deflected. It is also obvious that turning A B, so that it always takes up the same position relatively to N O, is only a means of causing the angle between the cord carrying W′ and N O to be constant, in order that the only change in the force exerted by the string carrying W′ may be that caused by the change of weight, not by any change in the direction of the pull. From this we conclude that in order that a force acting on a body turning on an axis may be directly proportional to the sine of the angle through which the body is deflected :

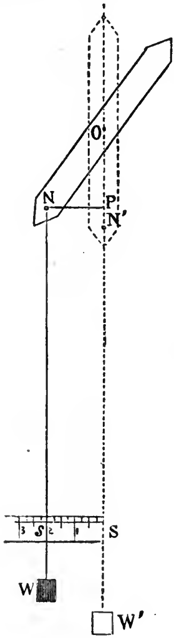

Fig 19.

1. *The controlling force must be constant in magnitude and direction.*

2. *The deflecting force, although variable in its direction in space, must be fixed in direction relatively to the deflected body.*

29. Preceding Conditions are Fulfilled in the Sine Galvanometer.—In any galvanometer in which the con-

trolling force is produced by the earth's magnetism, or by any *distant* fixed magnet, this force will be constant in magnitude and direction, and independent of the needle changing its position; also the deflecting force produced by the current passing round the bobbin, can be made to have an invariable direction relatively to the needle, if the bobbin, or the framework of the instrument to which the bobbin is attached, be turned round after the deflected needle; for it will be found that, although on turning the bobbin the needle turns away from the bobbin, it does not turn as fast as the bobbin. Under these circumstances, the sine of the angle through which the needle has been deflected from the position of rest which it had when no current was passing through the bobbin, will be directly proportional to the current strength. Now, if the coil be placed so as to have a fixed position relatively to the needle, both when no current passes through the coil and when a given current passes through the coil, then the angle through which the coil has to be turned from the first position to the second, is the same as the angle through which the needle has been deflected; and hence, in the so-called sine galvanometers, there is, in addition to the scale moving with the bobbin, an independent *fixed* scale, to show through what angle the coil has been turned. This, however, is not absolutely necessary, since, if, after the coil has been turned until it has the fixed position relatively to the needle, the current be interrupted, without the position of the instrument being disturbed, then the needle will swing back, and, after a few oscillations, will take up its original undeflected position, the angle between which and its deflected position will be the angle of which the sine has to be taken.

As a current passing through a coil has usually the greatest effect on a magnetic needle suspended inside it when the axis of the needle is perpendicular to the axis of the coil, this is the fixed position of the coil relatively to the needle usually adopted, and the one in which the pointer stands at 0° on the movable scale. But this particular

position is not at all necessary for the fulfilment of the sine law, and therefore special precautions need not be adopted, as in the case of the tangent galvanometer (*see ante*, page 45), to insure the axes of the needle and of the coil being at right angles when the pointer stands at zero on the scale.

Any galvanometer which is controlled by a distant magnet, and which can be turned round a point that is approximately the centre of the needle, can be used as a sine galvanometer, and, therefore, can be calibrated by the employment of the sine principle. All that is necessary to be done to make a measurement is as follows:—Place the instrument so that the pointer points to some fixed mark on the scale; 0° is a convenient mark, but not a necessary one; then send any convenient current through the galvanometer, obtaining a deflection of, say, $d_1°$. Turn the instrument until the pointer again points to the fixed mark on the scale. Stop the current, and observe through what angle $D_1°$ the needle comes back. $D_1°$ will, of course, be larger than $d_1°$. Now turn the instrument round, so that the pointer points to its original mark on the scale, 0° for example, and repeat with a second current, obtaining in the same way deflections $d_2°$, $D_2°$. Then the currents producing the deflections $d_1°$ and $d_2°$ respectively with the galvanometer, are proportional to the sines of $D_1°$ and $D_2°$.

30. Calibrating a Galvanometer by the Sine Method.—Fig. 20 shows an apparatus arranged for calibrating the galvanometer in this way. Three little blocks of wood, two only of which, *c c*, can be seen in the figure, are temporarily fixed so as to allow the galvanometer to be turned round without shifting its position, a precaution of practically no consequence if the controlling force be due to the earth's magnetism alone, but desirable if the whole or part of the controlling force is produced by a not very distant magnet. Of course the magnet must be so far away that neither the magnitude nor direction of its attraction on the suspended needle is altered by the

turning of the needle; but this need not be very far, unless the needle employed is long. V is a vessel containing two zinc plates for adjusting the strength of the current in the manner described in a previous experiment. W is one of the wires leading to the current generator, and T is the terminal to which the other is attached.

To calibrate a galvanometer by the employment of the sine principle, requires the current in each case to remain constant long enough for the instrument to be

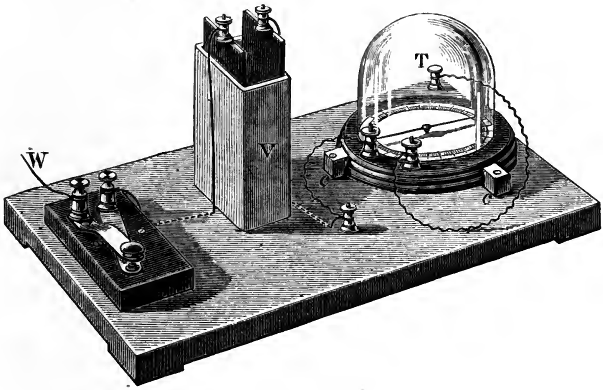

Fig. 20.

turned round after the needle, until the two are in a fixed position relatively to one another. But when once the calibration curve has been drawn, a galvanometer so calibrated can, of course, be used to measure currents as transient as a galvanometer calibrated in any other way.

31. Calibration by the Sine Method of the Higher Parts of the Scale.—If the first deflection is more than about 45° it is found impossible to use the sine principle in the ordinary way, because, on attempting to turn the coil after the deflected needle, so as to bring the fixed mark on the scale under the pointer, the needle moves so far round in advance of the coil that at last the

attraction of the earth or other controlling magnet begins to assist the current instead of opposing it. The equilibrium then becomes unstable, and the needle swings right round. The calibration of the higher parts of the scale, however, may be effected by the sine method, by using currents which produce a first deflection of less than 45°, in the following way :—Select some other starting-point, say 40° on the scale, for the zero, that is, let the galvanometer be turned, so that the pointer points to + 40°, when no current is flowing; now send a current through the galvanometer, deflecting the pointer to, say, + 60° (Fig. 21). Next, turn the galvanometer round until the 0° division, or whatever fixed mark was previously used in §§ 29 and 30, comes under the pointer. Lastly, stop the current and let the pointer now take up a position—30° say; then, when the galvanometer is placed in the ordinary position, so that the pointer points, say, to 0°, when no current is passing, the current that will deflect the pointer to 60° will be

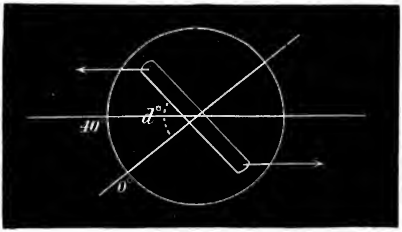

Fig. 21.

$$\frac{\sin. 30° \times \sin. 60°}{\sin. (60°—40°)},$$

or, generally, the current that will deflect the pointer to any angle $d°$ will be

$$\frac{\sin. \mathrm{D}° \times \sin. d°}{\sin. (d°—40°)},$$

where D° is the angle through which the pointer comes back on stopping the current.

After experiments have been made in the way described in §§ 29 and 30, and a curve drawn with the values of $d°$ as abscissæ, and of D° as ordinates for values of $d°$ up to about 45°, experiments may be made in the way just described, and the curve extended by using for the ordinates the values of

$$\frac{\sin. \mathrm{D}° \times \sin. d°}{\sin. (d°—40°)}.$$

The reasoning of this extended method of calibration is as follows :—From Fig. 19 we see that when a needle is controlled by

a uniform magnetic field, the moment of the controlling force* is proportional to P N, that is, to the sine of the angle through which the needle is deflected. If, then, a galvanometer is so placed that the pointer points to 0° when no current is passing, it follows that, in order that a current shall produce a deflection of $d°$, it must produce a force whose moment is proportional to sin. $d°$. When, however, the instrument is turned, as shown in Fig. 21, the current which is deflecting the needle to $d°$ produces a force whose moment is proportional to sin. $(d°—40°)$. Now, what is the relative strength of this current measured by the method described in §§ 29 and 30? It is proportional to the sin. D°. Hence, a current proportional to sin. D° deflects the needle to $d°$ when the controlling force has a moment proportional to sin. $(d°—40°)$. Consequently, a current proportional to

$$\frac{\sin.\ \mathrm{D}° \times \sin.\ d°}{\sin.\ (d°—40°)}$$

will deflect the pointer to $d°$ when the controlling force has a moment proportional to sin. $d°$, that is, when the pointer points to 0° when no current is passing.

32. Calibration by the Sine Method with a Constant Current.—The following, due to Mr. Mather, is perhaps the neatest of the methods of calibrating a galvanometer on the sine principle, since, by means of it, the calibration can be effected throughout the whole range of the scale, and no other apparatus than the galvanometer to be calibrated, and a current generator, such as a "*Daniell's cell*," which will give fairly constant currents, is required. Send a current through the galvanometer, such as will produce a deflection of about 30° when the galvanometer is so placed that the pointer points to 0° when no current is passing. Next, without varying the current, turn the galvanometer until the pointer points to about 35°. Stop the current and observe the position taken up by the pointer when it comes to rest. Turn the galvanometer round farther and farther, and repeat, observing in each case the position of the pointer when the current is flowing, and the position the pointer takes up when the current has been broken. Also make a series of observations with the galvanometer placed in such positions that the first deflection is less than 30°. In some one position of the galvanometer let $d°$ be the angular deflection from 0° when the current is flowing, and $z°$ when the current has been interrupted; then it follows, from what was stated in § 31, that this current, which we may call our unit current, passing round the galvano-

* The "*moment of a force about a point*" is the product of the magnitude of the force into the length of the perpendicular let fall from the point on the line representing the direction of the force.

meter coils, is able to produce a deflecting force whose moment is proportional to sin. $(d^\circ - z^\circ)$ when the needle is deflected to d°. Hence it follows that the current which would be necessary to produce a force whose moment should be proportional to sin. d° for the same position of the needle must be $\frac{\sin. d.^\circ}{\sin. (d^\circ - z^\circ)}$ times our unit current, that is, must be proportional to

$$\frac{\sin. d^\circ}{\sin. (d^\circ - z^\circ)},$$

but such a current would deflect the pointer to d° when the galvanometer was so placed that the pointer pointed to 0° for no current passing. Hence, to obtain the calibration curve, we have simply to plot values of d° for the abscissæ, and the corresponding values of

$$\frac{\sin. d^\circ}{\sin. (d^\circ - z^\circ)}$$

for the ordinates.

33. Method of Making a Sine Scale.—Instead of finding in a table of sines the sines of the various angles through

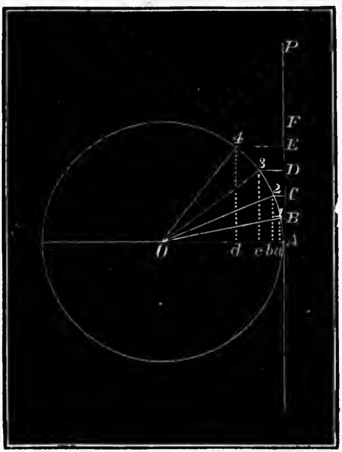

Fig. 22.

which the needle swings back, we may construct a sine scale in the following way :—On A P, Fig. 22, any tangent of the circle

on which the scale is to be made, mark off equal parts A B, B C, C D, &c. From B, C, D, &c., draw perpendiculars to A P, B 1, C 2, D 3, &c., meeting the circle in 1, 2, 3, &c.

Then the sines of the angles A O 1, A O 2, A O 3, &c., are proportional to the numbers 1, 2, 3, &c. For drop perpendiculars 1 *a*, 2 *b*, 3 *c*, &c., on O A:

$$\text{Then sin. } \mathrm{AO1} = \frac{1\,a}{\mathrm{O\,1}}$$

$$= \frac{\mathrm{B\,A}}{\mathrm{O\,A}}$$

since B A equals 1*a*, and O A equals O 1.

$$\text{Similarly sin. } \mathrm{AO2} = \frac{\mathrm{C\,A}}{\mathrm{O\,A}}$$

and so on.

Therefore, the sines of the angles are proportional to A B, A C, A D, &c.

Therefore, they are proportional to the numbers, 1, 2, 3, &c.

If we wish to divide the whole quadrant into an *exact* number of subdivisions in this way, we must commence by marking off on the tangent A P a length A F, equal to the radius of the circle, and then subdivide A F into any desired number of *equal* parts instead of taking A B, B C, &c., any equal lengths.

If, when using this scale, it be found on sending two currents through the galvanometer that the needle deflects through the angles A O 2, A O 3 respectively, the mistake must not be made of considering that the currents are in the proportion of two to three, for this will only be the case when A O 2, A O 3 are the angles through which the needle swings back after the galvanometer has been turned in each case.

34. Portable Galvanometer with Approximately Invariable Absolute Calibration.—A type of portable galvanometer (Fig. 23), to which was attached a very powerful "*permanent magnet,*" having its needle made of a number of small pieces of soft iron, was made and calibrated absolutely by M. Deprez, in 1880. The scale was divided simply into degrees, and a table of numbers giving the value in amperes of the various deflections was attached to the instrument. This instrument rendered considerable service in the early days of commercial electric lighting, but there were two disadvantages in connection with its use: first, as the scale was divided simply into degrees, the deflection with-

out the use of the table of values gave no indication of the strength of the current measured; and, secondly, it was necessary to refer to this table twice over when

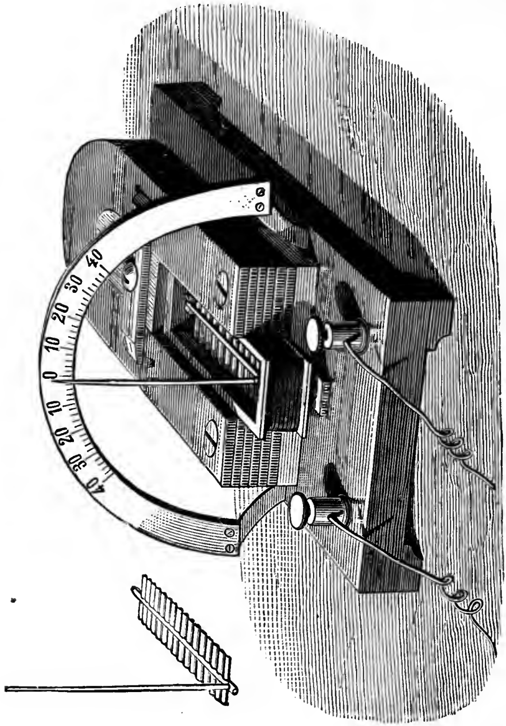

Fig. 23

measuring two different currents, as the deflection was not directly proportional to the current. The current, in fact, increased more rapidly than the angular deflection, a result which is generally found to occur in ordinary galvanometers, and which arises from the deflection

of the needle causing it to move into a position in which the current passing round the coil acts with less force on the needle than when it is in the zero position or parallel to the plane of the coil.

35. Construction of Galvanometers in which the Angular Deflection is Proportional to the Current. —We have already seen (page 43) that the current is proportional to the tangent of the deflection of the galvanometer needle, when neither the magnitude nor direction of the controlling force is altered as the needle moves into a new position on being deflected, and when, in addition, the direction of the controlling force is at right angles to the direction of the force with which the current passing round the coil acts on the needle.

In order, therefore, that the angular deflection may be directly proportional to the current, we must either cause the needle on being deflected to move into a position in which the current passing round the coil acts more powerfully on it, or into a position in which the controlling force becomes weaker; or we may arrange that both these results may be produced.

The first condition may be obtained in a rough way by employing the very defect of construction previously referred to in the adjustment of the tangent galvanometer, and which made the deflection on one side of the zero larger than that produced by the same current on the other—viz., not putting the coil so that its plane was parallel to the suspended magnet when no current was passing through the coil. The needle, when deflected to that side on which the greater deflection is obtained, will, instead of moving from a stronger to a weaker part of the magnetic field produced by the current, move at first into a stronger part, and then afterwards into a slightly weaker part. The effect of this arrangement is to make the proportional law connecting current and deflection true for a much larger deflection from the undeflected position of the needle than if we commenced with the needle parallel to the plane of the coil for no current.

But this arrangement has the disadvantage that it can only be used for currents deflecting the needle to one side of the scale, for, if the current be flowing in the opposite direction, the defect of want of proportionality between current strength and deflection will be increased.

This plan, by means of which the proportionality on one side of the scale is sacrificed to increase that on the other, has been employed by the author, and later on by

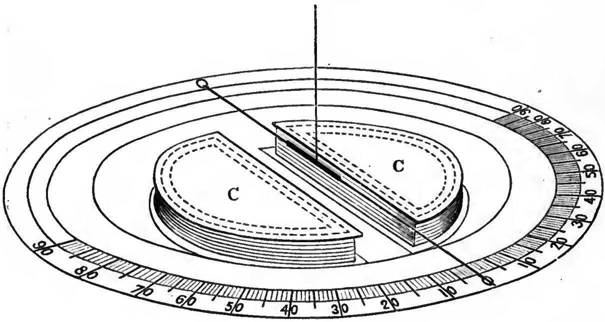

Fig. 24.

MM. Carpentier and Deprez, for making proportional galvanometers.

If the "*controlling field*" be a uniform field, such as is produced by the earth's magnetism, that is, if the controlling force acting on the pole of a given magnet is the same both in magnitude and direction at all points, then the arrangement shown in Fig. 24, and which has been worked out by Messrs. Walmsley and Mather, two of the assistants at the Finsbury Technical College, may be employed. The instrument consists of two coils shaped as shown, and the special device consists in fixing them so that they are separated by a distance a little less than the length of the needle. The instrument is placed so that when no current is passing through the coils the

needle hangs symmetrically between them, and it is found that direct proportionality of current and deflection up to 45° to 50° is obtained from the fact that, with the arrangement indicated, the needle, on being deflected, moves into a position in which the current acts more powerfully on it, or shortly into a more powerful part of the "*deflecting field.*" Galvanometers of this type are shown in use in Figs. 15 and 20.

36. Shielding Galvanometers from Extraneous Magnetic Disturbance.—If, however, the instrument is to be portable, and if it be desired that the deflections of the needle should be unaffected by the moving about of neighbouring magnets or pieces of iron, the galvanometer must be "*shielded*," and this, as stated in § 25, can be done by attaching a powerful permanent magnet to the instrument, the action of which on the suspended magnet is far stronger than that likely to be caused by any other neighbouring magnet. When using such a permanent magnet, there are two well-defined ways employed by the author for obtaining direct proportionality. The first consists in winding the insulated wire on the two halves of a brass bobbin A (Fig. 25), separated by a brass tube T, in which the pivoted soft iron needle carrying the pointer moves, and attaching soft iron pole-pieces P P, hollowed out as shown in the figure, to the permanent magnet M M. The wire is wound on the bobbin (which in the figure is shown unwound), much as cotton is wound on a reel; none is wound on the tube T, and the coils on the two halves of A are electrically connected with a wire passing by the side of T; into the ends of the brass bobbin, soft iron cores F F are screwed, the outer ends of which are seen in the figure. The other ends of these soft iron cores project a considerable distance into the brass tube, and the result is that on the needle being deflected from the position it occupies when no current is passing round the coils, and which is along a diameter of the tube T at right angles to the axis of A A, its ends come nearer the noses of these soft iron cores

inside the bobbin A A. Hence the deflecting force grows much stronger as the soft iron needle is deflected. The alteration in the strength of the controlling force depends on the exact curvature given to the ends of the soft iron pole-pieces P P, which embrace the brass tube T.

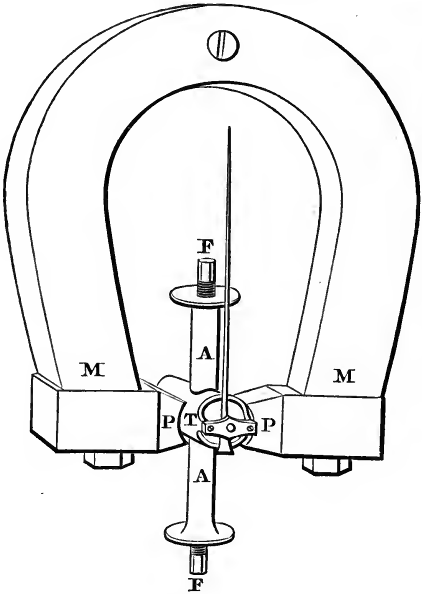

Fig 25.

If the curvature of the pole-pieces is much greater than that of the tube T, and the pointed edges of the pole-pieces be pressed against the tube so as to approach the needle as nearly as possible, then the action of the *controlling field* will somewhat increase in strength when the needle is deflected, since the ends of the needle will come nearer the iron of the pole-pieces when the needle

is deflected; whereas, if the curvature of the ends of the pole-pieces be much less than that of the tube—if, in fact, the ends of the pole-pieces be nearly flat—then the action of the controlling field will become weaker as the needle is deflected.

When no soft iron cores F F are employed, the "*straight line*" or "*proportional*" law can be produced by taking advantage of the fact that the deflecting field increases in strength as the needle is deflected, in consequence of its poles entering more into the coils wound on the two halves of the bobbin A A. In that case the ends of the pole-pieces P P should only be very slightly curved. For the purpose, however, of making the final adjustment for sensibility, to be described a little farther on, the use of the soft iron cores F F screwed, more or less, into the ends of the bobbin is found to be very convenient, and, as already explained, their presence leads to the deflecting force much increasing in strength as the needle is deflected. The result of this is that the correction is too great, that is to say, instead of the angular deflection increasing less rapidly than the current, which is the ordinary result obtained with galvanometers, the deflection would increase much more rapidly than the current, giving a flat instead of a steep calibration curve. To avoid this over-correction the curvature of the pole-pieces must be considerable.

The final result then obtained is as follows:—If the cores F F are too far in, the calibration curve is flat, that is, the angular deflection increases more rapidly than the current; if too far out, the calibration curve is steep, or the angular deflection increases less rapidly than the current; but between these two limits there are several positions of the cores giving nearly perfect proportionality between deflection and current. Within these limits the cores may be adjusted, and the sensibility of the instrument altered. If they be screwed out, it will require a larger current to produce the same deflection; while, on the other hand, if they be screwed in, the opposite effect

will be produced. Hence, within these limits, any deflection may be made to correspond permanently with any current.

37. Direct-Reading Galvanometers.—Hence, by the

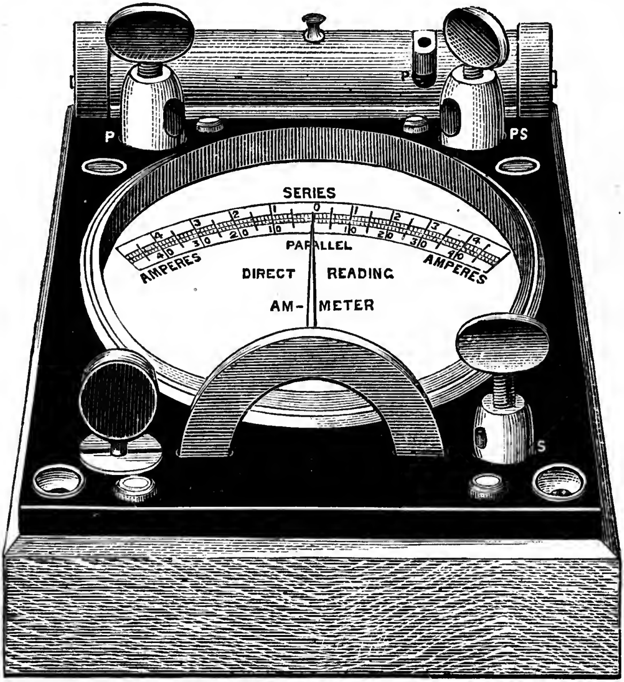

Fig. 26.

employment of these cores, we can not only construct an instrument in which the deflection shall be directly proportional to the current, but we can use a dial graduated in amperes instead of in degrees, and so obtain a "*direct-reading galvanometer*" as shown in

Fig. 26.* For, although it would be very difficult to fill the bobbin with a particular gauge of wire, so that with a particular controlling magnet a given number of amperes shall produce exactly a particular deflection, it

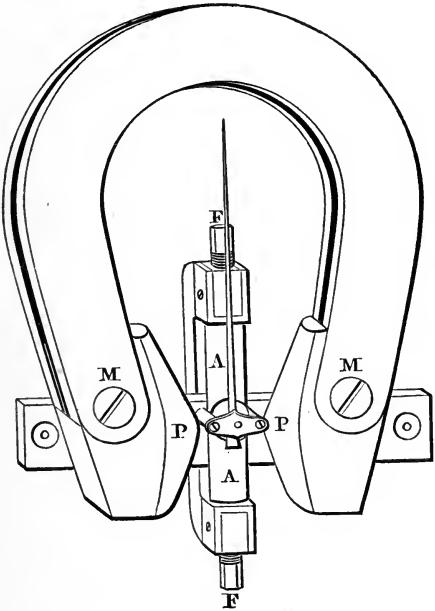

Fig. 27.

is easy by trial to approximate to this, and then finally adjust the instrument by screwing the soft iron cores a little in or out until any particular deflection on the dial is produced by exactly the number of amperes of currents marked opposite that deflection on the dial.

* *See* § 221, page 431, for further details regarding the double scale and commutator P shown in Fig. 26.

And should the permanent magnet lose its strength from time to time, when the instrument will of course become more sensitive, we can, by screwing out the cores, re-adjust it so that it will still continue to be a correct *direct-reading galvanometer*.

Another plan, and probably a still better one for obtaining all the above results, is to make the opening in the bobbin A through which the pivoted needle is inserted, in the construction of the instrument, much smaller, as shown in Fig. 27, so that the wire can be coiled almost continuously from one end of the bobbin to the other without the gap in the bobbin necessitated by the tube T (Fig. 25). With the arrangement of Fig. 27 the deflective force is but slightly increased as the needle is deflected; hence, to obtain the proportional line the controlling force must be made to diminish as the needle turns, which result can be obtained by curving the ends of the soft iron pole-pieces in the way shown in P P (Fig. 27) that is, by making them convex instead of concave to the coil, as was done with the previous arrangement.

38. Advantages of the Previous Types of Galvanometers.—All these instruments have the advantages that their indications are "*shielded*," that is, are not seriously affected by the presence of neighbouring magnets or pieces of iron; secondly, if the needle is well balanced the instrument can be used in any position without any error being introduced in the readings; and, thirdly, as the needle is very light (or, more strictly, has only a very small "*moment of inertia*")* and as it is moving in a very powerful magnetic field, the oscillations of the needle are very quick, and die out very rapidly, so that if the current that is being measured has a sudden change in its strength, the needle moves sharply from one point of the scale to another point, where it stops dead in

* The *moment of inertia* of a body about any axis is found by imagining the body divided up into a large number of very small parts, and taking the sum of the products of the mass of each part into the square of its distance from the axis.

response to the change in the current strength, or the instrument is "*dead-beat*"; whereas, if a needle of large moment of inertia were employed moving in a weak magnetic field, then on any change taking place in the current strength the needle would simply begin to oscillate over the scale, and many changes might take place in the current strength, the current even remaining constant at each of its various values for a very decided time before the needle would come to rest and allow any measurements to be taken. The advantage of employing a *dead-beat* instrument is very marked when the current produced by a dynamo worked by a gas-engine has to be measured. If the instrument is *dead-beat* every change in the current produced by the slight change of speed of the gas-engine at each explosion of the gas is accurately recorded; indeed, the slight change of speed that occurs each time the joint in the driving belt, if it be a "*lap-joint*,"* and not a "*butt-joint*," passes over the driving pulley, is observed; whereas, if the instrument be not *dead-beat* these fluctuations in the current merely cause the needle to keep up a constant vibration over the scale, and so prevent any accurate readings being taken.

Other forms of current galvanometers are given farther on (page 377), and another method of *shielding* by putting the galvanometer in an iron box, with very thick sides, is considered in § 53, page 103, and by giving the needle a motion of *translation* in § 202, page 390.

39. Ammeter.—Such a *dead-beat direct-reading* galvanometer is frequently called an "*ammeter*," hence the name on the dial of Fig. 26, and we may temporarily regard such an *ammeter* as our commercial instrument for measuring current strengths in amperes.

Other, and more modern, types of ammeters are described farther on (page 382), where also are stated the advantages and disadvantages of some of the most important kinds.

* A lap-joint is made by putting one end of the leather belt *over* the other, and lacing, or riveting, them together; while in a butt-joint the ends are simply brought together, but not put one over the other.

CHAPTER III.

DIFFERENCE OF POTENTIALS, ELECTRIC QUANTITY, DENSITY, AND THEIR MEASUREMENT.

40. Difference of Potentials.—When a current of electricity is flowing through a wire it has the same strength at all cross-sections of the wire; if, for example, the wire be cut anywhere, and a galvanometer be put in the circuit, the galvanometer will always show the same deflection while the same current is flowing. In the same way in the case of a water-pipe, the quantity of

water passing every cross-section of the pipe per second is exactly the same as soon as the flow of water becomes a "*steady*" one. Just at the commencement, when, for example, some water has entered at one end of the pipe, and none has flowed out at the other—when the pipe is filling, in fact—the flow at different cross-sections may be different; so also, in many cases, just at the moment after completing an electric circuit, the current will differ at different cross-sections. But as soon as the flow in each case becomes a steady one this difference disappears, and the strength of the water current, that is, the number of gallons of water passing per minute (not, of course, the velocity of the particles of water) is the same at all parts of the pipe, even if the pipe be broad at some points and narrow at others, so also the strength of the electric current flowing through a single circuit is "*uniform*"* at all parts of the circuit, independently of the thickness of the conductor and of the material of which it is made.

But, although the stream of water is the same at all parts of the pipe, the pressure per square inch of the water is by no means the same, even if the pipe be quite horizontal and of uniform gauge. This pressure per square inch of the water on the pipe, which is the same as the pressure per square inch of one portion of the water on another portion at the same part of the pipe, becomes less and less as we proceed in the direction of the flow, along a horizontal pipe of uniform sectional area. It is, in fact, this difference of pressure, or "*loss of head,*" as it is sometimes called, that causes the flow to take place against the friction of the pipe, the difference of pressure at any two points in the case of a steady flow through a horizontal pipe of uniform sectional area being equal to

* *Uniform* refers to space, *constant* to time. The height of the houses in a street is generally not uniform, but it is constant as long as there is no change made in the height of the houses. If water be run out of a cistern the level at all parts of the surface of the water is uniform, but it is not constant, since it steadily falls as the water runs out.

the frictional resistance of that length of pipe for that particular flow.

Quite analogous with this there is, in the case of an electric current flowing through a conductor, a "*difference of potentials*" at two points in the conductor, and this *difference of potentials* is necessary to overcome the "*resistance*" of the conductor, or opposition that it offers to the passage of an electric current through it.

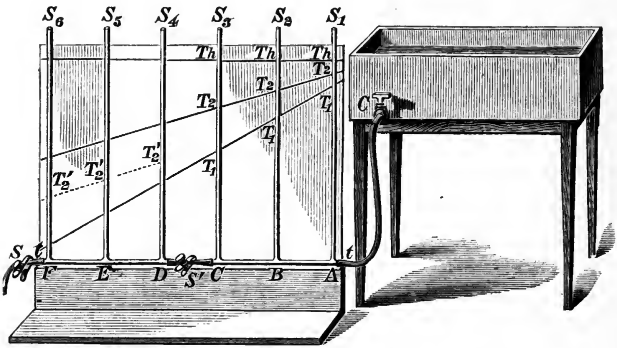

Fig. 28.

The pressure per square inch of the water at any point in a tube conveying a stream can be ascertained by attaching a vertical stand-pipe to the tube, and seeing to what height the water is forced up in this stand-pipe, and if at a number of points A, B, C, D, E, F (Fig. 28) in a glass tube *t t*, conveying a stream of water, a series of vertical glass stand-pipes S_1, S_2, &c., be fixed, the height to which the water is forced up in them will show the distribution of pressure along the pipe. If the tube *t t* be horizontal, straight, and of uniform cross-section, and if the flow of water be a *steady* one, then the tops of the water in the stand-pipes will be found to all lie in one straight line, from which it follows that the difference

of pressure between any two points is proportional to the distance between the points.

If the screw pinch-cock s be fully, or nearly fully, open, and that at s′ fully open, the stream of water through the tube *t t* will be rapid, and the tops of the columns of water in the stand-pipes will lie in a straight line such as $T_1 T_1 T_1$. If the cock s be screwed up a little so as to squeeze the bit of indiarubber tube (that at s′ still remaining fully open) the flow will be diminished, and the line joining the tops of the columns of water in the stand-pipes will make a less angle with the horizontal, or occupy a position $T_2 T_2 T_2$. As the cock s is screwed up more and more the line is tilted up more and more, until at last, when the cock is shut and the water turned off altogether, the line becomes horizontal, $T_h T_h T_h$, and is at the same level as the top of the water in the cistern. The inclination of this line to the horizontal, therefore, diminishes as the flow of water diminishes, and becomes nought when the flow ceases altogether.

So, in the same way, the "*electric potential*" at different points of a wire conveying a current can be measured statically by an apparatus that will be described farther on (§75, page 130), and if a number of measurements be made of the potential at different points of a circuit conveying a current, it will be found that the results are smaller and smaller as we proceed in one direction; and, farther, if the conductor be all of uniform gauge, and made of the same material, and the electric current be a *steady* one, it will be found that the difference of potential between any two points is proportional to the length of the conductor between these points.

This analogy between the distribution of water-pressure and of electric potential, is a very useful one for students in enabling them to grasp the idea of electric potential; but, like many other analogies, it must not be pressed too far; for example, a bend in a pipe, even with a steady flow of water, is found to cause a falling off in the water-pressure; whereas, a bend in a wire has no

effect on the electric potential if a steady current is flowing; or, again, if there be a sudden expansion or contraction in a pipe, there is a sudden alteration of the water-pressure, which has no analogy in any sudden alteration of the electric potential at a point in a circuit where the sectional area of the conductor changes abruptly.

In fact, the flow of water or of gas in a pipe can be diminished to any extent by a contraction of one point *only*, which may be practically effected by partially closing a tap. For example, if the screw pinch-cock s′ be partially closed, a great resistance to the flow of the water will be introduced at this point, shown by the fact that the line joining the tops of the columns of water in the stand-pipes now breaks up into two portions $T_2\,T_2\,T_2$ and $T_2'\,T_2'\,T_2'$, parallel to one another, but the one much below the other; whereas, if an electric circuit consist of many yards of wire, no appreciable alteration of the current will be produced by making only half an inch of the wire have, say, one-tenth of its previous sectional area. If, however, the current be so strong as to fuse the wire, then the current will become nought, just as the stream of water or gas becomes nought on the tap being entirely closed, and the analogy of fluid and electric flow will again hold.

41. Potential of the Earth Arbitrarily taken as Nought.—Unfortunately the statical measurement of electric potential is not nearly as simple as the statical measurement of fluid pressure, in consequence of the forces produced by the mutual attractions of any two ordinary bodies charged with electricity being very small. Potential has also to be measured relatively, in the way that temperature is usually measured, and not from a zero, or starting-point, as can be employed in the measurement of length or weight. The same length may be called one yard, or three feet, or thirty-six inches, or 91·44 centimetres, but a length that is nought on any one of these systems of measurement is nought on them all;

whereas, not only is the temperature which is called 15° on the Centigrade scale called 59° on the Fahrenheit, but the temperature that is called 0° on the former is called 32° on the latter. In the measurement of temperature, then, we take the temperature of some definite body and call it 0°, and we do not imply by doing so that no lower temperature can be obtained; so, in the measurement of potential we take the potential of a certain body and call that potential nought—the electric potential that is arbitrarily taken as nought being that of the earth.

In thus taking the potential of the earth as the *potential level* to measure from, no assumption is made as to the earth having no charge of electricity on it; indeed, so far from that, experiment shows that the earth produces exactly the same electrical effects as it would if it were "*negatively*" or "*resinously*" electrified: that is, electrified in the same way as is a piece of ebonite after being rubbed with a piece of dry flannel, and oppositely electrified to a piece of dry smooth glass, which, after being rubbed with a piece of dry silk, is said to be "*positively*," or "*vitreously*," electrified.

Measuring potentials relatively to that of the earth is simply like measuring heights above the Trinity watermark, or measuring longitude east or west of Greenwich.

42. The Difference of Potentials between Two Conductors does not Measure the Difference in their Electric Charges.—The fact that two conductors differ in potential tells us nothing about the quantities of electricity in either of them, nor whether these quantities are positive or negative, nor even whether either of the bodies is charged with electricity at all (*see* 8, § 69, page 124). All that we can deduce from the fact that two conductors, made of the same material, differ in potential is that if they be joined by a wire there will be a flow of electricity, or a current from one to the other, until this difference of potential is destroyed; and we say that the one from which "*positive*" electricity flows has the "*higher potential*," or a "*positive potential*," relatively to the other. In the same way, by

knowing the fact that the pressure of the gas in two gas-holders is different, we have no information as to the quantities of gas in either of the vessels, but we merely are sure that, if the vessels be joined by a pipe, gas will flow from the vessel in which the pressure is greater into that in which it is less as long as any difference in pressure remains. So, in the same way, if two vessels standing on the table contain water, and if we merely know that the level of the water in one of them is higher than that in the other, we can tell nothing about the number of gallons of water in the two vessels; but what we do know is, that quite irrespectively of the size of the vessels, or of the quantity of water in them, if the two vessels be joined together by a pipe anywhere below the lower water-level, water will flow from that in which the level is higher into that in which it is lower until this difference of level is destroyed.

So, again, we can form no conception from the fact that one body is hotter than another as to the amount of heat either will give out in cooling down to the freezing temperature, or even which of the two will give off the greater amount of heat when so cooled; the existence of a difference of temperature between two bodies only justifies us in concluding that if the bodies be so placed that heat can pass from one to the other, heat will pass from the hotter to the colder as long as any difference of temperature exists.

Difference of potential in electricity is therefore analogous with difference of pressure in gases, with difference of level in liquids, and with difference of temperature in heat.

From what has been said, it follows that if two conductors of the same material be in electric connection with one another, and if no current be flowing from one to the other, the potential of the two bodies must be the same. *Hence the potential at all points of a conductor on which electricity is at rest must be uniform.*

43. Volt.—If two conductors, having different electric

potentials, be brought into the immediate neighbourhood of one another, what is called "*inductive action*" will take place between them: that is to say, the presence of each will disturb the distribution of electricity on the other, and there will be an attractive force tending to make the bodies approach one another. The magnitude of this force is connected in a perfectly definite way with the difference of potentials between the bodies, their sizes and shapes, and their positions relatively to one another, but this connection is in general a complicated one. If, however, the opposed surfaces of the two conductors be planes parallel to one another, this force will be

$$\frac{4 \cdot 508 \times 10^{-10} \times V^2}{d^2} \text{ grammes}$$

for each square centimetre of the opposed surfaces, where V is the potential difference in "*volts*" between the conductors, and d the perpendicular distance in centimetres between the surfaces.

If the force be measured in grains, the distance in inches, and the unit of attracted area be one square inch, then the force becomes

$$\frac{6 \cdot 955 \times 10^{-9} \times V^2}{d^2}.$$

In order that this formula may be rigorously true, it is necessary that the bit of the plane surface on which we are considering the attraction should be situated at a distance from the edge of the plane which is large in comparison with d.

The particular values of the constants employed in the last two expressions have not been selected arbitrarily. The selection of special units for the measurement of force, distance, area, and potential difference determines the values of the constants in each particular case, so that while the first set applies to grammes, centimetres, and volts, the second set applies to grains, inches, and

volts. For a certain set of units of force, distance, area, and potential difference (viz., dynes, centimetres, square centimetres, and absolute electrostatic units of potential difference), the constants become still simpler, and, indeed, the magnitude of the electrostatic unit of potential difference was selected so as to make the fundamental equations of attraction as simple as possible. This unit of potential

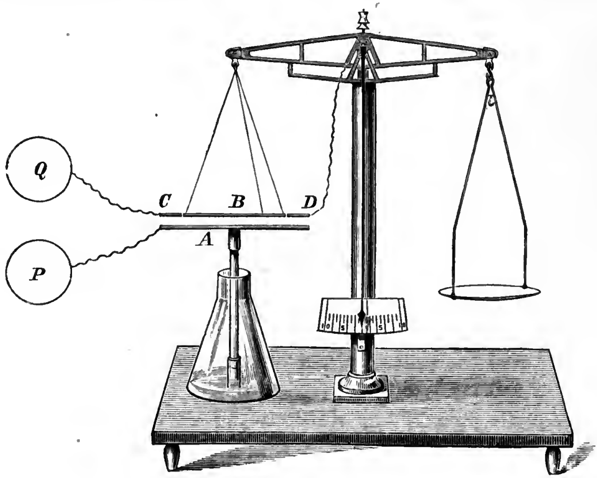

Fig. 29.

difference, however, is not used practically for several reasons, one of which is that it is much too large for such purposes; hence, the equations just given, and which are expressed in what are called engineer's units, contain what, at first sight, might appear to be arbitrary constants.

44. Measuring Potential Difference by Weighing.—We can, therefore, measure the potential difference between two conductors by weighing the attraction, and Fig. 29 shows a rough lecture model of a "*weight electro-*

meter" for effecting this result. A is a metallic plate insulated from the ground, but in electric connection with any conductor P, and therefore having the potential of P. B is a plate suspended by fine wires from one end of the beam of a balance which is well insulated from the ground, but in metallic connection with C and D, and with a body Q. B, C, and D have therefore the potential of Q. C D is in reality a square or circular plate, with a hole cut in it, which is nearly filled up by B, as seen in Fig. 30, the distance between the outer edge of B and the inner edge of C D being about three-quarters of a millimetre, or 0·03 of an inch. The use of the "*guard ring*," as it is called, C D, is to cause the law given above to be accurately true for all parts of B when the lower surface of B is in the same plane as the lower surface of C D (see the last paragraph but one, page 87); and the instrument is so adjusted that when the pointer points to nought on the scale, that is, when the balance indicates the equality of the weight in the right-hand scale-pan and the attraction of B, the lower surfaces of B and C D are in one plane.

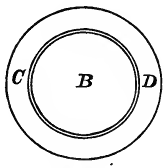

Fig. 30.

Such an apparatus can be used to measure a large difference of potential absolutely in volts, and we might define 2,000 volts as the difference of potential between A and B when, the distance between A and B being half a centimetre, and the area of B 100 square centimetres, the force acting on B was 0·72128 grammes. As will be seen, however, later on (§ 81, page 141), it is more convenient to define a volt in terms of the ampere (the standard of current) and the "*ohm*" (the standard of resistance).

Example 17.—If in the apparatus shown in Fig. 29 the suspended plate B were square, and its edge 1·4 centimetres long, and if the distance between it and the fixed plate A were 3 millimetres, what potential difference in volts must be maintained between A and B so that the attractive force may be 1 milligramme?

From what has preceded, we see that the attractive force on each square centimetre of the area of the suspended plate is

$$\frac{4 \cdot 508 \times 10^{-10} \mathrm{V}^2}{0 \cdot 3^2} \text{ grammes,}$$

therefore the force on the whole suspended plate is

$$1 \cdot 4^2 \times \frac{4 \cdot 508 \times 10^{-10} \mathrm{V}^2}{0 \cdot 3^2} \text{ grammes,}$$

and this, by the question, has to be equal to 0·001 grammes. Hence

$$\mathrm{V} = \frac{0 \cdot 3 \times 10^5}{1 \cdot 4} \sqrt{\frac{0 \cdot 001}{4 \cdot 508}}$$

Answer.—319·2 volts.

Example 18.—If the movable plate be circular, what must be its diameter so that when at a distance of 1 millimetre from the fixed plate a difference of potentials of 10 volts shall produce an attraction of $\frac{1}{100}$ gramme?

Let x be the diameter of the circular movable plate in centimetres, then its area equals

$$\frac{\pi x^2}{4}.$$

Hence, as the potential difference is 10 volts, the force is

$$\frac{\pi x^2}{4} \times \frac{4 \cdot 508 \times 10^{-10} \times 10^2}{0 \cdot 1^2} \text{ grammes,}$$

and this is to be equal to $\frac{1}{100}$; therefore

$$x = 2 \times 0 \cdot 1 \times 10^4 \sqrt{\frac{\cdot 01}{\pi \times 4 \cdot 508}}$$

Answer.—53·14 centimetres.

Example 19.—If the suspended plate be 3·5 square centimetres in area, what must be its distance from the

fixed plate so that 120 volts may produce an attraction of $\frac{1}{500}$ gramme?

If x be the distance,

$$3{\cdot}5 \times \frac{4{\cdot}508 \times 10^{-10} \times 120^2}{x^2} = \frac{1}{500},$$

$$\therefore \quad x = 120 \times 10^{-5} \sqrt{3{\cdot}5 \times 4{\cdot}508 \times 500}$$

Answer.—1·066 millimetre.

Example 20.—What force will be produced on a movable plate of 4·3 square centimetres 4 millimetres distant from the fixed plate, if the potential difference between them is 75 volts?

Answer—0·06816 milligrammes.

45. Increasing the Sensibility of the Weight Electrometer by using an Auxiliary High Potential.—It would be, however, quite impossible with such an apparatus to measure a potential difference of one or two volts, since unless the distance between the plates was very small—in which case want of perfect parallelism of the plates would introduce a serious error—the force of attraction even with a fairly large suspended plate would be extremely small. By employing the following device, however, the distance between the plates may be several millimetres, and the force of attraction some grains when a potential difference of one or two volts between the bodies P and Q (Fig. 29) has to be measured.

Let the fixed plate A be charged permanently to a very high and constant potential, V volts, by being connected with a body R which is at that potential, V being measured relatively to a metallic case (not shown in the figure) which encloses the apparatus. First let the suspended plate B and the guard ring C D be connected with one of the bodies P, having a potential v_1, in volts, relatively to the case of the apparatus, then if f_1 is the force in grains when the suspended plate of area a square inches is in the plane of the guard ring, and at a distance d inches from the fixed plate,

$$f_1 = 6{\cdot}955 \times 10^{-9} \times \frac{a\,(V - v_1)^2}{d^2}.$$

From this equation it will be seen that even if f_1 is larger than it was when P and Q were connected with A and B respectively, d may now be very much larger than the distances previously employed to separate the plates, since $V - v_1$ is very great compared with v_1.

Next connect Q with the suspended plate B and the guard ring C D, then if the potential of Q be v_2 volts relatively to the outside of the apparatus, and if f_2 be the attraction in grains for the same distance d between the plates,

$$f_2 = 6{\cdot}955 \times 10^{-9} \times \frac{a\,(V - v_2)^2}{d^2}.$$

Hence

$$V - v_2 - (V - v_1) \text{ or } v_1 - v_2 = \frac{d}{\sqrt{6{\cdot}955 \times 10^{-9} \times a}}\left(\sqrt{f_2} - \sqrt{f_1}\right).$$

If f_1 and f_2 be measured in grammes, d in centimetres, and a in square centimetres, then reasoning in the same way, we obtain

$$v_1 - v_2 = \frac{d}{\sqrt{4{\cdot}508 \times 10^{-10} \times a}}\left(\sqrt{f_2} - \sqrt{f_1}\right).$$

Of course $\sqrt{f_2} - \sqrt{f_1}$ will be no larger than would have been the square root of the force of attraction if P and Q had been respectively connected simply, one with the fixed plate A, and the other with the movable plate and guard ring, and if the high potential of R had not been used; but f_1 and f_2, the two forces, will be each large, and can be accurately measured, and what is especially important, d will be large, and the error arising from want of perfect parallelism of the plates entirely eliminated.

Another and simpler method of using the preceding apparatus consists in keeping the attractive force constant, and in varying, by means of a micrometer screw, the distance between the fixed and movable plates, so that this constant force (which must of course be known in grains or grammes) is exerted between the plates when the lower surface of the movable one is in the same plane as the lower surface of the guard ring. If then d_1 and d_2 be the distances in centimetres respectively when the same force f in grains is produced when B is connected respectively with P and Q, A being connected with R,

$$f = 6{\cdot}955 \times 10^{-9}\,\frac{a\,(V - v_1)^2}{d_1^2}.$$

$$f = 6{\cdot}955 \times 10^{-9}\,\frac{a\,(V - v_2)^2}{d_2^2}.$$

$$\therefore \quad v_1 - v_2 = (d_2 - d_1)\sqrt{\frac{f}{6{\cdot}955 \times 10^{-9} \times a}}.$$

If $v_1 - v_2$ is very small, so also will be $d_2 - d_1$, but d_1 and d_2 will

themselves be large, so that no error will be produced on account of want of perfect parallelism of the fixed and movable plates.

Two electrometers on this principle have been invented by Sir William Thomson; in the one, the "*absolute electrometer*," the force exerted on the movable plate B (Fig. 29) is known in grammes or grains, so that the potential difference is measured absolutely in volts; in the other, the "*portable electrometer*," the value of this force is not known, but it is always the same when the lower surface of the movable plate B is in the same plane as the lower surface of the guard ring C D. With this latter arrangement we cannot determine a potential difference $v_1 - v_2$ absolutely in volts, but we can use the instrument as a relative electrometer, and measure the ratio of v_1 to v_2 by taking a third or earth reading, obtained by reducing the potential of A B to nought by connecting it to the metallic case of the instrument: then if d_3 is the distance in inches between the fixed and movable plates,

$$f = 6{\cdot}955 \times 10^{-9} \frac{a\,(V - 0)^2}{{d_3}^2} \text{ grains.}$$

Combining this with the two other equations for f, we have

$$v_1 - 0 = (d_1 - d_3) \sqrt{\frac{f}{6{\cdot}955 \times 10^{-9} \times a}}.$$

$$v_2 - 0 = (d_2 - d_3) \sqrt{\frac{f}{6{\cdot}955 \times 10^{-9} \times a}}.$$

$$\therefore \qquad \frac{v_1}{v_2} = \frac{d_1 - d_3}{d_2 - d_3}.$$

With Sir William Thomson's absolute and portable electrometers, a potential difference of one volt can just be measured.

A far more sensitive *relative* electrometer, but one which is not at all portable, as hitherto constructed, is Sir William Thomson's "*quadrant electrometer*," which owes its great sensibility to the fact that, unlike the last two instruments, the sensibility of the quadrant electrometer is increased by increasing the potential of the auxiliary electrified body. The quadrant electrometer in its most perfect form is too complicated an instrument to be employed by a beginner, but a description of the details of the construction of a simplified type is given in § 75, page 130.

46. Rough Electrometer.—A "*gold-leaf electroscope*" is a rough electrometer or potential difference measurer. This instrument, as generally made, has a variety of defects, which will be referred to later on, but a form devised by the author, and in which these defects are eliminated, is shown in Fig. 31. It consists of a glass shade G G resting on a wooden base, and covered inside with strips of tin-foil T so as to leave only sufficient of the glass bare to enable the gold-leaves to be visible. These strips of tin-foil are bent round the bottom of the glass shade, and connected electrically with a brass ring round the bottom of the outside of the shade. To this ring three horizontal brass legs are attached for fixing the shade to the base, and in one of them is a binding-screw S for holding any wire which we wish to electrically connect with the tin-foil coating. Inside the shade G G, a thin rod of flint-glass *g g*, shaped as shown, is cemented into two holes in the base, and at the centre of this rod is cemented a little metallic tube *t t*, carrying a thick wire W W, and the gold-leaves L. This wire W W passes through the top of the instrument *without touching* it, and may carry at its top a little knob or a little binding-screw. V is a vessel containing pumice-stone soaked in strong sulphuric acid, which has the effect of keeping the

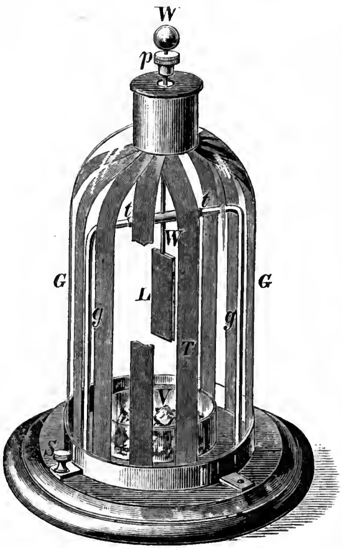

Fig. 31.

interior, and consequently the glass rod *g g*, quite dry. When the instrument is not in use, the little ebonite stopper *p*, sliding a little stiffly on the wire, is pushed down, and so closes the hole in the top of the instrument.

47. Action of a Gold-leaf Electroscope.—It has been stated (§ 43, page 87) that when two conductors in the immediate neighbourhood of one another are at different potentials they tend to approach one another with a force which depends solely on the potential difference, and on the shape and relative position of the conductors. Hence it follows that when the gold-leaves and the tin-foil coating of the electroscope are at different potentials, there will be for each potential difference a certain definite force pulling each gold-leaf towards the tin-foil coating on its own side. This causes the gold-leaves to diverge, and consequently to be slightly raised until the forces due to their weight exactly balance the forces of attraction between them and the tin-foil coating.

For a given gold-leaf electroscope, then, *the divergence of the gold-leaves depends simply on the potential difference between the gold-leaves* L, *and the tin-foil coating* T; and the value of any particular divergence of the leaves, noted on a fixed graduated scale attached to the electroscope, but not shown in the figure, can be ascertained in volts for any particular electroscope by comparison with a weight electrometer previously described, or it can be calibrated by the method described in § 191, page 354.

Experiment shows that a well-made electroscope, with the leaves made of thin pure gold—not "Dutch gold," which is often employed for this purpose—will show a perceptible divergence for a potential difference of about 100 volts.

If W W be connected with the screw S by means of a piece of wire, no difference of potentials can be set up between the gold-leaves and the outside, hence no divergence of the gold-leaves can be produced even by putting the electroscope on an insulating stand, and charging it

so that sparks can be drawn from any part of the electroscope on the finger being approached.

If the wire W W be connected with any body A, and the binding-screw S with any body B, then the divergence of the gold-leaves serves to show the potential difference between A and B in accordance with the absolute calibration curve of the particular instrument. If then B be a gas- or water-pipe in connection with the earth, the potential of the tin-foil coating will be nought, and the divergence of the gold-leaves will measure simply the potential of A.

In a moist country like England the divergence of the gold-leaves will approximately measure the potential of W W, or of any conductor electrically connected with W W, relatively to the earth without connecting S with the earth by means of a wire, since the film of moisture which condenses on the dusty wooden base makes a more or less good electric connection between S and the ground, so that, unless special precautions be taken to insulate the wooden base from the ground, the tin-foil coating may be regarded as being approximately at the potential of the earth.

48. Objections to the Ordinary Methods of Constructing Gold-leaf Electroscopes.—In the gold-leaf electroscopes commonly met with in shops, the rod W W, carrying the gold-leaves L, is supported from the top of the instrument, as if the sliding-plug *p* (Fig. 31) were permanently kept pressed down, and the glass rod *g g* removed. The consequence is that there is a great tendency for electricity to leak down the outside of the glass shade, on account of the moisture and dust on it. And farther, even if the inside of the glass shade were clean and dry, and had no tin-foil pasted on, much more electricity would leak along its surface than would leak along the surface of the thin flint-glass rod *g g*. For the breadth of the surface at right angles to the direction of leakage is much greater in the case of the shade than in the case of the rod, or simply the width of the road

along which leakage takes place is much greater for the surface of the glass shade than for the surface of the rod. To avoid this leakage, it is the practice of electrical instrument makers to endeavour to render the surface of the shade as insulating as possible by coating it with shellac varnish, which is less hygroscopic, or attractive of moisture, than the glass, and by not using any tin-foil. But the effect of rendering the glass shade insulating is to cause some conductor outside the instrument (the table, or the walls of the room, or it may be the body of the experimenter) to replace electrically the tin-foil coating T seen on the glass shade in Fig. 31. Hence, the gold-leaf electroscope, when constructed of the form usually met with in shops, measures when dry the difference of potentials between the gold-leaves and some vague body outside the apparatus. And whenever we use it, we are landed on the horns of a dilemma—if we leave the outside of the shade damp (as it frequently will be in England unless it be dried near a fire), the potential of the outside of the glass becomes practically that of the earth, and the indications of the instrument have a definite meaning. But the insulation of the glass being much lowered by this coating of moisture, the mere connecting of any charged body by a wire with the knob of the electroscope tends to discharge the body, or lower its potential. On the other hand, if we take precautions to clean and dry both surfaces of the glass shade, this leakage difficulty may be overcome, but then a most serious vagueness is introduced as to which of the various conductors outside the electroscope is the one with whose potential the potential of the body under test is being compared. (*See* § 57, page 108.)

49. Conduction and Induction.—A conductor can be electrified either by a *transfer* of electricity between it and another conductor, or merely by an *alteration in the distribution* of the electricity on its surface without any transfer of electricity to another conductor. In the former case the body is said to be electrified "*by*

conduction," or "*conductively;*" in the latter "*by induction*," or "*inductively*." Loading or unloading a ship would be analogous with electric conduction, while shifting some of the cargo from the bow to the stern would be analogous with induction. Acting inductively on a charged insulated conductor neither increases nor diminishes the charge on the conductor as a whole, although it alters the distribution of the charge (*see 1*—*7*, § 69, page 123). If the conductor be previously uncharged, then acting inductively on it produces no charge on it as a whole, but merely induces equal and opposite charges on its two sides or ends (*see 8*, § 69, page 124). An inductive method may, however, be conveniently employed to charge a conductor by connecting it with the earth by a wire, while an electrified body is held near it, then removing the earth connection, and lastly, the electrified body. If this electrified body has a positive potential, the charge induced in the conductor will be negative. Instead of connecting the conductor with the earth by a wire, one's own body may be used, and the conductor touched with the finger.

When the gold-leaves of an electroscope are charged inductively in this way, care must be taken not to induce too great a charge in the knob, as otherwise on removing the electrified body, the leaves will diverge so widely as to be torn asunder.

50. Potential Uniform at All Points Inside a Closed Conductor.—We have seen that when electricity is at rest on a conductor the potential at all points of the conductor is the same. The following experiment will show that not only is this the case, but that *the potential at all points inside a closed hollow conductor is uniform, and has the same value as at any point on the surface of the conductor*:—Attach one end of a fine wire to the knob of the electroscope, and the other to the end of a clean dry glass rod, which is to be used as an insulating handle for holding the end of the wire by. Then, if this end be touched against the outer surface of a conductor,

charged conductively or inductively, or, after being introduced inside the conductor through a hole in its surface, it be first touched against the inside surface, and then be held merely inside the hollow conductor without touching it, or be moved about inside the hollow conductor, the divergence of the gold-leaves will be exactly the same, proving what is stated above. The hole in the surface of the conductor through which the test wire is introduced may be fairly large—as large, for example, as the opening at the top of a coffee-pot—without altering what has just been stated, excepting for points in the air just inside the pot close to the opening, where the potential will be somewhat different from the uniform potential inside the pot. If, however, the opening be small, then the potential even just inside the opening will be found to be the same as the uniform potential of the pot, so that if the metallic surface of the conductor be not continuous, but be *made of wire gauze, or even of bits of wire like a bird-cage, the potential is found to be uniform inside*, unless the meshes of the wire gauze be very large.

51. No Force Inside a Closed Conductor Due to Exterior Electrification.—Since the potential at all points inside a hollow closed conductor is uniform and equal to the potential of the surface of the conductor, as far as exterior electrification is concerned, it follows that if there be electrified bodies inside a hollow conductor, either some or all insulated from the conductor, the raising or lowering of the potential of the conductor relatively to the earth will not alter in the slightest the potential difference between any two bodies inside. In fact, no matter what electrified bodies there may be inside the conductor, the relative internal distribution of potential will be quite unaffected by electrifying the conductor outside, either conductively or inductively. This experiment was first tried by Faraday on a large scale; he found on taking his most delicate electrical apparatus inside a room which he had had built of wood

twelve feet cube, covered with tin-foil to make it conducting, and insulated so that it could be charged, that he was totally unable to detect the slightest evidence of this room being electrified outside, even when it was so powerfully electrified that sparks were being given off by the walls of the room, nor could he detect any evidence of any electric disturbance produced outside the room. This important fact may be expressed by saying that *there is no electric force inside a conductor due to exterior electrification, or a metallic shell, no matter how thin, completely screens inside bodies from exterior electrification.*

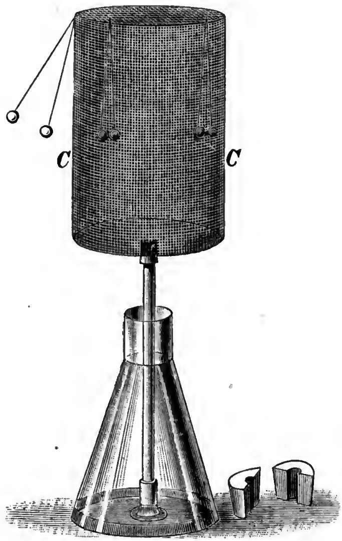

Fig. 32.

This fact may be tried experimentally, thus—c c (Fig. 32) is a cage made of rather fine wire gauze, and supported on an insulating stand. Inside this cage are suspended one pair of pith balls, by means of silk fibres, which are fairly insulating, and one pair by pieces of cotton, which is relatively a fairly good conducting substance. Outside the cage one or more pairs of pith balls are suspended by pieces of cotton. Then it will be found that, whereas the pith balls outside the cage can be made to diverge from one another, either by bringing an electrified body near the cage so as to electrify it inductively, or by giving it a charge, it is impossible by any method to produce

the slightest divergence of either of the pairs of the pith balls inside the cage.

The converse of this, however, is *not universally* true, that is, a metallic box may or may not screen bodies placed *outside* it from the action of an electrified body put *inside* the box. Four distinct cases must be considered.

1. If the box be connected with the earth the screening action will be perfect whether the box be small or large.

2. If the metallic box be not connected with the earth, and be not much larger than the electrified body inside it, the screening action will be very small.

3. If the box be not connected with the earth, but if the dimensions of the side, which is between the body inside it and the body outside, be fairly large compared with the distance between the bodies, the screening action will be considerable.

4. If the dimensions of the side referred to in 3 be very large, then the screening action will be as perfect as with arrangement 1.

52. A Metallic Box not a Magnetic Screen unless made of Very Thick Iron.—Contrasted with the experiment made with the apparatus shown in Fig. 32, the following may be tried :—B B (Fig. 33) is a wooden stand covered with a glass shade, and having inside it a small magnetic needle *m*, suspended by a fibre of unspun silk from a fixed wire bridge. Attached to the needle is a long pointer *p p*, by means of which the deflection of the needle is read off on a scale fastened at the base of the instrument. The magnetic needle takes up a particular position due to the earth's magnetic attraction, from which it may be deflected by means of the magnet M, which can be fixed in any desired position. If, now, when the needle *m* has been deflected 30° or 40° from the position it occupied due to the earth, screens of copper-wire *c c*, brass wire *b b*, &c., be successively put over the stand and glass shade B B, and thus interposed between M and *m*, it

will be found that not the slightest change will be produced in the deflection of m, or, in other words, the insertion of these screens does not in any way diminish the magnetic attraction between M and m. And this will be found to be still the case even when a screen made of iron wire is interposed. In making this latter experiment it is sometimes found that the interposition of a screen made of iron wire does vary the deflection, but on examination it will be found that this variation is due to the iron wire itself having been previously magnetised, and having

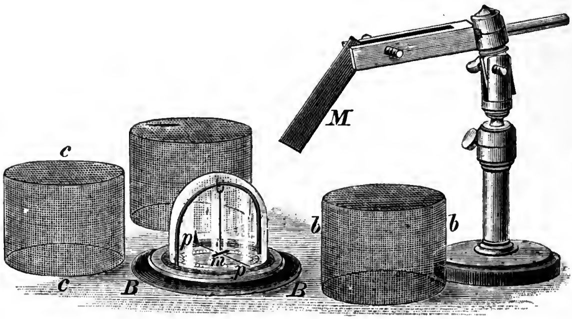

Fig. 33.

retained some of its previous magnetism from its being hard, and not to its shielding m from M. The proof of this is that turning round the screen will alter the deflection of m, and hence that, while with one position of the iron screen the deflection of m is diminished, with another it will be much increased. This disturbing effect arising from residual magnetism on the screen, can be avoided by constructing the screen of *soft* iron wire, and making it red-hot just before the experiment.

If, however, a wide plate of thick soft iron be inserted between M and m, the deflection of m from its position due to the earth's magnetism will be diminished, and if B B be inserted inside an iron box, whose sides

have the thickness of the sides of an ordinary iron safe, then not merely will this box screen *m* from the action of M, but also from the earth's magnetic action.

53. Marine Galvanometer.—This plan of screening a suspended magnetic needle from outside magnetic attraction, by inserting the former in an iron box with very thick sides to it, has been employed by Sir W. Thomson in his "*marine galvanometer*," an instrument intended to be used on board steam-ships, where the motion of the large masses of iron composing the engines, the shaft of the screw, &c., would seriously disturb the deflection of an ordinary unshielded galvanometer. Oscillations of the needle that might be produced by the rolling of the ship are avoided by suspending the needle by a fibre attached above and below, and passing through the centre of gravity of the needle, as described in § 27, page 60.

54. Reflecting Galvanometers. — With the marine galvanometer, and generally with all Sir W. Thomson's

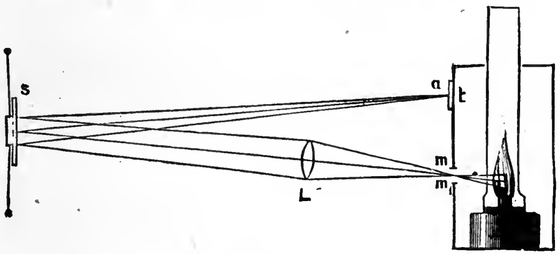

Fig. 34.

galvanometers, a very small deflection of the needle can be observed without the employment of a long pointer (which would be unwieldy, and by adding to the moment of inertia of the suspended arrangement, would render the needle sluggish), as well as without the employment of a microscope, by the reflection of a ray of light from a small piece of looking-glass fastened to the

magnetic needle, and turning with it. In Fig. 34, *s* is the mirror, reflecting a ray of light from a lamp on to a scale *t*, shown more in detail in Fig. 35, the double convex lens L being for the purpose of making an image of the slit *m m*, on the scale *t*, which could not be done by a *plane* mirror *s*, as shown in the figure. To avoid the direct light of the lamp producing a general illumination

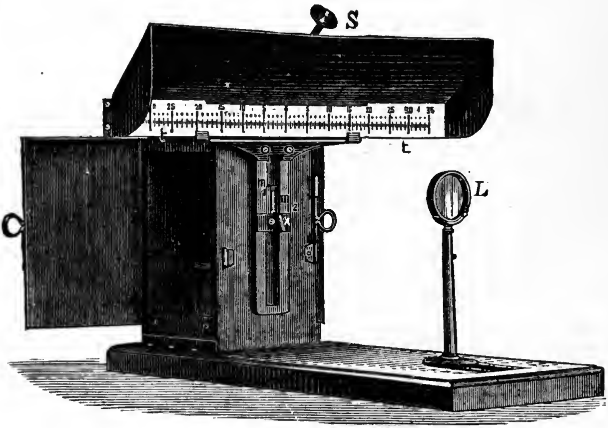

Fig. 35.

of the scale, and preventing the reflected image being clearly seen, the lamp is sometimes shut up in a box as shown, but a complete box is not absolutely necessary, the mere front of the box, as seen in Fig. 36, being sufficient to keep off direct light.

The handle S (Fig. 35) works a rack and pinion for moving the scale horizontally, so as to bring the zero mark on the scale opposite the spot of light or image. If a slit m_1, m_2 (Fig. 35) alone be employed, it must, of course, be made very narrow so as to obtain a sharp line of light on the screen; but a better plan is to use a wide slit, or,

rather, a round hole, and to stretch a fine wire across it vertically, the image of this wire on the screen, and not the edges of the spot of light, being used to read by. Because not merely can the spot of light be large, in which case the numbers on the graduated scale can be easily seen by it, but any flickering of the flame, produced by a draught, although causing the spot of light on the scale to flicker in a corresponding manner, does not produce any flickering of the image of the wire.

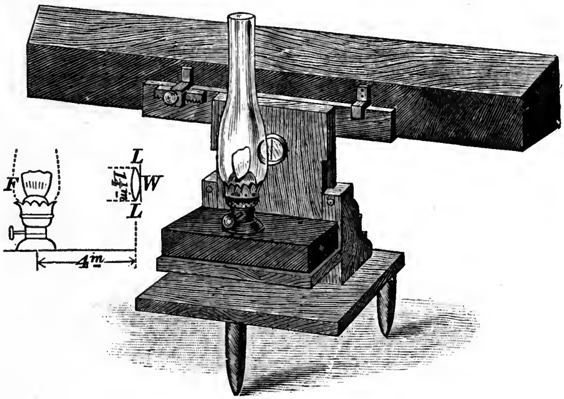

Fig. 36.

An objection to the use of a plane mirror *s* and the lens L, is that the image on the scale is necessarily very much larger than the object, and hence not nearly as well illuminated. A better plan is to use a concave mirror, with which an image can be formed on the scale without the use of a lens at all, the distance between the lamp and the mirror being then equal to the radius of the mirror. But, perhaps, the best method is that due to Mr. Mudford, a former student of the Finsbury Technical College, which consists in using the concave mirror and putting a double convex lens L L *between* the wire W and the flame F, as shown in Fig. 36. With this arrangement a good

image is obtained with a comparatively small flame. The lens should be placed close behind the wire, and the flame should be at about the principal focus of the lens, so that the effect is to produce a general illumination of the lens, which is found to give very good results if it has a focal length of about four inches. Instead of a wire, Mr. Mather has found that a vertical scratch on the lens produces a very good image, and may be employed instead of the wire placed just in front of the lens.

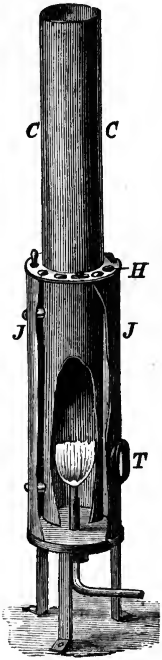

Fig. 37.

A paraffin lamp, with an ordinary flat flame, is commonly employed with reflecting instruments, the edge of the flame being turned towards the lens; but a gas-jet, shown partially in section in Fig. 37, and constructed by Mr. Mudford, may be conveniently substituted for the paraffin lamp. To obtain a fairly intense light, this burner is constructed on the *regenerative* principle, that is, the air is heated before coming in contact with the flame. This result is obtained by having no opening for the air at the bottom, and causing it after entering the holes H to pass down between the outer cylinder J J and the hot inner cylinder C C, at the bottom of which a ring is cut away to allow it to get to the flame. The ray of light passes out through a small disc of glass at T, and to avoid the glass being blackened by the flame being accidentally turned up too high, the burner should be governed, a Suggs's two cubic feet steatite float burner answering well for this purpose.*

55. Angular Motion of the Reflected Ray is Twice the Angular Motion of the Mirror.—Let I O (Fig. 38) be

* A flat albo-carbon burner with a special form of chimney has also been used by the author with good results.

the incident ray, and O R, O R′ the reflected rays when the mirror is in the positions $s\,s$ and $s'\,s'$ respectively. Let O P, O P′ be perpendicular to the mirror when it is in these two positions. Then by the law of reflection,

$$\text{angle I O P} = \text{angle R O P},$$
$$\text{and angle I O P}' = \text{angle R}'\text{ O P}';$$

therefore, subtracting the first from the second, we have

$$\text{angle P}'\text{ O P} = \text{angle R}'\text{ O R} - \text{angle P}'\text{ O P},$$
$$\text{or angle R}'\text{ O R} = 2\text{ angle P}'\text{ O P};$$

but R′ O R is the angle through which the reflected ray is deflected, and P′ O P is the angle between the perpendiculars to the mirror in

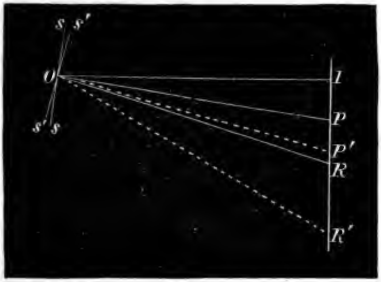

Fig. 38.

its two positions, and is, therefore, the angle through which the mirror is turned; hence, *when a mirror is turned through any angle, the reflected ray turns through twice that angle.*

56. Connection between the Motion of the Image on a Plane Scale and the Angular Deflection of the Mirror. —Let the mirror be parallel to the scale when no current is passing, and let the image be reflected to R and R′ for currents C and C′ respectively; then, since the deflection of the magnet in a mirror galvanometer is always *small*, and since we have seen (§ 20, page 46) that for small deflections the current is always proportional to the tangent of the deflection, no matter what be the shape of the coil or the shape or size of the needle, provided its magnetic axis is parallel to the plane of the coil when no current is passing, it follows that (Fig. 38)

$$C : C' :: \tan. \frac{\text{I O R}}{2} : \tan. \frac{\text{I O R}'}{2},$$

$$\therefore \quad :: \frac{\sqrt{1+\tan.^2 \text{IOR}}-1}{\tan. \text{IOR}} : \frac{\sqrt{1+\tan.^2 \text{IOR}'}-1}{\tan. \text{IOR}'},$$

$$\text{or} \quad :: \sqrt{\frac{1+\left(\frac{\text{IR}}{\text{OI}}\right)^2-1}{\frac{\text{IR}}{\text{OI}}}} : \sqrt{\frac{1+\left(\frac{\text{IR}'}{\text{OI}}\right)^2-1}{\frac{\text{IR}'}{\text{OI}}}}.$$

Hence, when IR and IR′ are nearly equal, we may say that

$$\frac{\text{C}}{\text{C}'} = \frac{\text{IR}}{\text{IR}'}, \text{ approximately,}$$

but for very accurate observations this approximation must not be employed.

57. Static Electrical Apparatus should be Enclosed in a Metallic Case.—In constructing static electrical apparatus, we must carefully consider what are the actions we wish to take place, and what to avoid; for example, in the case of a gold-leaf electroscope we wish the divergence of the gold-leaves to measure the potential difference between one conductor attached to the knob W (Fig. 31), and another attached to the screw S. If, then, W and S be joined by a piece of wire so as to be at the same potential, we wish that no divergence of the leaves shall be able to be produced either by electrifying the electroscope as a whole conductively, or by electrifying it inductively by bringing a charged body near it. And it will be found, if the tin-foil coating T cover nearly all the glass shade, only just sufficient space being left without tin-foil to see the gold-leaves through, that it is impossible in any way to produce a divergence when W is electrically connected with S; whereas, if there be not in-foil, or if the tin-foil only cover a portion of the shade, that a divergence of the leaves can be easily produced.

Want of care in this particular prevented Piazzi Smyth from being able to determine, by his experiments on atmospheric electricity, made on the Peak of Teneriffe, even whether this electricity was positive or negative.

58. Quantity of Electricity.—We have seen that it is possible to electrify a non-conductor, such as ebonite, by rubbing it with a piece of dry clean flannel, and experiment shows that it can be either highly electrified by a prolonged rubbing, so that the gold-leaves of the electroscope diverge widely when the ebonite is held at a foot or two away from the knob of the electroscope, or it may be only slightly electrified by being only just touched with the flannel, in which case the ebonite may be brought quite close to the knob, or may even be made to touch the knob, without any perceptible divergence of the leaves being produced. The rubbed ebonite may, therefore, be said to possess a greater or smaller "*electric charge*," or the "*quantity of electricity*" in the ebonite in the first case may be said to be greater than in the second. Strictly speaking, however, as we have no conception of the existence of electricity apart from the body which is said to be electrified (as we have of a pint of water apart from the pint pot), it is more correct to speak of the "*amount of a body's electrification*" than of its charge of electricity, or of the quantity of electricity in it. But just as it is very convenient to speak of an electric current, as if it had an independent existence apart from the conductor through which it is said to be flowing, so it is

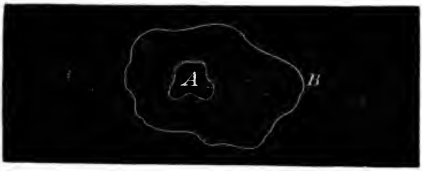

Fig. 39.

convenient to speak of a charge, or a quantity of electricity, as if electricity existed independently.

In order to decide what we mean by saying that one quantity of electricity is two or three times as great as another quantity, or simply one quantity is two or three

times another, we shall adopt the following arbitrary definition :—

When one conducting body A *is entirely surrounded by another conducting body* B (Fig. 39), *the quantity of electricity in* A, *or the electric charge in* A, *is directly proportional to the potential difference between* A *and* B *as long as the position of* A, *relatively to* B, *is absolutely fixed.*

For example, if A be an insulated conducting body suspended in a room B, the walls, ceiling, and floor of which are made of conducting material, then the quantity of electricity on A is directly proportional to the potential difference between A and B as long as the position of A in the room is unaltered.

If not only A be inside the room B, but if in addition there be another electrified body C fixed in position in the room, as in Fig. 134, page 341, it can be shown that, if the potential difference between A and B be represented by A B, and the potential difference between C and B be represented by C B, the total charges on A and on C may each be regarded as being composed of two parts—the total charge on A being equal to the charge A would have if the potential difference between it and B were A B, and C were connected with B, *plus* the charge A would have if it were connected with B, and if the potential difference between C and B were C B, C being now, of course, insulated from B. Also the total charge on C is the charge C would have if the potential difference between it and B were C B, and A were connected with B, plus the charge C would have if it were connected with B, and the potential difference between A and B were A B, A being now, of course, insulated from B.

If, however, A be moved about inside B, then the potential difference between A and B gives us no indication of the relative charges on A. Or, again, even if A and C be at rest inside B, the potential differences between A and B, and between C and B, give by themselves no idea of the relative amounts of electricity on A and on C. In exactly the same way, although the pressure of gas in a given vessel, at a constant temperature, is proportional to the weight of gas in the vessel, the pressure of gas in a vessel whose temperature is varied in some unknown way, or the pressures of the gas in different vessels of unknown volumes, give no indications of the

various weights of the gases. The height of the barometer, for example, tells us, by itself, nothing about the total weight of air in the room.

59. Comparison of Quantities of Electricity.—In order that the indications of a difference of potential measurer may be directly proportional to the charge on a body connected with it, or rather to the charge on the body in excess of what it might have inductively when its potential is nought, the body must be fixed in size and shape, and in its position relatively to other bodies. So, in the same way, in order that the indications of a pressure gauge may be directly proportional to the weight of a gas, it is necessary that the vessel containing it should be fixed in size and kept at a constant temperature. In order, therefore, to compare the weights of the same kind of gas in different vessels at different temperatures by means of measurements of pressure, we must first equalise the temperatures, and then successively *entirely* empty the gas in each vessel into a standard vessel, and measure the pressure that each of the quantities of gas, when put into the standard vessel, will produce by itself.

To empty all the gas out of a vessel into a standard gas-holder, to which the pressure gauge is attached, for the purposes of thus ascertaining the weight of gas in the first vessel, would be an extremely difficult and inconvenient process; whereas, to empty all the electricity out of a body into a standard body, attached to an electroscope, is an extremely simple one. Because, since there is no electricity at the bottom of the inside of a conducting pot (*see* § 64, page 118), it follows that if a charged body be put inside a conducting pot and touched against the bottom, it will give up all its charge to the pot, and when drawn out, without touching the sides of the pot, will be found to be completely discharged.

Hence, using this principle, we can, with the apparatus shown in Fig. 40, compare the electric charges that are given, say, to the metallic bodies B, B, when hung up

by their silk cords, and charged, say, to the same potential. All that has to be done is to put first one of them inside the insulated tin-pot P, touch it against P near the bottom, and observe the divergence d_1, of the gold-leaves

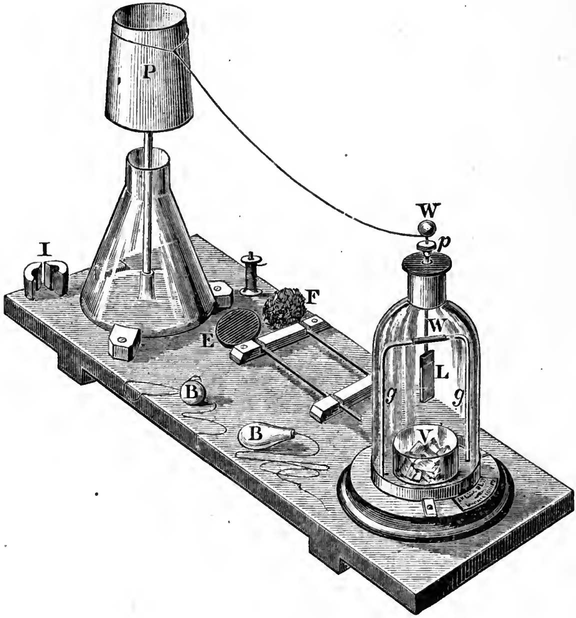

Fig. 40.

of the electroscope. Then, after withdrawing the first body and discharging the electroscope, place the second one in the metal pot P, touch it, as in the case of the other body, near the inside of the bottom of P, obtaining a divergence of the gold-leaves, say d_2. Then d_1 and d_2 will, according to the proper calibration curve of the electroscope, measure the potentials of the pot P in the

two cases, and hence will measure the relative quantities of electricity on the two bodies B.

From what has been said it will be seen that if either of the bodies had touched P on the *outside*, this result would *not* have been obtained; also that we must not, for example, stand close to P when making the first measurement, and not close to P when making the second, since the essence of the test is that the charges on the two bodies B shall be successively *entirely transferred* to the conductor P, and that P shall be *absolutely fixed in external shape and in position relatively to other bodies.*

Further information regarding the unit of electric quantity, and more exact modes of measuring quantities of electricity, will be found in Chapter VII., § 155, page 289.

60. Quantity of Electricity produced by Rubbing Two Bodies Together.— On putting the insulated charged body B, in the last experiment, into the pot P, it is noticed that *after* B *has been lowered so far into the pot that it is well under cover of the sides (which occurs when* B *cannot be easily seen from outside), no further increase is produced in the divergence of the gold-leaves by further lowering* B, *or even by touching* B *against the sides or bottom of the pot.* Hence, in order to measure the charge on a body, it is not absolutely necessary to discharge that body into P, since experiment shows that the potential of P remains the same whether B is discharged into P, or whether B is merely well inside P. The fact is that as soon as B is well under cover of the sides of P, there is, as was first shown by Faraday, *a charge induced on the inside of the pot* P, *exactly equal to the charge on* B, *but of the opposite sign, and another charge on the outside of the pot, also equal to the charge on* B, *but of the same sign.* This latter charge remains unaffected by touching B against the pot, as this has only the effect, if B be a conductor, of allowing the charge on B to neutralise the charge which has been induced on the inside of the pot equal to that on B, but opposite in sign.

This important fact that, as soon as B has been lowered a certain distance into the pot, the potential of the pot becomes equal to what it would have been if all the charge on B had been given up to the pot, enables us to measure the charge on an insulator, which charge could not easily be all communicated to P, even on touching the insulator against P.

Consequently this apparatus may be conveniently employed for testing the amounts of positive and negative electricity that are simultaneously produced when two bodies are rubbed together. E and F (Fig. 40) are respectively discs of ebonite and of wood, the latter being covered with cat's-fur. The ebonite is a good insulator; the cat's-fur and wood make but poor insulators; both discs are, however, as seen in the figure, mounted on long, thin, insulating glass handles. If, now, the glass handles be cleaned and dried, and if the ends of them be held in the hands, the two discs may be rubbed together without practically any of the charge of electricity produced in the ebonite or in the cat's fur being lost. When either of these discs is held inside the metal pot P, it is found that the gold-leaves will diverge; but there is this difference between the divergence that, whereas when the divergence is produced by the rubbed cat's-fur being held inside the pot, this divergence can be *increased* by bringing either near the pot, or near the knob of the electroscope, or near the wire connecting them a piece of dry clean *glass* rod that has been previously rubbed on dry silk, on the other hand, if the gold-leaves are diverging because the rubbed ebonite is held inside the pot, the divergence of the gold-leaves is *diminished* by the approach of the piece of rubbed dry clean *glass*. Hence, the electricities are of the opposite sign, that on the rubbed cat's-fur being like the electricity on rubbed glass, which, as already stated in § 41, page 85, is called vitreous or positive, while that on the rubbed ebonite is called resinous or negative.

But more than that, experiment shows that if, by

means of the insulating handles, both the rubbed discs be held *well inside* the pot, either both not touching the pot, or both touching it, or one or other touching it, or touching one another, the divergence of the gold-leaves is absolutely nought. Hence we conclude that *the charges of electricity in the ebonite and cat's-fur, which have been rubbed together, are not only opposite in kind, but are equal in amount.*

Before trying this experiment it is well to make sure that there is no residual charge of electricity in the ebonite disc. This can be ascertained by seeing whether any divergence of the gold-leaves is produced on inserting the disc into the pot before it is rubbed with the cat's-fur. If it is found that such a divergence is produced, then the disc should be discharged by being passed through the flame of a spirit-lamp before it is rubbed with the disc of cat's-fur.

When this apparatus is not in use, the plug *p* at the top of the electroscope should be pushed down to prevent dust and moisture entering the electroscope; and the two halves of the indiarubber stopper I should be inserted in the neck of the glass bottle belonging to the insulating stand, to prevent dust and moisture settling on the glass rod of this stand.

61. Object of Rubbing Two Bodies Together to Produce Electrification.—The sole object of *rubbing* together the two bodies when one or both of them is more or less a non-conductor, is to bring the various parts of the surfaces of the two bodies successively into intimate contact. The energy expended in the friction is not only far greater than the electric energy developed, but is in no way a measure of the latter. This may be experimentally seen from the fact that if, after rubbing a rod of ebonite with a piece of cat's-fur, the two be brought together towards the knob of the electroscope with the fur wrapped round the ebonite as it is during the operation of rubbing, practically no divergence of the gold-leaves will be observed; whereas if the ebonite and

the cat's-fur be separated after being rubbed together, the ebonite will produce a marked divergence. In fact, as will be more clearly seen later on (§ 189, page 352), *the electric energy stored up in the rubbed ebonite after being separated from the fur is not the equivalent of the work done in the rubbing, but of the small amount of work done in the separation against the electric attraction of the negative electricity in the ebonite for the positive in the fur.*

If the bodies are both conductors, simply touching them together without rubbing is all that is necessary to produce the full electrification, and no increase in the charges will be produced by rubbing the two bodies

Fig. 41.

together. Of course, if the bodies are conductors, one or both of them must be held by insulating handles, otherwise the charges of positive and negative electricity residing in them respectively during contact will flow together through the body of the operator, and neutralise one another on the conductors being separated.

62. Proof-plane.—The preceding experiments for measuring potential differences and the charges of electricity in bodies, must be carefully distinguished from another experiment, with which the student is probably more familiar—viz., that of successively touching various parts of the surface of a charged conductor with a small disc of metal M fixed at the end of an insulating handle H, shown in Fig. 41, and called a "*proof-plane*," and testing the various electric states of this proof-plane by touching it against the knob of the electroscope each time after it has been touched against some particular part of the surface of the charged conductor.

63. Electric Density.—What this experiment decides is the various potentials of the proof-plane at the different times when it is being touched against the knob of the electroscope, and not the potentials of the various parts of the surface of the conductor against which it has been touched. The proof-plane when touching the charged conductor has the potential of the conductor; and, further, if when in contact with the conductor it be pressed flat against the surface, the quantity of electricity that was previously on the bit of the surface of the conductor now covered by the proof-plane rests on the surface of the proof-plane, instead of on the surface of the conductor. When the proof-plane is removed by the insulating handle, it will carry away with it the charge of electricity, provided that in taking the proof-plane away it be moved *without tilting* along a line perpendicular to the surface. But its potential alters as it is being moved, so that while when the proof-plane is in contact with the charged conductor, its potential, quite irrespectively of the quantity of electricity that happens to be on it, is simply that of the charged conductor, its potential, but not its charge of electricity, varies as it is moved; and, finally, when the proof-plane has been moved out of the influence of the charged conductor, and is then put into contact with the knob of the electroscope, its potential becomes simply proportional to the charge of electricity on it.

Hence the divergence of the gold-leaves, which (according to the calibration curve of the electroscope) *measures directly the potential of the proof-plane, measures indirectly the electric charge residing on it, and which previously resided on that small bit of the surface of the charged conductor that was covered up by the proof-plane.* This quantity of electricity is proportional to the "*electric density*," or the quantity of electricity residing on a unit of area at that part of the surface of the charged conductor touched by the proof-plane. And the *density* is called positive or negative, according as the

charge taken away on the proof-plane is positive or negative.

Experiments made thus with a proof-plane show that, in the case of an electrified flat sheet of metal which is far away from other conductors, the density is very much larger near the edges than it is at points far removed from the edges, and is less and less the farther the point is from the edge. If, however, two flat sheets of metal such as A and B (Fig. 29, page 88) be placed parallel to one another, and near together, the density at *any* point on either of the opposed surfaces is found to be the same in value, but is positive on the surface of one of the plates and negative on the other. At points near the edge of the upper surface of A the density will be a little less than when it is nearer the middle of that surface, but, if the potential of B and of the guard ring C D be the same, the density at all points on the lower surface of B will be *absolutely* the same.

In the case of a charged conically-shaped conductor, such as is shown in Fig. 118, page 316, the density is very great at the pointed end, and comparatively small at the rounded end. The use of the special apparatus on which the conical body is supported for enabling accurate experiments on density to be made is described in § 171, page 316.

64. Density is Nought on the Inner Surface of a Closed Conductor.—Experiments made with a proof-plane in the way just described show that the density is nought on the inner surface of a nearly closed hollow conductor, and even when the conductor is only partially closed the density is found to be nought at any point on the inner surface from which bodies outside the conductor are not easily visible. For example, the density on the parts near the *bottom* of the interior of a charged metal coffee-pot, or even on the parts near the bottom of the interior of a charged shallow metal tea-pot *with the lid open*, is practically nought, but will be no longer nought if one end of a metal rod, say the end of a

poker, be held inside the pot without touching it. And not merely on the inner surface of a pot made of continuous metal will the density be found to be nought, but in the case of a pot made of wire-gauze, even with fairly wide meshes, the density is also nought at all parts on the inner surface except close to any very large opening. But in this case, as in the other, if a metal rod be held partly inside and partly outside the pot, the distribution of density will be quite altered.

From the preceding experiments we see *that electricity at rest resides only on the surface of a conductor*, and therefore, as far as the effects of electricity *at rest* are concerned, it is immaterial whether our conductors are of solid or hollow metal or whether they be simply made of wood and coated with tin-foil or gold-leaf.

65. Potential of a Conductor Depends Partly on the Amount of Electricity on it.—This is easily seen from the fact that the divergence of the gold-leaves can be varied by charging more or less a conductor in electric connection with them.

66. Potential of a Conductor Depends Partly on its Shape.—That altering the shape of a conductor alters its potential may be proved thus:—P (Fig. 42) is a metal plate fixed to the wire W W of the electroscope in place of the knob, and M is an insulated piece of metal carried by a clean dry glass handle H, by means of which M may be laid on P, or separated more or less from P. If now M be laid on P, and a charge given to P and M as one conductor, the leaves will diverge, indicating the common potential of P and M; and it will be found that on sliding M over P, or tilting M up, without in either case separating M from P, the divergence of the gold-leaves diminishes. But on putting M back into its original position, the divergence of the gold-leaves regains its original value, proving that the alteration of the form of the compound body M P, without altering the amount of electricity on it, alters its potential.

67. Potential of a Conductor Depends Partly on

its Position.—The fact that the potential of a conductor can be changed by varying its position relatively to other bodies can be proved also with the apparatus shown in Fig. 42. If, M having been removed to some distance from P, a charge be given to P, it will be found that on approaching M, held by its insulating handle, towards P, the divergence of the gold-leaves or the potential of P

Fig. 42.

diminishes. Further, if, when M is near P, M be connected with the tin-foil coating of the electroscope, or with the earth with which the tin-foil coating is already connected, the divergence of the gold-leaves will diminish much more. And, lastly, if M, still in connection with the tin-foil coating, be placed very near the plate P, and parallel to P, but without touching it, the divergence of the gold-leaves will be almost nought, showing that the potential of P has practically become that of the tin-foil coating, or nought. On removing M, the divergence will regain its original value, showing that the potential of

P was diminished, not by P having been discharged (which is also, of course, one way of diminishing its potential, and, therefore, care must be taken that M does not touch P) but by the mere approximation of the piece of metal M connected with the tin-foil coating.

68. The Potential of a Conductor can therefore be Varied by—

1. *Altering the charge of electricity on it.*
2. *Altering the external shape of the conductor without altering the charge of electricity on it.*
3. *Altering its position relatively to other bodies.*

In the same sort of way the pressure of a gas (say oxygen) in a gasometer can be varied by—

1. *Altering the weight of the oxygen in the gasometer.*
2. *Altering the size of the gasometer without altering the weight of oxygen in it.*
3. *Altering the temperature.*

69. Examples showing the Difference between Potential, Density, and Quantity.—To familiarise the student with the difference between potential, density, and quantity, the following examples may be considered. A (Fig. 43) is an insulated piece of metal charged positively, and far away from other bodies, so as to be beyond the range of their inductive action; then its potential, the density on its two sides, and the quantity of electricity, or charge on it (the approximate modes for measuring which have been described, § 59, page 112), are given in the following table; a positive potential meaning that if the body were joined to the ground by a wire, or "*put to earth,*" as it is technically called, positive electricity would flow to the ground from this body.

Fig. 43.

Now, let a large body B, in metallic connection with the earth (Fig. 44), be brought near A on its right side, then *2* will represent the electric state of A. Let B be brought nearer to A; A's state will now be given by *3*.

If, on the other hand, A and B be separated more and more, A's state will be more and more like that given in *1*.

Next let a large positively charged body, C (Fig. 45), be

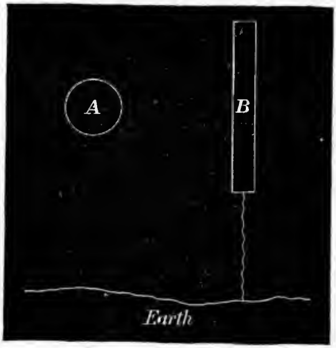

Fig. 44.

brought near A on its left side, *4* will then represent A's state. Bring C nearer to A, but not so near that a spark or a brush discharge * can pass between A and C; A's state

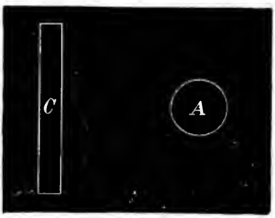

Fig. 45.

will be changed to *5*. Now, while C is near A, let A be connected electrically with the ground (Fig. 46); positive electricity will pass from A to the ground, and *6* will

* *See* notes to § 192, page 358, and § 196, page 369.

then be the potential, density, and charge of A. Lastly, let A be disconnected from the ground, and then let C be removed to a great distance from A, when *7* will be arrived at.

STATE OF THE CONDUCTOR A.

Number.	Figure.	Potential.	Density.		Charge.
			Right side.	Left side.	
1	43	+	+	+	+Q, say
2	44	+, but less than in *1*.	+, but greater than in *1*.	+, but less than in *1*.	+ Q as before.
3		+, but small.	+, and much greater than in *1*.	+, but much less than in *1*.	+ Q.
4	45	+, and greater than in *1*.	+, and greater than in *1*.	+, but less than in *1*.	+ Q.
5		+, and still greater than in *1*.	+, and much greater than in *1*.	Almost nought.	+ Q.
6	46	Nought.	Nought.	—	—q, say
7		Negative.	Negative.	—, but less than in *6*.	— q as in *6*.

Bringing up the positively charged body C near the body A in Fig. 45 has exactly the same sort of effect as heating considerably the left end of an elongated gas-holder, and slightly cooling the right end. The pressure of the gas at all points in the gas-holder is of course uniform, but greater than before any heat was applied, just as the potential of all parts of A in *4* is uniform, but greater than in *1*. The quantity of gas in the gas-holder, like the quantity of electricity in A, remains unaltered, whereas the density or weight of a cubic inch of the gas at the cold end is greater than

before, while the density at the hot end is less than before, just as the density at the right side of A is greater, and at the left side less than in *1.*

8. Next let P, an insulated uncharged conductor, be brought near M, a negatively charged body, then the potential of P is negative, since negative electricity would go from it to the ground if it were put to earth by a conducting wire; the density on the side next M is positive, and on the side away from M is negative, and

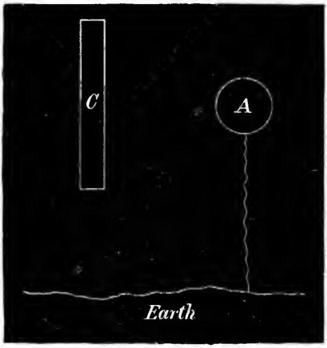

Fig. 46.

the charge on P is nought, since no electricity has been put into it or taken away from it.

9. Without moving P or M, let P be connected with the earth, then its potential is nought, the density on the side next M is positive and greater than before, nought on the side away from M, and the charge on P is positive, + Q, say.

10. Now let the wire connecting P with the ground be removed, and let P and M be separated slightly, then the potential of P is positive; the density on the side next M is positive, but not so great as it was before P and M were separated; on the side of P away from M there is a slight positive density, and the charge on P remains + Q.

11. Let M be brought nearer P than in *9*, then the potential of P becomes negative, because negative electricity will go out of P if it be put to earth; the density on the side next M will be positive, and greater than in *9*, while the density on the side away from M will be slightly negative, and the charge, as before, + Q.

P is therefore in such a condition that its potential will be nought without being connected with the ground if M be brought to the same distance from P that it was in *9*; its potential will be positive if M is farther away, as in *10*, and negative if M is nearer, as in *11*.

All this can be very well seen experimentally if M (Fig. 42) be charged negatively, and P be connected with the tin-foil coating of the electroscope for a moment and then insulated when M is at a certain distance *d* from P, and parallel to P. Then, when M is at a greater distance than *d* from P, the gold-leaves will diverge with positive electricity, or the potential of P is positive; whereas if M be at a less distance than *d* from P, the gold-leaves will diverge with negative electricity, or the potential of P is negative; and when M is at a distance *d* from P, the gold-leaves will not diverge at all, or the potential of P is nought. In the above, M is supposed to be moved parallel to itself, and along a line perpendicular to P, otherwise the distance from P will not accurately determine its position relatively to that of P.

70. Static and Current Methods of Measuring Potential Differences Compared.—To measure the pressure of steam or of water, a static pressure gauge is a very convenient and sensitive instrument; whereas, on account of the extreme smallness of the forces produced by the attractions of ordinary charges of electricity, a static method of measuring a small electric potential is either most insensitive, or requires the employment of a delicate piece of apparatus that can only well be used in a laboratory, hence such a measurement cannot at present be performed with any portable apparatus. In fact, a static portable electrometer, that

will measure accurately a small fraction of a volt, is at present a great desideratum.

But just as the pressure of water at any given point in the side of a vessel containing it can be ascertained by measuring the flow of water that is produced through a particular pipe inserted in an opening in the side of the vessel at the point in question, so the potential difference between two bodies can be ascertained by measuring the current that is produced through a particular wire used to electrically connect these two bodies; for it can be shown experimentally (*see* § 75, page 135) that if the current passing through a particular wire be measured in amperes, and the potential difference maintained at the ends of the wire be measured in volts, by means of, say, a quadrant electrometer, the number of amperes is directly proportioned to the number of volts. In the case of water this current method would be most troublesome to carry out practically, on account of the alteration of flow produced by bends and irregularities in the sectional area of the pipe, and especially because slight changes in the mode in which the water enters the pipe, arising from slight differences in the way in which the pipe is attached to the vessel, produce decided changes in the current. But in the case of electricity this current method of measuring potential difference is most convenient, since for a given potential difference the current flowing through a wire depends only on the wire and on its temperature, and not at all on the shape the wire is made to assume, or on the form of the coil in which the wire is wound; nor does the current depend on the exact way in which the ends of the wire are joined to the two bodies, provided only that the contact at each end is a clean metallic one. *A galvanometer, then, which directly measures current may be used to indirectly measure potential difference.*

Both in the case of measuring water-pressure and electric potential, the production of a current through

the test-pipe or wire tends to diminish the very thing we desire to measure. Hence, unless there be some efficient means of keeping up the water-pressure, or the electric potential difference, we must be content to employ only a small current, and use a proportionately delicate instrument to measure it. In some cases—as, for example, with two insulated ordinary metallic bodies charged to a different electric potential—the current method of measuring this potential difference would be practically impossible, as the potential difference which it was our object to measure would, by joining the bodies together with the wire of a galvanometer, be entirely neutralised before the needle of the most delicate galvanometer began to move. In such a case the static method is the only one that can be employed.

71. When a Potential Difference Galvanometer may be Employed.—In all cases, however, where there exists some means of keeping the potential difference constant between two bodies even after they are allowed to discharge one into the other through the coil of a galvanometer, this galvanometric method of measuring potential difference can be employed. If the coil of the galvanometer is made of a long fine wire, there is much less chance of the potential difference being altered by the application of the galvanometer than if it were made of a short thick wire, and for that reason *potential difference galvanometers are wound with a long fine wire.* In certain special cases, before the application of our galvanometers, the two bodies whose potential difference we desire to measure are already joined by a short thick wire—as, for example, two parts near together in a circuit carrying a current—and in such cases the wire used for the coil of the galvanometer employed to measure the potential difference between these two points need not be very long or fine. Generally, however, a long fine wire must be used in making a potential difference galvanometer.

For practical purposes a potential difference galvano-

meter must, like an ammeter, be calibrated absolutely; only in this case it is not the number of amperes, or fraction of an ampere, passing through the instrument, and producing any particular deflection, that we desire to know, but the number of volts that must be maintained at the terminals of the instrument to produce this current.

72. Voltmeter.—The permanent magnet proportional galvanometer described in § 37, page 76, may be wound with fine wire instead of with thick, and calibrated in volts by ascertaining, by means of a standard electrometer, for example, the number of volts necessary to be maintained at its terminals to produce various deflections of its needle; such a dead-beat potential galvanometer when *direct-reading* is called a "*voltmeter*," and it may be taken temporarily as our commercial instrument for measuring potential differences.

Other and more modern forms of potential difference galvanometers are described in Chapter VIII., and the advantages and disadvantages of some of the various types entered into. Methods of practically calibrating *voltmeters* are also given in § 213, page 408, to § 215.

CHAPTER IV.

RESISTANCE AND ITS MEASUREMENT.

73. Resistance.—Whenever an electric current is passing through a circuit, a certain amount of obstruction, or "*resistance,*" is offered to the current, and we have seen that, by the insertion of a longer or shorter piece of wire, or of a longer or shorter column of liquid into a circuit, the current can be diminished or increased in strength. *Any number of amperes can be sent through*

any body, provided that we have a sufficiently powerful generator, and provided that the body is not fused or otherwise destroyed by the current before the current has reached the required strength. Hence, we cannot measure the magnitude of the electric resistance of a body by the smallness of the current strength unless we know something about the power of the generator, just as the number of gallons of water per minute passing through a pipe furnishes no indication of the resistance of the pipe unless we know the difference of pressure maintained at the two ends which is driving the water through the pipe. If, however, the same electric potential difference be maintained at the ends of one wire A as is maintained at the ends of another B, then *the resistances of these wires will be inversely proportioned to the number of amperes flowing through them respectively;* or more generally, the resistance is proportional to the ratio of the potential difference maintained at the ends of the wire to the strength of the current flowing through it.

74. Ohm's Law.—Experiments originally made by Ohm in 1827, and verified to a high degree of accuracy by an elaborate series of experiments made at the Cavendish Laboratory at Cambridge some years ago, show that *this ratio of potential difference to current is absolutely constant for a definite piece of metal at a constant temperature*, and may be called simply the "*resistance*" of that piece of metal.

75. Experimental Proof of Ohm's Law.—To test Ohm's law it is necessary to employ a more delicate *statical* potential difference measurer than a gold leaf electroscope, and a form of Sir W. Thomson's quadrant electrometer, constructed by Dr. Edelmann, of Munich, and shown in the following figures, may be conveniently employed for this purpose. The instrument rests on a metallic bracket L (Figs. 47, 48), screwed to the wall, and is levelled by means of the three levelling screws. G G (Figs. 47 and 48) are four quarters of a brass cylinder insulated from one another,

and held in position by ebonite collars R R and S S (Fig. 48). These quarter cylinders are connected together

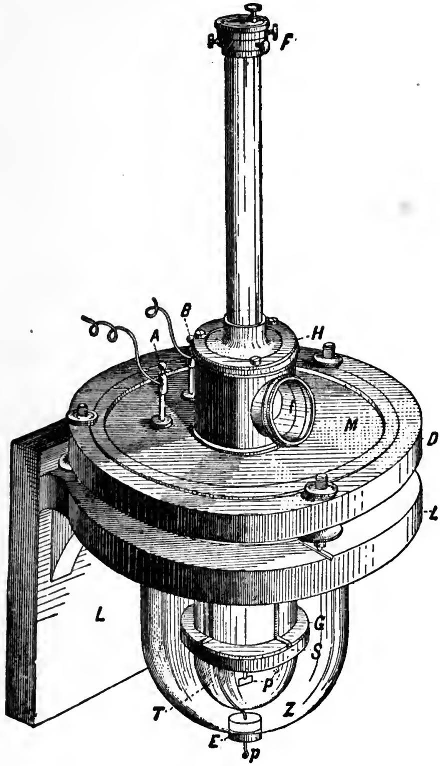

Fig. 47.

in opposite pairs by means of two pieces of wire, the first and third being also attached to the "*electrode*"* A (Fig. 47), and the second and fourth to the *electrode* B.

* "*Electrode*" is the name given to a wire or rod by means of which a current enters or leaves a piece of apparatus.

Suspended inside this system of quarter cylinders, there hangs, by means of a fibre of unspun silk, a movable

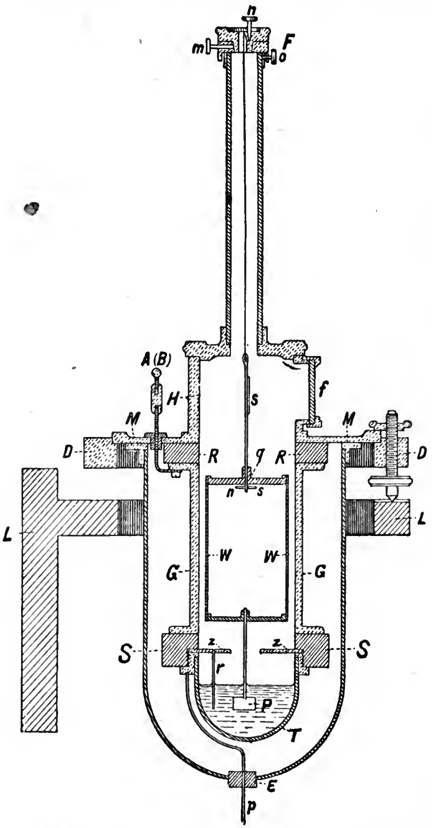

Fig. 48.

piece of aluminium w w, shaped as shown in elevation in Fig. 48, in plan in Fig. 49, and in perspective in Fig. 50, and which may be called the needle. This movable

arrangement, or needle, has attached to its bottom a looped platinum wire, to which is fastened a small piece of sheet platinum, P (Figs. 47, 48, and 50), dipping into a small quantity of sulphuric acid, contained in the glass vessel T, and electrically connected with the wire *p* by means of the platinum wire *r*, which dips into the acid. Into a small collar *q* (Fig. 50), at the top of this needle, there is fixed a little stem of tortoise-shell, carrying the mirror *s*, which reflects a ray of light through the window *f* (Figs. 47 and 48), on to a distant scale, in accordance with Sir W. Thomson's reflecting arrangement already described in § 54, page 103.

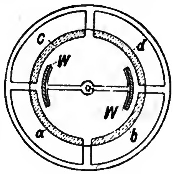

Fig. 49.

The movable arrangement W W is kept at a high potential by one end of what is called a "*dry pile*" (*see* § 197, page 372) being attached to the wire *p*, which passes through a collar E, let into the outer glass vessel Z, the other end of the dry pile being attached to the brass framework D D of the instrument. When all the four quarter cylinders are brought to the same potential by connecting the electrodes A and B together with a piece of wire, then no matter how highly the needle be charged, it will, except for the extremely small torsion produced by the silk fibre, which can be made insignificant by turning round the head F, rest in any position if turned round a vertical axis passing through its centre. But when all the four quarter cylinders are at the same potential, we want the spot of light to stand at nought on the scale, hence it is necessary to give directive force to the needle; this is done by means of a small magnet *n s* fastened to it, as seen in Figs. 48 and 50, and a controlling magnet which turns the needle so that it rests in the symmetrical position shown in Fig. 49, when all the four quarter cylinders are at one potential. The deflection of the needle, or the motion of the spot of light on the scale, which is proportional to this deflection,

is very nearly directly proportional to the difference of potential between the opposite pairs of quarter cylinders as long as the potential difference between the needle W W and the outside of the instrument is constant, and the magnetic controlling force produced by the outside controlling magnet is unaltered in magnitude or direction.

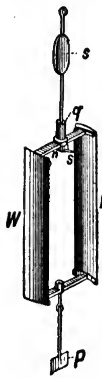

Fig. 50.

The complete formula for a given position of the controlling magnet may be proved to be as follows:—Let N be the potential difference between the needle and the framework of the instrument, P the potential of one pair of quarter cylinders relatively to the framework D, and Q the potential of the opposite pair of quarter cylinders also relatively to the framework, d the deflection of the spot of light on the scale from the zero position, then

$$d \propto (P - Q)\left\{N - \tfrac{1}{2}(P+Q)\right\},$$

from which it follows, first, that the sensibility of the instrument increases as N increases; secondly, that d becomes more and more nearly proportional to P—Q as N becomes larger and larger.

This formula is calculated on the supposition that the vertical edges of the needle W are *never* very near the vertical edges of the stationary quarter cylinders. With such a short needle as is shown in the figure, and which correctly illustrates the apparatus as made by Dr. Edelmann, this condition is far from being fulfilled when the needle is deflected. Hence the instrument would be improved if each half of the needle were made broader, even though the moment of inertia would thereby be increased, the consideration which has probably influenced Dr. Edelmann in making it so narrow. Another improvement would consist in supporting the glass vessel T from ebonite rods instead of by the ebonite ring S S, since leakage takes place from the sulphuric acid in the vessel, over the surface of the ebonite ring S S, to the quarter cylinders G, G, and, consequently, if either pair be left entirely insulated, even for a short time, the spot of light rapidly moves off the scale, from the potential of this insulated pair of quarter cylinders being raised by the electricity leaking into them.

Fig. 51 shows diagrammatically the quarter cylinders CC, C′C′ of the Edelmann electrometer joined to the

terminals T, T′ of a resistance R through which a current is sent by the battery B, and its strength measured by the

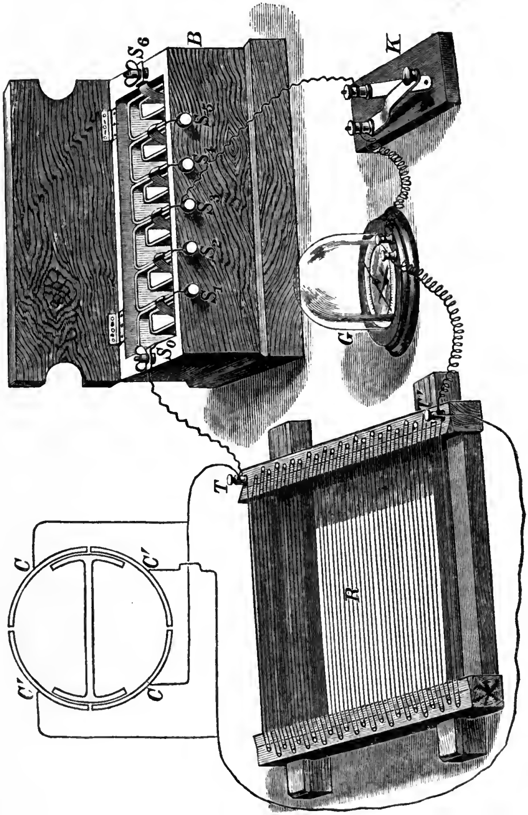

Fig. 51.

galvanometer G. As the wire R is long and rather fine, in order that the potential difference at its terminals may be large enough to be measured with the electrometer, it

would be necessary, if we wished to vary the current considerably, by increasing the resistance in circuit, to introduce a resistance in the circuit several times as great as the resistance offered by the wire R. A simpler plan than employing such large resistances consists in varying the number of cells used to send the current, and this is easily done by keeping one wire attached to the binding screw S_0 and attaching the other wire to the screw S_1 or S_2, &c., according as we wish the current to be sent by one or two, &c. cells. If the current be varied it will be found that if simultaneous readings of the electrometer and galvanometer are taken for the different currents that *the ratio of potential difference to current is constant.*

76. Comparing Resistances. — The simplest way of insuring that the same potential difference—that is, the same number of volts—shall be maintained at the ends of two wires is to join the wires in parallel circuit, as shown in Fig. 52, or what may be called simply "*in parallel.*" The number of amperes flowing in the two circuits can be measured, of course, by properly calibrated galvanometers put in the two circuits, but the coils of each of these galvanometers must be made of such a short piece of thick wire that the insertion of the galvanometer in either of the circuits does not weaken the current in that circuit, otherwise the number of amperes will not be inversely proportional simply to the resistances of the wires A and B, but to the resistances of the two circuits, increased by the addition of the resistances of the respective galvanometers.

It may here be noticed that a properly calibrated galvanometer always measures the current flowing through the circuit in which it has been placed. But it does not, of course, follow that the current is the same as it was before the insertion of the galvanometer, therefore if it is the latter we desire to measure care must be taken that the insertion of the galvanometer shall not diminish the current. Just in the same way when measuring temperature, a thermometer put into a vessel

of liquid always measures quite accurately the joint temperature of the liquid and thermometer; but except in the very exceptional case of the thermometer bulb and the liquid being at the same temperature before the insertion of the thermometer bulb (so that the mercury neither rises nor falls when the thermometer is inserted), the thermometer will either slightly raise or slightly lower the previous temperature of the liquid, unless the volume of the bulb be very small compared with the volume of the liquid, or, more accurately, unless the thermal capacity of the bulb and liquid in it is very

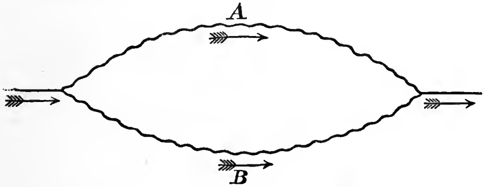

Fig. 52.

small compared with that of the liquid and vessel combined into which it is placed. And for that reason, thermometers with extremely small bulbs, containing very little mercury, have frequently to be employed.

So a current galvanometer should have as small a resistance as possible, and, for a similar reason, as we have already seen, a potential difference galvanometer should have as high a resistance as possible, so as to fulfil the general law which must be carefully attended to in all experiments—*the test must not alter the thing tested.*

By comparing, then, the currents sent through two wires, at the ends of which the same potential difference is maintained, their resistances can be compared, and in this way two resistances can be made, for example, equal to one another. But as the insertion of the galvanometer will generally increase the resistance of the circuit in which it is placed, two galvanometers of known

resistances and with known absolute calibration curves should be employed.

77. Simple Substitution Method of Comparing Resistances.—The following "*simple substitution method*" is, however, much simpler to be used when one resistance has to be made equal to another, as it requires the employment of only one galvanometer of unknown resistance, and of which even the relative calibration need not be known, much less its absolute calibration; in fact, a simple galvanoscope, that merely indicates more or less as regards the current, is all that is needed. Put any convenient electric generator in circuit with a galvanoscope and a wire whose resistance we wish to reproduce, and observe the deflection. Next remove this wire, and put in its place another wire, with which a smaller deflection is obtained, on the same galvanoscope using the same generator. Now gradually diminish the length of the second wire until the original deflection is reproduced, then the resistance of the new wire will be exactly equal to that of the old. In making the experiment, it is desirable to select for the second wire one which, as already stated, gives a smaller deflection, and therefore has a larger resistance than the first, so that by shortening it its resistance may be made equal to the first. We shall see, however, later on, that even if the deflection with the second wire be too large instead of too small, so that it has too small and not too large a resistance, the resistance of the second may be increased and made equal to that of the first wire by passing it through a draw-plate, so that it becomes thinner and of smaller diameter. But this is not nearly so easy an adjustment as shortening a wire that has been selected with too great a length.

To detect any possible change in the sensibility of the galvanoscope, or in the power of the generator during the test—a change in either of which would, of course, destroy the accuracy of the test—it is well after the second wire has been altered, until the first deflection on the

galvanoscope has been nearly reproduced, to substitute the first wire for the second, and see whether the deflection now obtained with the first wire in circuit is exactly the same as was originally obtained. If it be found to be slightly different, then the final adjustment of the second wire must, of course, be made with the new deflection of the galvanometer obtained with the first wire in circuit, and not with the deflection that was originally obtained when the first wire was in circuit. While making the preceding test, care must be taken not to alter the sensibility of the galvanometer by accidentally

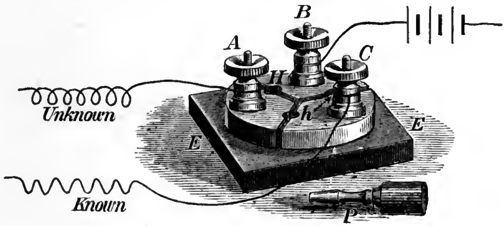

Fig. 53.

moving the controlling magnet, and it is well not to keep the current flowing continuously for too long a time, as the battery is liable to become what is called "*polarised,*" and the current in consequence diminished.

The preceding method of comparing the equality of two resistances is exactly analogous with what is known as Borda's method of double weighing, by means of which the true weight of a body can be accurately determined, no matter how unequal be the lengths of the two portions of the beam, or how unequal the weights of the pans of the balance employed.

78. Plug Key.—In order to connect the galvanoscope and current generator quickly and conveniently with either the known or the unknown resistance, the plug key, shown in Fig. 53, may be employed. It consists of three pieces of brass A, B, and C fastened to a slab of ebonite

or wood E E. By inserting the conical brass plug P into the hole H, the current produced by the battery, one end of which is attached to B, will pass through the unknown resistance, whereas if it be inserted in *h* it will pass through the known resistance and not through the unknown.

79. Potential Difference Method of Comparing Resistances.—Another method of comparing two resistances, depending directly on the definition of resistance, consists in sending a current through the two wires A B and C D placed "*in series*," or end on (Fig. 54), and comparing, by means of a suitable galvanometer, the potential difference between A and B with that between C and D. For since the same current passes through these two

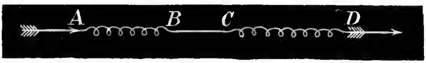

Fig. 54.

wires, and since resistance is the ratio of potential difference to current, it follows that—

$$\frac{\text{resistance of AB}}{\text{resistance of CD}} = \frac{\text{potential difference between A and B}}{\text{potential difference between C and D}}.$$

80. Ohm.—The legal unit of resistance, as settled by the International Electrical Congress, at their meeting held in Paris in 1884, is *that of a column of pure mercury* 106 *centimetres long*, 1 *square millimetre in sectional area, at a temperature of* 0° *C.* This is called the "*ohm*," and is the only one of the electrical units that has yet been legalised. All the others have, however, been accurately defined in terms of the ohm and the ampere, but as the exact rate of chemical action corresponding with the ampere (although now generally accepted as being that given in § 6, page 11) has not yet been defined *legally*, it cannot be said that a practical unit of current has yet been *legally* adopted, and the same remark applies to the volt and to all the electrical units depending on the ampere.

81. Volt, Practical Definition of.—A volt is the difference of potentials that must be maintained at the ends of a wire of one ohm resistance, so that a current of one ampere may pass through it; or generally, if V be the potential difference in volts maintained at the end of a conductor having a resistance of o ohms, and if A be the current in amperes flowing through it

$$A = \frac{V}{o}.$$

82. British Association Unit of Resistance.—Previous to 1884, the unit of resistance used most extensively in Great Britain and elsewhere was the *British Association*, or "*B. A.*" unit, called also previously to 1884 an ohm. The name ohm is, however, now restricted to the legal unit, and the older one is called a B. A. unit. The value of this latter was decided on by the Electrical Committee of the British Association, after years of extremely careful and painstaking work, and copies of the standard were first issued in 1865, since which time they have been multiplied almost indefinitely. The ideal B. A. unit (as distinguished from the actual one, which, as will be explained farther on, is slightly wrong) is a *derived* unit, and not an *arbitrary* one, that is to say, it is selected so that the equations connecting current, resistance, potential difference, work, &c., shall be of the simplest kind. without arbitrary co-efficients. The great value of this so-called *absolute*, or British Association, system of electrical units was fully accepted at the meeting of the International Electrical Congress at Paris in 1881, and it was decided that for purposes of reference, that particular length of a column of mercury one millimetre square in section which at a temperature 0° Centigrade was found to have most nearly the true B. A. unit of resistance, should be called the ohm, and legalised. Doubts having arisen as early as 1878 as to whether there had not been some mistake made by the British Association Committee in their original determination,

the whole work was repeated, and it was eventually agreed, at the meeting of the Conference in 1884, that the length of mercury which, having one square millimetre in section, had at 0° C. one ohm resistance should be internationally accepted as 106 *centimetres*, the decimal of a centimetre which required to be added to make this length perfectly accurate being left for further experiment and consideration. And in England it has been also decided that for the purposes of issuing practical standards of electrical resistance, the number of B. A. units adopted, from the means of a large number of experiments, as the resistance of a column of mercury 100 centimetres 1 square millimetre, at 0° Centigrade, which is the "*Siemens' unit of resistance*," shall be 0·9540.

Therefore it follows that

1 legal ohm = 1·0112 B. A. units.
1 B. A. unit = 0·9889 legal ohm.

Example 21.—With a potential difference of 108 volts maintained at the terminals of an Edison incandescent lamp, 0·75 ampere passes through it, what is the lamp resistance? *Answer.*—144 ohms.

Example 22.—If the potential difference be reduced to 105 volts, and the resistance of the lamp remain the same, what current will now pass through it?

Answer.—0·729 ampere.

Example 23.—If a wire have 127·4 B.A. units' resistance, what is its resistance in legal ohms?

Answer.—126·0 ohms.

Example 24.—If a wire of uniform section have 27 B. A. units' resistance, how much per cent. must be cut off it so that it may have 26 ohms' resistance?

27 B. A. units = 27 × 0·9889 ohm.

Answer.—26·7 ohms.

Answer.—To reduce to the 26 ohms we must cut off $\frac{7}{267}$, or about 2·6 per cent.

Example 25.—What percentage error would be made in assuming that the B. A. unit was the same as the legal ohm?

Answer.—The resistance would be assumed to be about 1·1 per cent. larger than it really was.

To familiarise the student with the practical value of

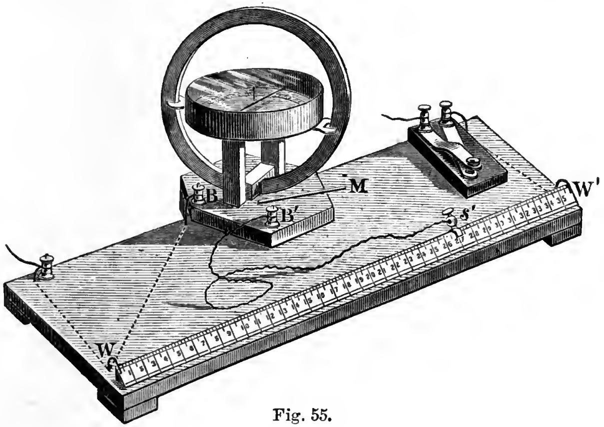

Fig. 55.

an ohm, it may be mentioned that a copper wire one foot long, $\frac{1}{1000}$th of an inch in diameter, has roughly 10 ohms' resistance, which is also roughly the resistance possessed by a mile of iron wire one-fifth of an inch in diameter.

83. Variation of Resistance with Length.—The apparatus shown in Fig. 55 is adapted for ascertaining this, and consists of a thin platinum wire of uniform sectional area, stretched along the graduated bar between the two points w, w′, and through which, on pressing down the key, a constant current flows, produced by some current generator attached to the two wires which come from the binding screws at the farther side of the figure.

To one end W is joined one terminal B of a tangent galvanometer, the coil of which is wound with a very fine wire, and to the other terminal B′ is attached a flexible wire, by which it can be electrically attached to any other point of the stretched platinum wire by means of the binding screw s'. Experiment shows, that if the sensibility of the tangent galvanometer is kept unchanged by the adjusting magnet M not being moved, the tangent

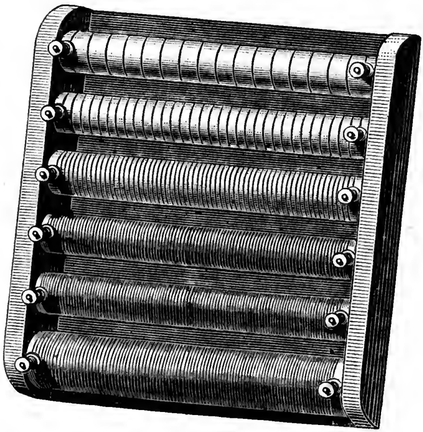

Fig. 56.

of the deflection is directly proportional to the distance W s'. Now, the resistance of the wire forming the coil of the tangent galvanometer is very great compared with that of the stretched platinum wire W W′, hence it follows (*see* § 71, page 127) that the potential difference between the points W and s' of the stretched wire is unaffected by the presence of the galvanometer. Consequently we may conclude that the tangent of the deflection measures the potential difference that would exist between the points W and s' if the galvanometer were not present. Hence, when a constant current is flowing through a particular

wire, the potential difference between two points is directly proportional to the length of wire between those two points, so that potential difference divided by current which we have defined as the measure of resistance, is directly proportional to the length of wire.

This experiment can be performed for greater lengths of wire by replacing the stretched wire shown in the last figure by lengths of the same wire wound for convenience round in a screw groove turned on a wooden cylinder. Fig. 56 shows such an arrangement, consisting of six coils of iron wire of lengths, say 5, 10, 20, 30, 40, and 50 feet respectively, all the wire being drawn to have exactly the same diameter, say 0·0095 inch.

From what has preceded it follows that, if distances O A, O B, &c. (Fig. 57) measured horizontally from a point

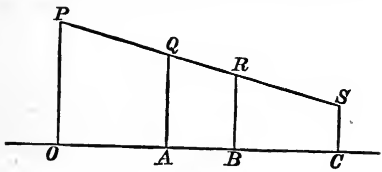

Fig. 57.

O, represent the resistance of a circuit from some fixed point up to various points of the circuit, and if vertical distances O P, A Q, &c., represent the potentials at these points, *the points* P, Q, R, S, *&c., will lie in one straight line when the current is steady*, and the tangent of the angle this line makes with O C will measure the strength of the current, this strength being in amperes if the resistances are measured in ohms, and the potentials in volts.

84. Construction of Coils; Multiples of the Ohm.—We are now in a position, if we have a single wire having one ohm resistance to start with, to construct, in the following way, by the simple substitution method, coils having a resistance of any number of ohms we please. First, make a second coil having one ohm resistance, then

put these two ohm coils in series as in Fig. 54, page 140, when the resistance of the two will be, as we have seen, two ohms. Now make a single coil, having two ohms' resistance by comparison, then using this in series with one of the one-ohm coils, we shall have a resistance equal to three ohms, compared with which we can then make a single coil having three ohms' resistance, and so on.

85. Variation of Resistance with Sectional Area.—For the purpose of testing experimentally how the resistance of a wire depends on its sectional area, which may be done by the simple substitution method, a board somewhat like that shown in Fig. 56 is employed, but having wires of exactly the same length (say twenty-one feet) and the same material (iron) wound round each of the cylinders. The sectional areas of these wires are however different, being proportional to the squares of the diameters, which may be 0·0195, 0·0158, 0·0136, 0·0106, 0·009, 0·0078 of an inch.

86. Variation of Resistance with the Material.—On the cylinders of a third board are wound wires of exactly the same length (say twenty-one feet), and drawn to have exactly the same diameter (say 0·012 of an inch), but made of the following materials: copper, platinum, brass, iron, lead, and German silver, from which the effect of difference of material can be ascertained.

As in selecting a piece of wire there are *three distinct things* that have to be considered—its length, its thickness, and the material of which it is made—it is important that the change in the resistance produced by a change in each of these three things should be *separately* measured; and generally, in experimenting, *when it is possible to change several of the conditions under which the experiment is made, it is of the utmost importance that only one of the conditions should be varied at one time.* The effect produced by the variation of one condition should be fully inquired into before any one of the other conditions is in any way altered, otherwise it will be generally quite impossible after-

wards to gather from the results what portion of the variation in the effect is produced by any particular change in the conditions.

87. Variation of Resistance with Temperature.— We have already said that the resistance of a wire

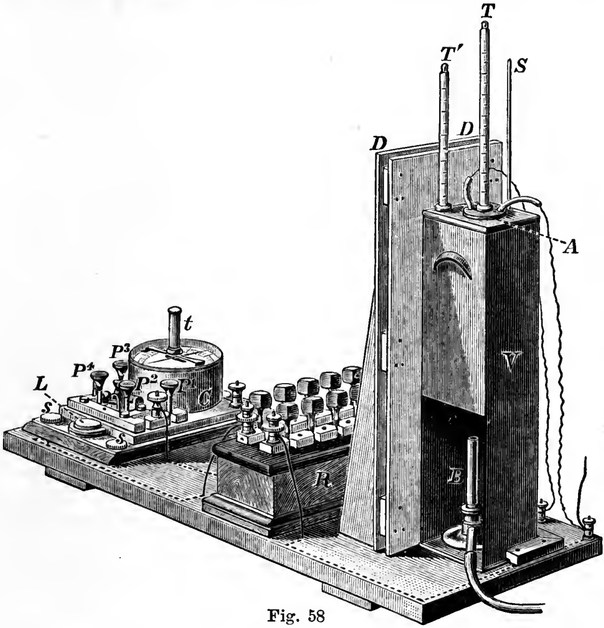

Fig. 58

depends on its temperature, and the apparatus shown in Fig. 58 is arranged especially for testing this. A coil of silk-covered iron wire is wound on a long, thin, hollow wooden bobbin, the top of which is seen at A. This bobbin is placed in a long thin glass tube, which itself is placed in water contained in the vessel V, the temperature of which can be raised by the Bunsen gas-burner B. S is the top of a piece of stout brass wire attached to a flat

piece of wood in the vessel V, and by means of which the water can be stirred up and its temperature made fairly uniform throughout. The temperature of the coil of wire is shown by the thermometer T, the bulb of which is inside the thin hollow wooden bobbin; but as even with this arrangement there may be a difference of temperature between the wire and the thermometer bulb, if the heating of the water is performed rapidly, it is better, before making a measurement of the resistance in the manner about to be described, to withdraw the Bunsen lamp, and wait a few minutes for the interior of the water-bath all to settle down to a uniform temperature, which is indicated by the two thermometers T inside the wooden bobbin, and T′ in the water-bath outside the bobbin indicating the same temperature. The double screen D D is for the purpose of preventing the heat radiated from the lamp warming the apparatus used for measuring the resistance, the action of which is based on the mode of measuring resistance shown in Fig. 52, page 137. From what was there said, it follows that if the currents flowing through A and B are equal, then the resistances of A and B are also equal. This equality of the currents might be ascertained from the deflections of two galvanometers placed in the circuits A and B, these deflections not being necessarily equal, but having values which the absolute calibration curves of the galvanometers show to correspond with equal currents.

This test could, however, more easily be made if, instead of using two separate galvanometers, a galvanometer were employed containing two distinct coils C, C′ (Fig. 59), one placed in the circuit A, and the other in the circuit B, and if the positions of these coils relatively to a suspended magnetic needle were so adjusted, that on equal currents passing through them their effects on this needle exactly balanced one another, so that the resultant deflection of the needle was nought. With such an arrangement a deflection nought of the needle would indicate that the resistances of the complete circuit A, including

that of the coil C, was equal to the resistance of B, including that of the coil C′. Further, if these coils not only had equal and opposite effects on the needle when equal currents were passing through them, but had also equal resistances, then a deflection nought of the needle would indicate not merely that the resistances of the circuits A and B, but also that the resistances of the remainders of the two circuits A and B, after excluding the resistances of the two coils C and C′, were also equal.

Hence, with the conditions of *equal magnetic effect and equal resistance* of the two coils C and C′, it follows that when there is no deflection of the galvanometer needle, the two wires, A and B, short or long, used to join the point P with the ends of the coils, have equal resistances.

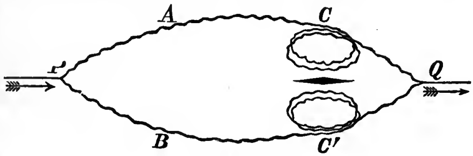

Fig. 59.

The instrument for measuring resistance, constructed on this principle, is called a "*differential galvanometer*," and such a galvanometer is seen to the left of Fig. 58.

In the apparatus shown in Fig. 58, these two wires, A and B of Fig. 59, are our experimental coil of iron wire in the water-bath, and the wire in the resistance box R, hence, as the resistance of the wire in the water-bath varies by being warmed, we can, by varying the resistance in R so as to always obtain no deflection of the needle of the differential galvanometer, measure the change of resistance produced by the variation of temperature.

88. Construction of a Differential Galvanometer.—The actual way in which the two conditions, *equality of magnetic effects, and equality of resistance* of the wires of the two coils of the differential galvanometer are fulfilled, is as follows:—Two reels of silk-covered copper wire are chosen, so that the diameter of the

wire on each is as nearly as possible the same, and the two wires are wound side by side on the galvanometer bobbin until it is nearly full; the wires are then tested and cut, so that the resistance, but not of course necessarily the length, of each wire is the same. A current is now sent in opposite directions through the two coils in series, when it will be found that, although the wires have been wound on side by side, one of them will have a greater magnetic effect than the other, partly perhaps because, being a trifle thicker, it has to be longer than the other, so as to have the same resistance, or partly because it is, on the whole, nearer the suspended needle than the other. To remedy this, a small portion of the wire having the greater magnetic effect is unwound, and without being cut, which would of course destroy the equality of the resistances of the two coils, the portion so unwound is coiled up out of the way in the base of the instrument. In this way, by unwinding more or less from the coil that was magnetically the more powerful, a very good balance can be obtained. In the use of differential galvanometers in which the needle is suspended by a silk fibre (as, for example, it is in Fig. 58, where the silk fibre is inside the tube *t*), a final and most delicate adjustment can be obtained by raising or lowering one of the levelling screws *s s* slightly, so as to tilt the needle nearer to or farther from one of the coils. And the spirit-level L should then be permanently adjusted so that the bubble is in the centre of the glass cover of the level, after the instrument has been tilted in the manner just described. The plugs P^3, P^4, seen in the figure, are for the purpose of enabling the two coils of this differential galvanometer, which is known as Latimer Clark's differential galvanometer, to be joined so as to oppose one another's effect, or to assist one another when it is desired to use the instrument as an ordinary galvanometer instead of a differential one, and the plugs P^1, P^2 are for the purpose of shunting either coil of the differential galvanometer (*see* § 107, page 185).

89. Construction of Plug Resistance Boxes.—The general construction of a resistance box was explained in § 12, page 28; but in the one shown in Fig. 58, the coils used to connect the various pieces of brass on the top of the box are not equal, but may conveniently have the following values going round them consecutively, starting from one of the binding screws:

$$0{\cdot}1,\ 0{\cdot}2,\ 0{\cdot}2,\ 0{\cdot}5,\ 1,\ 2,\ 4,\ 10,\ 20 \text{ ohms.}$$

There is also an "*infinity plug*," that is, two of the pieces of brass are not connected by a coil at all. Hence, if we take out the first and second plugs, the rest being left in, the resistance in the box will be $0{\cdot}1 + 0{\cdot}2$ or $0{\cdot}3$ ohms; if we take out the first and fourth, replacing the second, it will be $0{\cdot}1 + 0{\cdot}5$ or $0{\cdot}6$ ohms, &c. So that with the coils above-mentioned, any resistance between $0{\cdot}1$ and 38 ohms can be obtained with the nine coils. The brass plugs and the holes into which they fit are made *conical*, and the plugs should be well *ground into the holes* during manufacture. To prevent a resistance being introduced between the plug and the two pieces of brass on each side of it, a good contact is necessary, and to insure this, a plug, when put into the hole, should receive a slight screwing motion, when it will be found, with well-made plugs, that, although there is no screw thread on the plug, the whole resistance box can be easily lifted up by taking hold of one plug after it has properly been put in. Such closeness of contact it would be extremely difficult to secure by simply pressing down the plug, unless a large downward pressure were employed, and a corresponding tugging when taking it out, which would soon wrench off the ebonite head. The ebonite heads are usually screwed on to the tops of the brass plugs, but to prevent the head unscrewing in use, a pin should always be driven through the ebonite top and the head of the brass plug after they have been fitted together.

The holes in the figure, seen in the brass pieces themselves, are for the purpose of holding the plugs,

when they are not placed between the pieces of brass to short-circuit the intervening coil ; but the use of the holes in the brass pieces cannot be recommended, since, when the resistances corresponding with the holes that are unplugged are being rapidly counted, a plug stuck in one of the pieces of brass is liable to be mistaken for a plug between two pieces of brass, and hence coils which are actually in circuit are liable to be missed out in the counting up. Further, unless the pieces of brass are very large, the ebonite head of a plug stuck into one of them prevents the next plug being properly inserted, or removed, when the resistance of the next coil is to be subtracted from or added to the resistance in circuit.

90. Law of the Variation of Resistance with Temperature.—Experiments made with the apparatus seen in Fig. 58, show that the resistance of copper increases about 0·388 per cent. per 1° C., or 1 per cent. for a rise of temperature of 2°·57 C. This increase of resistance is not due simply to the wire becoming longer, for if the change of resistance were due merely to alteration of size, then, since the co-efficient of increase of length by temperature is the same as the co-efficient of increase of diameter, and as the resistance is directly proportional to the length, and inversely proportional to the square of the diameter, it follows that as far as mere size is concerned, increase of temperature should diminish the resistance. The fact, however, that the expansion of a metal by heating has the effect of separating all the particles of which the metal is composed from one another, may have something to do with the greater difficulty a current has in passing through a hot wire than through a cold one. But even this rough figurative explanation must be received with caution, since the resistance of a liquid which also expands in all directions with increase of temperature, diminishes as the temperature rises instead of increasing as is the case with metals.

Very careful experiments made on the increase of resistance of metals with temperature, show that the

increase, although roughly proportional, is not absolutely proportional to the increase of temperature, the resistance increasing in fact more rapidly than the temperature for all pure metals except mercury, so that the expression connecting resistance with temperature must contain a term, involving at least the square of the temperature. The actual result obtained by Dr. Matthiessen for most pure metals, excepting iron, is approximately

$$R = r\,(1 + 0{\cdot}003824\,t + 0{\cdot}00000126\,t^2),$$

where r is the resistance at 0° C., and R the resistance at any temperature t° C.

For mercury the formula is

$$R = r\,(1 + 0{\cdot}0007485\,t - 0{\cdot}000000398\,t^2);$$

for the gold-silver alloy in Table I.,

$$R = r\,(1 + 0{\cdot}0006999\,t - 0{\cdot}000000062\,t^2);$$

for German silver,

$$R = r\,(1 + 0{\cdot}0004433\,t + 0{\cdot}000000152\,t^2);$$

for the platinum-silver alloy in Table I.,

$$R = r\,(1 + 0{\cdot}00031\,t).$$

Carbon is an exception to the otherwise universal law, that the resistance of elementary substances, as distinguished from compounds, increases as the temperature rises. This fact is a reason for thinking that very possibly carbon is really a compound body.

91. Resistance of Metals per Cubic Centimetre and per Cubic Inch.—The following table, deduced from Dr. Matthiessen's results, and expressed in terms of the 1884 legal standard (see § 80, page 140), gives the value at 0° C. of the resistance in microhms, or millionths of an ohm, of a cubic centimetre and of a cubic inch, which means the resistance from one face to the opposite face across the cube.

TABLE No. I.

Chemically Pure Substances arranged in order of Increasing Resistance for the same Length and Sectional Area.

LEGAL MICROHMS.

Name of Metal.	Resistance in Microhms at 0° Centigrade.		Relative Resistance.
	Cubic Centimetre.	Cubic inch.	
Silver, annealed	1·504	0·5921	1
Copper, annealed	1·598	0·6292	1·063
Silver, hard drawn	1·634	0·6433	1·086
Copper, hard drawn	1·634	0·6433	1·086
Gold, annealed	2·058	0·8102	1·369
Gold, hard drawn	2·094	0·8247	1·393
Aluminium, annealed	2·912	1·147	1·935
Zinc, pressed	5·626	2·215	3·741
Platinum, annealed	9·057	3·565	6·022
Iron, annealed	9·716	3·825	6·460
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard, or annealed	10·87	4·281	7·228
Nickel, annealed	12·47	4·907	8·285
Tin, pressed	13·21	5·202	8·784
Lead, pressed	19·63	7·728	13·05
German silver, hard, or annealed	20·93	8·240	13·92
Platinum-silver alloy (1 oz. platinum, 2 oz. silver), hard, or annealed	24·39	9·603	16·21
Antimony, pressed	35·50	13·98	23·60
Mercury	94·32	37·15	62·73
Bismuth, pressed	131·2	51·65	87·23

From the preceding table we see that of the various metals, *annealed silver* is the one having the *least*, and *bismuth* the one having the *greatest*, resistance for a *given length* and *sectional area.*

The resistances of "commercial" metals are always higher than the values given in the preceding table, and the

difference is often very considerable. As copper can, however, now be easily obtained having as much as 95 per cent. of the "*conductivity*" of pure copper (which means that the resistance of a wire of commercial copper exceeds that of a wire of the same length and sectional area made of pure copper by not more than 5·3 per cent.), copper of less conducting power than this should not be bought for electrical purposes.

Conductivity is the reciprocal of resistance, so that if r_1 and r_2 be the resistances, and c_1 and c_2 the conductivities,

$$\frac{c_1}{c_2}=\frac{r_2}{r_1}.$$

From the preceding table, the resistance of a wire of any length and sectional area, at 0° C., can be easily found, by employing the formulæ given.

Example 26.—To find the resistance of a wire 52 metres long, 1 square millimetre in section at 22° C., made of pure copper, hard drawn.

$$\left.\begin{array}{c}\text{Resistance required}\\ \text{in ohms}\end{array}\right\}=\frac{1{\cdot}634}{10^6}\times\frac{52\times 100}{\frac{1}{100}}$$

$$\times\,(1+0{\cdot}003824\times 22+0{\cdot}00000126\times 22^2).$$

Answer.—0·9221 ohms.

Example 27.—To find the resistance of a wire 110 feet long $\frac{1}{20}$th of an inch in diameter at 46° C., made of pure annealed platinum.

$$\left.\begin{array}{c}\text{Resistance required}\\ \text{in ohms}\end{array}\right\}=\frac{3{\cdot}565}{10^6}\times\frac{110\times 12}{\frac{\pi}{4}\times\frac{1}{20^2}}$$

$$\times\,(1+0{\cdot}003824\times 46+0{\cdot}00000126\times 46^2).$$

Answer.—2·825 ohms.

Example 28.—At what temperature will a wire $3\frac{1}{2}$

miles long $\frac{1}{12}$th of a square inch in section, made of German silver, have a resistance of 22·23 ohms?

$$22{\cdot}23 = \frac{8{\cdot}240}{10^6} \times \frac{3{\cdot}5 \times 5280 \times 12}{\frac{1}{12}}$$

$$\times (1 + 0{\cdot}000443\, t + 0{\cdot}000000152\; t^2).$$

Solving this quadratic equation for t, we find t equals 37°·5 C.

Example 29.—If the resistance of a sample of commercial metal is 97·5 ohms, whereas the resistance of the same piece of metal, if quite pure, would be 94·3 ohms at the same temperature, what is its percentage conductivity in terms of that of the pure metal?

The conductivity of the sample of commercial metal $\Big\} = \frac{1}{97{\cdot}5}$

The conductivity of the same if pure would $\Big\} = \frac{1}{94{\cdot}3}$;

∴ if x be the percentage conductivity,

$$\frac{1}{97{\cdot}5} = \frac{x}{100} \times \frac{1}{94{\cdot}3},$$

$$\therefore\ x = 96{\cdot}72.$$

Answer.—96·72 per cent. conductivity.

92.—Resistance of Metals for a given Length and Diameter, or for a given Length and Weight.—It is frequently convenient to know, not merely the resistance of a cubic centimetre, or of a cubic inch, but of a wire of a given length and diameter, or of a given length and weight. The following numbers, giving the resistance at 0° C. of pure substances, are deduced from Dr. Matthiessen's experiments, and are expressed in terms of the 1884 legal ohm. The substances are arranged in order of increasing resistance for the same length and weight, the order for increasing resistance for the same length and sectional area being that given in Table No. I., page 154.

TABLE No. II.

Chemically Pure Substances at 0° Centigrade, arranged in order of Increasing Resistance for the same Length and Weight.

LEGAL OHMS.

Name of Metals arranged in order of increasing resistance for the same length and weight.	Resistance of a wire 1 foot long, weighing 1 grain.	Resistance of a wire 1 foot long, $\frac{1}{1000}$th of an inch in diameter.	Resistance of a wire 1 metre long, weighing 1 gramme.	Resistance of a wire 1 metre long, 1 millimetre in diameter.
	Ohms.	Ohms.	Ohms.	Ohms.
Aluminium, annealed	0·1074	17·53	0·0749	0·03710
Copper, annealed . .	0·2041	9·612	0·1424	0·02034
Copper, hard drawn .	0·2083	9·831	0·1453	0·02081
Silver, annealed . .	0·2190	9·048	0·1527	0·01916
Silver, hard drawn .	0·2389	9·826	0·1662	0·02080
Zinc, pressed . . .	0·5766	33·85	0·4023	0·07163
Gold, annealed . . .	0·5785	12·38	0·4035	0·02620
Gold, hard drawn . .	0·5884	12·60	0·4104	0·02668
Iron, annealed . . .	1·085	58·45	0·7570	0·1237
Tin, pressed	1·380	79·47	0·9632	0·1682
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard, or annealed .	2·364	65·37	1·650	0·1384
German silver, hard, or annealed . . .	2·622	125·91	1·830	0·2666
Platinum, annealed .	2·779	54·49	1·938	0·1153
Lead, pressed . . .	3·200		2·232	0·2498
Antimony, pressed .	3·418	213·6	2·384	0·4521
Platinum-silver (1 oz. platinum, 2 oz. silver), hard, or annealed	4·197	146·70	2·924	0·3106
Bismuth, pressed . .	18·44	789·3	12·88	1·670
Mercury	18·51	572·3	12·91	1·211

From this we see that of the metals *aluminium* has the *least* resistance for a *given length* and *weight*, and *mercury* the *greatest;* whereas we saw from Table No. I, page 154, that for a *given length* and *sectional area* it was *annealed silver* that had the *least* resistance, and *bismuth* the *greatest.*

Example 30.—What will be the weight of an iron wire 100 yards long, having a resistance of 1 ohm at 0° C.?

An iron wire 1 ft. long weighing 1 grain has 1·085 ohms at 0° C., therefore an iron wire x ft. long weighing x grs. has $x \times 1{\cdot}085$ ohms at 0° C. Hence an iron wire x ft. long weighing y grs. has $\frac{x^2}{y} \times 1{\cdot}085$ ohms at 0° C.

In the question x is 300, and the resistance is 1 ohm. Therefore

$$\frac{300^2}{y} \times 1{\cdot}085 = 1;$$

$$\therefore\ y = 300^2 \times 1{\cdot}085 \text{ grs.}$$

Answer.—13 lbs. 15 oz.

Example 31.—What will be the length of a platinum wire weighing 2·8 grains, and having a resistance of 0·7891 ohms at 250° C.? *Answer.*—$7\frac{1}{2}$ inches.

Example 32.—Which has the greater resistance, a copper wire 20 feet long 0·015 inch in diameter, or a platinum-silver wire 10 feet long 0·037 inch in diameter, at 0° C.?

The resistance of the copper wire will be to that of the platinum as $\frac{20 \times 9{\cdot}612}{{\cdot}015^2}$ is to $\frac{10 \times 146{\cdot}7}{{\cdot}037^2}$, and as this ratio is 0·7973, it follows that the former has rather more than three-quarters of the resistance of the latter.

Example 33.—What will be the resistance, at 95° C., of a copper wire 20 metres long weighing 12 grammes, and having 92 per cent. of the conductivity of pure copper? *Answer.*—7·092 ohms.

93. Comparison of Electric and Heat Conductivities.—The reciprocals of the numbers given in column 4 of Table No. I. will express the relative electric conductivities of the metals for the same length and sectional area. These numbers are given in column 2 of

Table No. III. On comparing these with the conductivities of the metals for heat for the same length and sectional area as given in column 3 of Table No. III., and which are the numbers obtained by Wiedemann and Franz, we observe that the metals arrange themselves *approximately*, but not absolutely, in the same order for the two conductivities.

TABLE No. III.

Relative Conductivities per Cubic Unit.

Name of Metal.	Electric.	Heat.
Silver, annealed	100	100
Copper ,, ...	94·1	74·8
Gold ,, ...	73	54·8
Platinum	16·6	9·4
Iron	15·5	10·1
Tin, pressed	11·4	15·4
Lead	7·6	7·9
Bismuth	1·1	1·8

As we experiment with worse and worse conductors, we find that the electric conductivity diminishes much more rapidly than the heat conductivity. For example, the electric conductivity of copper is about 10^{20} times the conductivity of vulcanised indiarubber, whereas the heat conductivity of copper is only about 10^{4} times that of vulcanised indiarubber. *Hence, while we can obtain insulators for electricity, or bodies which relatively to the metals do not practically conduct electricity at all, insulators for heat are unknown.*

94. Material Used in Resistance Coils.—We see then that it is not merely sufficient to know the length and diameter of a wire as well as the material of which it is made, *but we must know also the temperature of the wire if we wish to be sure about its resistance.* Fixity of length, diameter, and material, are easy enough to obtain, but constancy of temperature it is much more difficult to secure, partly on account of changes of temperature of the room, and partly on account of the slight heating of

a coil of wire produced by a current passing through it. Consequently, in the construction of resistance coils it is important to use a metal of which the resistance changes as little as possible with temperature, and which is not too costly. To ascertain what that metal was, Dr. Matthiessen, in 1862 and 1863—that is, in the early days of resistance coils—made, on behalf of the Electrical Standards Committee of the British Association, a large number of very accurate experiments on the change of resistance with temperature, and a few of his results are contained in the following table.

TABLE No. IV.

Approximate Percentage Variation in Resistance per 1° C. at about 20° C.

Platinum-silver alloy (1 oz. platinum, 2 oz. silver), hard, or annealed	0·031
German silver, hard, or annealed	0·044
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard, or annealed	0·065
Mercury	0·072
Bismuth, pressed	0·354
Gold, annealed; Zinc, pressed; Tin, pressed	0·365
Silver, annealed	0·377
Lead, pressed	0·387
Copper, annealed	0·388
Antimony	0·389
Iron	about 0·5

From this we see that, whereas (of the substances experimented on by Dr. Matthiessen) an *alloy of platinum-silver,* hard or annealed, is the one of which the resistance changes *least* by temperature, *German silver,* which is a very much cheaper alloy, is nearly as good in this respect. *Hence, nearly all resistance coils are made of German silver,* except when greater lightness and portability are required, in which case the alloy of one part of platinum and two of silver by weight is employed.

A new alloy, called "*platinoid,*" consisting of German

silver, with one or two per cent. of metallic tungsten added, has been recently found by Mr. J. Bottomley to have a resistance per cubic centimetre of about 34 microhms, or about 60 per cent. higher than that possessed

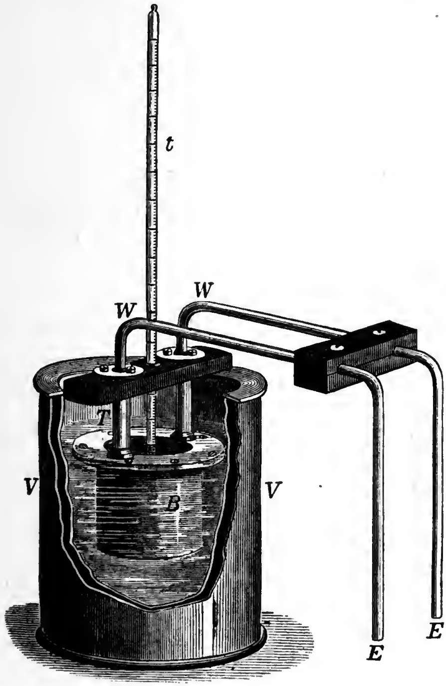

Fig. 60.

by German silver; and, what is still more important, its percentage variation of resistance per 1° C. is only about 0·021, or less than half that of German silver. We may, therefore, expect that platinoid will supersede both German silver and platinum-silver for resistance coils, if

its resistance be found to be equally unchanged by lapse of time.

Iron, we see, is the worst of the substances shown in the table to be used in the construction of resistance coils, as far as the temperature error is concerned; but it is not unfrequently used when cheap resistance coils are required for large currents, and when, as sometimes is the case, great constancy of resistance is not necessary.

The resistance coil, when used as an *accurate standard*, is wound inside a brass box B, shown in Fig. 60, so that it may be inserted in a vessel of water V V, and its temperature accurately noted by means of the thermometer *t*. The brass box B for holding the coil is made cylindrical inside and outside, with a large diameter and small thickness, so as to expose as much surface as possible to the water, in order that the coil inside may acquire the temperature of the water as quickly as possible; and the vessel V V containing the water may with advantage have double sides, with an air-space between them, as seen in the figure, to prevent transference of heat between the water and outside space.

The tubes T T are to prevent the coils being short circuited by water getting through the holes, by which the rods W W attached to the ends of the resistance coil are brought out. These tubes are made of brass, but they are lined with tubes of ebonite to prevent electric contact between these brass tubes and the rods W W. Electric connection with these rods is made by dipping their ends E E into little wooden cups containing mercury.

Example 34.—At what temperature, approximately, would a German silver coil, which had one British Association unit of resistance at 16° C., have the resistance of one legal ohm?

$$1 \text{ legal ohm} = 1{\cdot}0112 \text{ B.A. units},$$

therefore the temperature must be raised sufficiently to increase the resistance of the coil by 1·12 per cent. Therefore, since the resistance of German silver increases

0·044 per cent. per degree, as stated in the last table, if t be the temperature above 16° to which the coil must be raised,

$$0{\cdot}044 \times t = 1{\cdot}12,$$
$$\text{or } t = 25^\circ{\cdot}5 \text{ approximately.}$$

Answer.—The B.A. coil will have a resistance of one legal ohm at 41°·5 C.

Example 35.—A set of resistance coils made of platinum-silver are correct at 14° C. Between what limits of temperature approximately may they be used without correcting the results, if the temperature error is not to exceed $\frac{1}{4}$ per cent.?

The resistance of platinum-silver increases about 0·031 per cent. per 1° C., as stated in the last table; therefore, if t be the number of degrees above or below 14° C., within which the coils may be used without the error exceeding $\frac{1}{4}$ per cent.,

$$0{\cdot}031 \times t = 0{\cdot}25,$$
$$\therefore t = 8^\circ.$$

Answer.—The limits of temperature are approximately 6° and 22° C.

Example 36.—If the greatest change of temperature at some particular place between summer and winter is from −8° to 25° C. in the shade, what is the greatest percentage variation in the resistance of a set of German silver coils? *Answer.*—1·45 per cent. approximately.

Example 37.—At what temperature would a metre of mercury one square millimetre in section have one ohm resistance? *Answer.*—83°·3 C.

95. Mode of Winding Resistance Coils.—Not only must a special metal be employed in making resistance coils, but the wire must not be wound on the bobbin in the ordinary way. If it were wound on the bobbin as cotton is on a reel, then each bobbin in a resistance box would act as a magnet when a current passed through

it, and a box full of electro-magnets would be a most inconvenient thing to have near a delicate galvanometer used in testing resistances, since one would be constantly in doubt as to whether the deflection observed on putting on the current was due to want of adjustment in the resistance, or to the temporary magnetisation of the adjacent resistance box. Hence, the wire of a resistance coil is wound back on itself as shown in Fig. 7, page 28, so that the current, in passing through the wire, first goes several times round the bobbin in one direction, and then an equal number of times in the opposite direction, and the two magnetic effects neutralise one another.

The disturbing magnetic effect that might otherwise have arisen when using resistance coils, is overcome by this double mode of winding; but the magnetic action of a current passing round an ordinary reel of wire, or a coil wound for a galvanometer or for an electromagnet, &c., must be carefully taken into consideration when anything of this form has to be tested for resistance. As such coils are frequently wound before being tested, they must, when it is desired to test them, be placed so far away from the galvanometer that the mere passage of the current round the coil produces by itself no deflection of the galvanometer needle, when no current is allowed to pass through the galvanometer.

96. Calibrating a Galvanometer by Using Known Resistances.—From Ohm's law (§ 74, page 130), it follows that the current passing through any circuit is inversely proportional to its resistance if a constant potential difference be maintained at the ends of the circuit. Consequently if a constant potential difference be maintained at the terminals T T (one only of which is seen in Fig. 61) of the circuit, consisting of the key K, the detector D, and the resistance box R, the current passing through the detector will be inversely proportional to the sum of the resistances of the key, detector, and resistance box. Such a constant potential difference can be maintained, as will be seen in § 139, page 261, by attaching to the terminals T T

an accumulator or any galvanic cell, the resistance of which is small compared with the rest of the resistance in the circuit.

To perform the calibration, it is, perhaps, best to first employ such a resistance in the box R that the deflection on the detector is about $10°$; let this be r_1, and let the

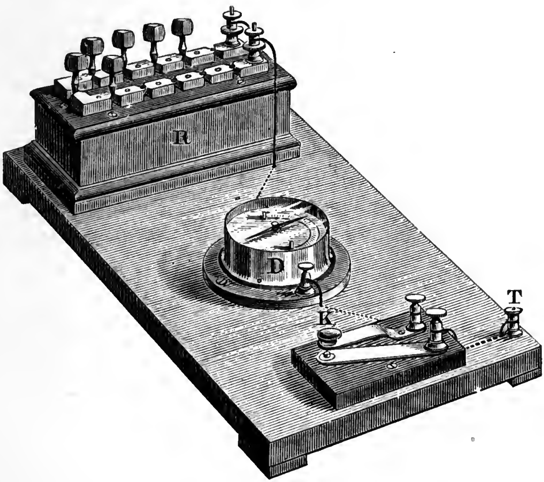

Fig. 61.

galvanometer resistance be g, and let the deflection be $d°_1$. Next employ a resistance r_2, such that

$$r_2 + g = \tfrac{1}{2}(r_1 + g),$$
$$\text{or} \quad r_2 = \tfrac{1}{2} r_1 - \tfrac{1}{2} g,$$

then the current will be doubled since the resistance of the key K is practically nought, *if the platinum contact points be cleaned by inserting a piece of paper between them, then pressing them together, and pulling out the paper with the points pressed together.* (Emery paper should not be used as it rubs away the platinum, and

still less should the contacts be scraped with a knife or a file.) Let the deflection, with this value of r_2, be $d°_2$. Next employ a resistance r_3, such that

$$r_3 + g = \tfrac{1}{3}(r_1 + g),$$
$$\text{or} \quad r_3 = \tfrac{1}{3}\, r_1 - \tfrac{2}{3}\, g,$$

then the current will be trebled. Let this produce a deflection of $d_3°$, &c. In this way a series of deflections will be obtained, corresponding with currents proportional to 1, 2, 3, 4, &c., and a relative calibration curve can be drawn in the way already described.

The Wheatstone Bridge.

97. **Wheatstone's Bridge.**—The differential galvanometer, in its simple form, is a very convenient apparatus

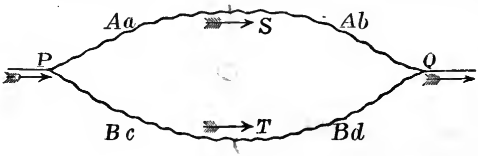

Fig. 62.

for testing the equality of two resistances, but there is a still better method for accurately and rapidly comparing any two resistances, which was originally devised by Mr. Christie, and brought into public notice by the late Sir Charles Wheatstone, and hence has been called a "*Wheatstone's bridge*," or a "*Wheatstone's balance.*"

The principle of the Wheatstone's bridge is seen from Fig. 62, and is as follows:—In passing from P to Q, either along the wire P S Q, or along P T Q, there are points having all potentials between the potential of P and that of Q, therefore it follows that for every point in the circuit P S Q, there must be a point on the circuit P T Q, having the same potential. Let S and T be two such points; then, if they were joined with a galvanometer, no current

would flow through it, or if joined to the opposite quarter cylinders of the electrometer described in § 75, page 130, there would be no deflection. Let A be the current flowing along P S, and which also must be the current flowing along S Q, since no current passes through the galvanometer, and B the current flowing along P T Q, and let a, b, c, d be the resistances respectively of P S, S Q, P T, T Q; then, since the potential difference between P and S is the same as the potential difference between P and T,

$$A\,a = B\,c.$$

Similarly, since the potential difference between S and Q is the same as the potential difference between T and Q,

$$A\,b = B\,d.$$

Therefore, combining these two equations, we have

$$\frac{a}{b} = \frac{c}{d},$$

which is the law of the Wheatstone's bridge.

The last equation may be written in the form

$$\frac{a}{c} = \frac{b}{d},$$

and this is the equation that we should have obtained for no current through the galvanometer, had its terminals joined P and Q, and the current generator been placed between S and T. Hence *when balance is obtained with a Wheatstone's bridge, the balance will not be disturbed by interchanging the galvanometer and battery.*

In order, then, to tell the value of one of the resistances, say a, by the Wheatstone's bridge method, we must know the value of either of the adjacent ones, say b, in ohms, and the ratio only of the other two, say c and d. Hence one mode of using the bridge to measure the resistance of a is to keep the ratio of c to d constant, and simply vary the resistance of b until no current passes through the galvanometer. Another method consists in keeping b

constant, and varying the ratio of c to d. For example, the resistances c and d may be the resistances of different lengths of the same kind of wire, in which case we know that c will be to d simply as the ratio of these lengths, whatever be the absolute resistance in ohms of the two parts. A form of Wheatstone's bridge, in which P T Q, of Fig. 62, was one piece of stretched wire, and the ratio of c to d varied by moving the connection of the wire leading to one terminal of the galvanometer, was originally employed by the Electrical Committee of the British Association, and is, for this reason, sometimes called the British Association bridge; at other times, the "*metre bridge*," from the stretched wire being a metre long. The wire may be made of platinum, or better still, of platinum-iridium, which, being very hard, prevents the wire being worn at any part.

A convenient form of metre bridge is shown in Fig. 63. It has three stretched wires $w\,w$, each a metre in length, and so arranged that either one of them alone, or two of them in series, or all three in series, can be made use of to form the two sides c and d of the Wheatstone's bridge (Fig. 62). When the plug E is, as in the figure, placed in the hole H, the current simply passes through the stretched wire which is nearest to the observer. If on the other hand the plug E be put in the hole h, then, since the brass plate P is permanently connected with the plate p by a thick copper strip under the base of the instrument, the middle stretched wire is short-circuited, and the wire nearest to the observer is in series with the one farthest from him. Lastly, if the plug be removed altogether the three wires are in series.

The object of thus lengthening the wire is to increase the sensibility of the test when desired, and a still further increase in the sensibility can be effected by removing the short-circuit pieces S_1 S_2, and inserting coils of known resistance in place of them. For example, suppose that the ratio of the unknown to the known resistance be $\frac{3}{2}$, then the slide K must be placed so as to divide the stretched

wire into two parts having this ratio. Hence, if one of the three wires only be used, the lengths of the two parts

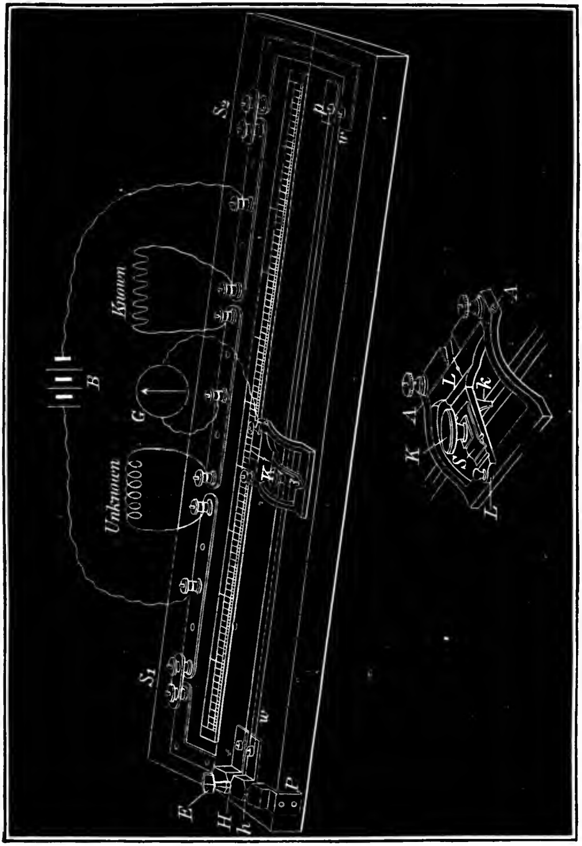

Fig. 63.

which will give exact balance will be 60 and 40 centimetres, and an error of 1 centimetre in the position of

the slider will correspond with an error in the determination of the ratio of

$$\frac{\frac{61}{39}-\frac{60}{40}}{1\cdot5}\times 100 \text{ per cent., or } 4 \text{ per cent.}$$

If, on the other hand, the three wires in series be employed, then the lengths into which the three metres of wire must be divided to obtain exact balance will be 180 and 120 centimetres, and an error of one centimetre in the position of the slider will correspond with an error in the determination of the ratio of

$$\frac{\frac{181}{119}-\frac{180}{120}}{1\cdot5}\times 100 \text{ per cent., or } 1\cdot4 \text{ per cent.}$$

If now two coils, each having a resistance equal to, say, 1,000 centimetres of the stretched wire be inserted in place of the short circuit pieces S_1 and S_2, an error of a centimetre in the position of the slider will only correspond with an error of

$$\frac{\frac{1381}{919}-\frac{1380}{920}}{1\cdot5}\times 100 \text{ per cent., or } 0\cdot18 \text{ per cent.}$$

Contact between the platinum-tipped knife-edge *k* and one or other of the stretched wires, is produced by depressing the knob K, which causes the lever to which this knife-edge is attached to turn on an axis A A. On removing the pressure, the lever is pressed up by a spring underneath it; and the slider should never be moved with the knife-edge *k* depressed, as this would scrape the stretched wire and alter its diameter. In order to enable *k* to make contact with either the first, second, or third wire, the knob K is not fastened rigidly to the lever, but can slide along it in a slot, and can be so placed that the near end of the spring S rests in either of the three

grooves on the top of the lever corresponding with the three positions of k when it is in contact with the three stretched wires respectively.

98. Superiority of the Wheatstone's Bridge over the Differential Galvanometer, and Conditions affecting the Sensibility of the Bridge.—*The Wheatstone's bridge is superior to the differential galvanometer*, in that not merely can two resistances be ascertained to be equal to one another, but the value of any resistance in terms of another can be exactly measured, so that if we possess one single resistance the value of which is known exactly in ohms, we can, without knowing the resistance of any other wire, measure, by means of the metre bridge, the value in ohms and fractions of an ohm of any unknown resistance.

Practically, however, the sensibility of the bridge is limited by the galvanometer not being sensitive enough to indicate the small current that passes through it when the ratio of a to b is not quite equal to that of c to d (Fig. 62, page 166), and when both ratios are far from unity. In fact it can be shown that *the bridge is most sensitive when all the four resistances, a, b, c, d, are equal to one another.* If, however, it is impossible to make them equal, then it is desirable to consider whether the galvanometer or the battery (*see* § 129, page 226) have the higher resistance, because *greater sensibility will be obtained by using the one that has the higher resistance to connect the junction of the two greater of a, b, c, d, with the junction of the two less*, than if the galvanometer and battery be joined up in the opposite way. For example, if

$$\begin{aligned} a &= 1 \text{ ohm} \\ b &= 100 \text{ ohms} \\ c &= 4 \text{ ohms} \\ d &= 400 \text{ ohms}, \end{aligned}$$

and the resistances of the galvanometer and battery be 37 ohms and 5 ohms respectively, one terminal of the galvanometer ought to be connected with the junction of a and c, and the other with the junction of b and d. (*See* also § 238, page 467.)

Further, it is important to consider whether we should select a galvanometer wound with fine wire or one wound with thick wire, in order to obtain the most accurate measurements with a Wheatstone's bridge. Calculation and experiment show that if *nothing* but the gauge of wire used in winding the bobbins of the galvanometer be varied, that is to say, if the bobbins and the space on them occupied by the covered wire remain the same, as well as

the strength and direction of the controlling field and the suspension of the galvanometer, then with a given testing battery, and with given values of the four "*arms*" of the bridge, a, b, c, d, the greatest deflection will be produced on a galvanometer on making a definite change in one of the four *arms*, say a, if the wire wound on the galvanometer bobbin be such that *the resistance of the galvanometer equals the product of the sum of the resistances of the two arms on one side of it into the sum of the resistances of the two arms on the other side of it, divided by the sum of the resistances of the four arms.* For example, if the galvanometer connect the junction of a and c with the junction of b and d, the wire used in winding the galvanometer bobbins ought to be selected of such a thickness that the galvanometer when wound has a resistance of

$$\frac{(a+b)\,(c+d)}{a+b+c+d}.$$

Of course this does not mean that a roughly-made pivot galvanometer having this resistance will give better results than a delicate fibre-suspended reflecting galvanometer with a much greater or a much less resistance. The formula can only be used on the assumption that *nothing but the gauge of wire employed in winding the galvanometer can be varied.* (*See* § 237, page 466.)

99. Commercial Form of Wheatstone's Bridge.—In the Wheatstone bridges, as commonly constructed, the resistances of all three branches are made up of coils, the values of which are known in ohms, and the apparatus is frequently made of the form shown in Fig. 64, where the c and d of Fig. 62 are each replaced by three coils of 10, 100, and 1,000 ohms, called the "*proportional coils*," and the b of Fig. 62 is made up of the following coils, 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, 1,000, 1,000, 2,000, 5,000. With these latter sixteen coils, any integral resistance between 1 and 10,000 may be formed, and this special arrangement, although not requiring the least number of coils to enable any resistance between 1 and 10,000 to be obtained, is found in practice to be the most convenient. With this bridge, then, we can measure any resistance between $\frac{10}{1110} \times 1$, or $\frac{1}{111}$th of an ohm, and $\frac{1110}{10} \times 10{,}000$, or one million one hundred and ten thousand ohms.

In Fig. 64, the battery seen at the left-hand side is indicated symbolically by three thin lines, which stand

for the copper plates, and by three shorter and thicker lines, which stand for the zinc plates or rods. The cells are understood to be coupled by the zinc plate, or rod, of the upper cell being joined to the copper plate of the second, and the zinc plate of the second to the copper plate of the third; so that the six lines in Fig. 64 are a symbolical representation of the battery shown in the next figure (Fig. 65). This symbolical representation,

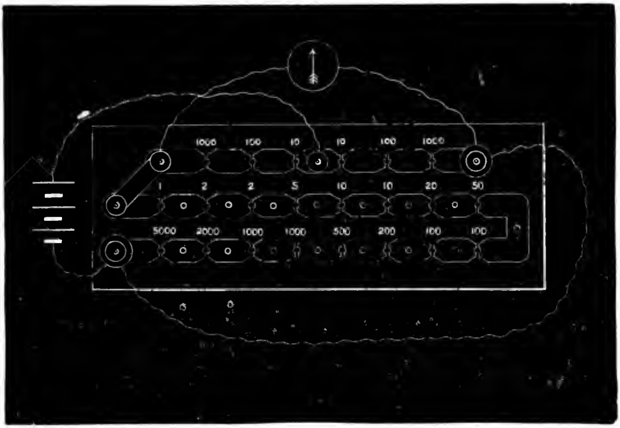

Fig. 64.

which is commonly used to stand for a battery, will be employed in the rest of this book, and will be found still further explained in § 135, page 240.

The resistance coils sold in boxes are always made so that the resistance of each is an exact number of ohms or certain special fraction of an ohm at the same temperature, which is specified on the box, and the trouble of adjusting a number of coils to fulfil this condition causes resistance boxes to be rather costly. It is undoubtedly more convenient that the resistance of each coil should be an exact number of ohms or a certain

special fraction, but it would be far cheaper if the coils were made approximately to have the resistance 1, 2, 5 ohms, &c., and their actual resistances in ohms and fractions of an ohm, when tested at some one temperature, were marked on the box.

100. Bridge Key.—In using a Wheatstone's bridge it is desirable to send the current through the four arms of the bridge *a, b, c, d* (Fig. 62), before it is allowed to pass through the galvanometer, and this is especially important when testing the resistance of the copper conductor of a long submarine cable, since the current in such a case takes an appreciable time to reach its maximum value and become steady, due to the cable acting as a "*condenser*" (*see* § 162, page 301). Hence, if the galvanometer circuit were completed when the battery was attached to the bridge, an instantaneous swing of the galvanometer would be produced, even if *a* bore to *b* the ratio of *c* to *d*. And although, since the ratio of resistances having been effected, the deflection of the galvanometer would become nought as soon as the current in the four branches of the bridge became steady, great delay in the testing would be caused by this first swing of the needle. A similar difficulty would occur in measuring the resistance of an electromagnet or even of any coil without an iron core, if it were not wound doubly as are the coils in resistance boxes (*see* Fig. 7, page 28); because whenever a coil is so wound that a current passing through it produces magnetic action, a short interval of time has to elapse, after putting on the battery, before the current reaches its maximum, or steady, value, arising from what is called the "*self-induction*" of the coil.

A key for sending the current through the four arms of the bridge before it is allowed to pass through the galvanometer, is shown at K (Fig. 65), and is a modification of the one originally employed by the Electrical Committee of the British Association. On pressing down the button, contact is first made between the flexible piece of brass A and the flexible piece of brass B.

This completes the battery circuit, and causes the current to flow through the four arms of the bridge shown symbolically in Fig. 65 by the spiral lines. On the button being still further pressed down, B is brought into contact with a little knob of ebonite E on the top of the flexible piece of brass C. This does not complete

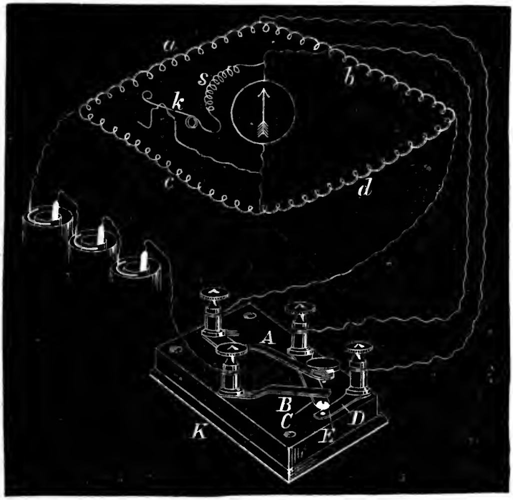

Fig. 65.

any other electric circuit; but on the button being still further depressed, C is brought into contact with D, and the galvanometer circuit is completed.

This form of key is to be preferred to the ordinary bridge key, because all the connections are above the base of the key and in sight, whereas when the connections are made under the base, it frequently happens that the pieces of guttapercha-covered wire used to make the connections are either badly insulated, or are loosely connected at their ends with the terminals of the key, and so introduce unnecessary resistance.

101. Use of a Shunt with the Bridge.—It is desirable to employ also another key *k* (Fig. 65), which may be quite simply made of a twisted bit of hard brass wire, bent so as to press up against a sort of bridge of hard brass wire, since the resistance at the contact is in this case of no consequence. When the key is not depressed, a portion of the current is shunted past the galvanometer through any convenient shunt *s*, the resistance of which need not be known, as it does not enter into the calculations. The object of this shunt is merely to diminish the sensibility of the galvanometer when the first approximation is being made to the value of the unknown resistance. As soon as this has been done the key *k* should be depressed, and all the current in the galvanometer circuit arising from want of perfect balance allowed to pass through the galvanometer itself, and the resistances adjusted until perfect balance is obtained. Another device to expedite the testing, and also to prevent powerful currents being sent through the galvanometer, consists in not holding the key K down when the first rough approximation is being made, but merely giving it a tap, which has the effect, when the balance is far from perfect, of giving the needle of the galvanometer a slight impulse to one side or the other, according as the ratio of *a* to *b* is larger or smaller than that of *c* to *d*, instead of causing the needle to violently swing against the stops on one side or the other as it would do if the key K were held down before balance was arrived at.

102. Meaning of the Deflection on a Bridge Galvanometer.—A considerable amount of time will be saved in testing if the meaning of a deflection of the galvanometer needle, say to the right, be once for all definitely ascertained, and a note be made whether it means that the ratio of *a* to *b* is too large or too small. The simplest way of recording this, if we assume, for example, *a* to be the unknown resistance, is to put the words "*increase b*" and "*diminish b*" one on each side of the galvanometer, these being the directions to be followed

according as the needle deflects towards one or other of them. The position of these two directions must, of course, be reversed if the terminals of the testing battery be reversed.

SHUNTS.

103. Shunts.—We have already seen, for example, in the apparatus shown in Fig. 17, page 59, and again when using a Wheatstone's bridge (§ 101, page 176), that it is sometimes convenient to use a wire as a by-path or shunt to convey a portion of the current, the remainder only passing through the galvanometer. We will now consider what must be the relative resistances of the shunt and galvanometer to allow any particular fraction of the whole current to pass through the galvanometer. Let s, g be the resistances in ohms of the shunt and galvanometer, and S, G the currents in amperes passing through them respectively; then, if V be the potential difference in volts at the terminals of the shunt and galvanometer, it follows from Ohm's law (§ 74, page 130) that

$$S = \frac{V}{s},$$

$$G = \frac{V}{g},$$

$$\therefore \qquad \frac{G}{S} = \frac{s}{g};$$

or the current strengths in the galvanometer and shunt are *inversely* as their resistances.

Also, by a well-known rule in proportion, it follows that

$$\frac{G}{S + G} = \frac{s}{s + g},$$

and

$$\frac{S}{S + G} = \frac{g}{s + g};$$

but S + G is the sum of the currents flowing through the

shunt and the galvanometer respectively, and therefore is equal to the whole current in the circuit, A amperes say, hence

$$\frac{G}{A} = \frac{s}{s+g},$$

and

$$\frac{S}{A} = \frac{g}{s+g}.$$

104. Multiplying Power of a Shunt.—

Since

$$A = \frac{s+g}{s} \times G,$$

the fraction $\frac{s+g}{s}$ is frequently called the "*multiplying power of the shunt*," that is, the quantity that the current flowing through the galvanometer must be multiplied by to obtain the total current.

As an example of the last equation, let us suppose that we desire that G shall be one-tenth of A, then

$$\frac{s}{s+g} = \frac{1}{10},$$

$$\text{or } s = \frac{1}{9}g;$$

or, again, if we wish that G shall be one-thousandth of A, then

$$\frac{s}{s+g} = \frac{1}{1000},$$

$$\text{or } s = \frac{1}{999}g.$$

105. Combined Resistance.—It would be, of course, possible to substitute for the two resistances s and g, which are in parallel, a single wire of resistance x such that *for the same potential difference*, V, *at its terminals*, the current flowing through it should be equal to the sum of the currents flowing through the two parallel circuits

To find x we have

the current that would flow through it $= \frac{V}{x}$,

the current flowing through s . . $= \frac{V}{s}$,

the current flowing through g . . $= \frac{V}{g}$,

$$\therefore \text{ since } \frac{V}{x} = \frac{V}{s} + \frac{V}{g},$$

$$x = \frac{sg}{s+g};$$

or if two wires be in parallel, then the product of their resistances divided by their sum represents the resistance of a single wire through which a current will pass, equal to the sum of the currents passing through the two wires, *for the same potential difference.* Such a single resistance is called the "*combined resistance,*" or the "*parallel resistance,*" of the two.

From what has preceded we see that when G is a tenth of A,

$$\frac{sg}{s+g} = \frac{1}{10}g,$$

or the combined resistance of the shunt and galvanometer is one-tenth of the resistance of the galvanometer.

In the same way, if there be any number of resistances a, b, c, d, &c., in parallel, and x be a single resistance, such that *with the same potential difference* at its terminals the current that will flow through x is equal to the sum of the currents that flow through all the resistances a, b, c, d, &c., the *combined resistance*

$$x = \frac{1}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \text{\&c.}}$$

If A, B, C, D, &c., be the currents flowing through the various circuits, and X be the total current, then

$$\frac{A}{X} = \frac{\frac{1}{a}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \text{\&c.}},$$

$$\frac{B}{X} = \frac{\frac{1}{b}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \text{\&c.}},$$

&c.

Example 38.—What must be the resistance of a shunt so that $\frac{4}{5}$ of the whole current shall pass through a galvanometer having 452 ohms' resistance?

Here $$\frac{s}{s+g} = \frac{4}{5},$$

$$\therefore \quad s = 4\,g.$$

Answer.—1,808 ohms.

Example 39.—If the resistance of a shunt be 1 ohm, and that of the galvanometer 2 ohms, what fraction of the total current passes through the galvanometer and what through the shunt?

We have $$\frac{G}{A} = \frac{s}{s+g},$$

therefore, substituting the values given,

$$\frac{G}{A} = \frac{1}{3}.$$

Answer.—One-third of the current passes through the galvanometer, and two-thirds through the shunt.

Example 40.—If a galvanometer have 1,980 ohms' resistance, and a shunt be attached so that the current passing through the galvanometer is only $\frac{1}{100}$th of the

total current, what will be the resistance of the shunt, and by how many ohms will the resistance of the circuit be diminished by employing the shunt?

Here $$\frac{s}{s+g} = \frac{1}{100},$$

$$\therefore \quad s = \frac{1}{99} g,$$

or, in this case, = 20 ohms;

$$\text{and } \frac{s\,g}{s+g} = 19{\cdot}8 \text{ ohms};$$

$\therefore$ the diminution of the resistance of the circuit produced by applying the shunt is 1,980 − 19·8, or 1,960·2 ohms.

106. Construction of a Shunt Box.—The three coils, having respectively the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of the resistance of the galvanometer, are usually inserted in a small box *b* (Fig. 66), which accompanies the galvanometer. The terminals of the galvanometer, as well as the two wires which connect the galvanometer with the rest of the circuit, are joined to the binding screws S S on the shunt box, and each of the three shunt coils has one of its ends connected with the brass piece C, while the other ends are connected respectively with the brass pieces D, E, and F. If, then, the brass plug P′ be inserted in the hole between the brass bar A B and the brass piece C, all the current will pass from A B to C, through the plug, and none through the galvanometer, since the resistance of A B to C through the plug is extremely small compared with that through the galvanometer. If, on the other hand, the plug be inserted in the hole between A B and D, as in the figure, the current will pass from A B to D through the plug, and from D to C through the coil in the shunt box, which connects with C. And as this coil has $\frac{1}{9}$th of the resistance of the galvanometer, $\frac{1}{10}$th of the total current will pass through the galvanometer. Simi-

larly, if the plug be inserted in the hole between A B and E, or A B and F, $\frac{1}{100}$th or $\frac{1}{1000}$th of the whole current will pass through the galvanometer.

In order to obtain very good "*surface insulation*" (*see* § 140, page 267), the brass pieces A, B, C, D, E, and F are,

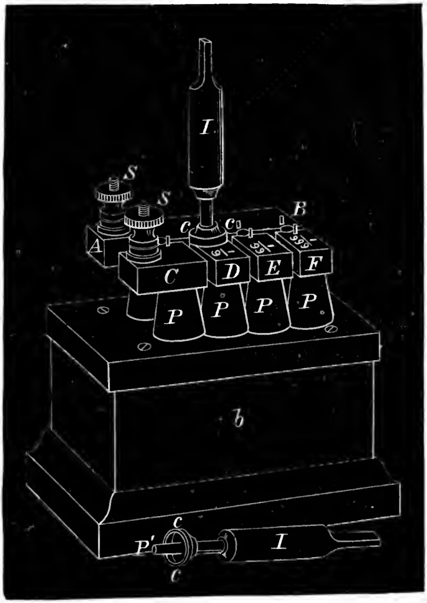

Fig. 66.

in the particular shunt box shown in the figure, mounted on ebonite pillars P, P, P, P, and to avoid the insertion of the plug into one or other of the holes pushing these pillars outwards, and so preventing the plug making firm contact with the pieces of brass on each side of it, there is a spring cap *c c*, sliding on the plug, which passes over the two vertical pins on each side of the hole, and so holds

the brass pieces together against the wedging action which tends to force them asunder when the plug is pressed in. The plug has a long ebonite handle I, which should be held by the flat part at the end to prevent leakage taking place along the surface of the handle and through the body of the experimenter to the ground.

107. Increase of the Total Current produced by the Employment of a Shunt.—The Use of Shunts with a Differential Galvanometer.—The insertion of a shunt diminishes the resistance of the circuit from g to $\frac{sg}{s+g}$. In some cases this produces practically no effect on the total current, so that the current flowing through the galvanometer will be $\frac{s}{s+g}$ of the current that *was* flowing through it *before the insertion of the shunt.* But in other cases this variation of the resistance in circuit materially affects the total current, so that, although G is always $\frac{s}{s+g}$ of the total current, this total current may be so increased by the diminution of the total resistance that the fraction $\frac{s}{s+g}$ of the new total current is practically as large as the previous total current, or, in other words, shunting the galvanometer may produce practically no diminution in the current passing through it.

This effect produced on applying a shunt, which is often entirely overlooked by beginners, may be experimentally investigated with the apparatus shown in Fig. 67. B is a battery consisting of six cells fitted with terminal binding screws, so that one, two, or any number of cells up to six can be used; M is a galvanometer of very small resistance, and R_1, R_2, R_3, R_4, resistance coils in the main circuit. G is a galvanometer of some 500 ohms' resistance, also in the main circuit, but fitted with a shunt S. Any one of the coils, R_1, R_2, R_3, or R_4, can be cut out of circuit by turning the handle h so that a small

bridge-piece *b* of flexible brass makes contact between two metallic buttons *k k*, which are attached respectively to the two ends of the coil.* The resistance in the shunt S can be varied either by taking out or inserting the plugs in its base in the usual way, or by turning the handle which varies the resistance in a way to be explained a little farther on. Then it is found that if the resistance in the main circuit is fairly large, say 1,000 ohms, altering the resistance of S alters the deflection of G, but does

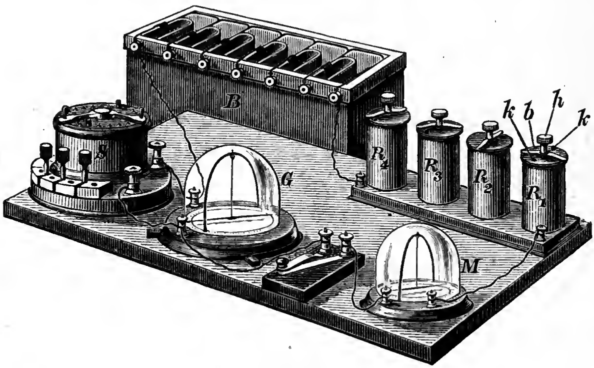

Fig. 67.

not sensibly alter that of M; while, on the other hand, if the resistance in the main circuit is small, that is, if the four bridge pieces at the tops of the four coils are turned so as to short-circuit all the four coils, then the value of S may be altered within wide limits without altering the value of the deflection of G, but the deflection of M will be large when the resistance in S is small, and small when the resistance in S is large. It is necessary to be able to vary the number of cells from one to six in order that

* This plan of cutting out a coil was the one originally employed by the late Sir Charles Wheatstone with the earliest forms of resistance coils.

in all the experiments, each made with a particular value of the resistance in the main circuit, and for a series of values of the shunt, the largest deflection of G, which is obtained when the galvanometer is unshunted, may be about the same.

We have merely referred to the two extreme cases, a very large and a very small resistance respectively in the main circuit; but readings should be taken of the deflection of G for a series of values of the resistance of the shunt, with each of several values of the resistance in the main circuit; and a series of curves should be drawn connecting deflections of G with values of S, each curve for a different resistance in the main circuit.

The mathematical working out of this experiment, together with the consideration of the construction of "*constant total current shunts*," will be found farther on (§ 137, page 253).

We have seen (§ 87, page 149) that if the two coils C and C′ (Fig. 59) of the differential galvanometer have equal resistances, and if, in addition, they be so adjusted relatively to the needle that no deflection is produced when equal currents flow round the coils, no deflection will be produced when A and B have equal resistances, and a difference of potentials is set up between P and Q by any convenient current generator. If, now, one of the coils, say C, be shunted with a shunt, having, say, one-ninth of the resistance of C, then the parallel resistance of C and its shunt will be one-tenth of the resistance of C alone. Therefore if the resistance of A be also diminished to one-tenth of what it was, the total resistance of the branch P A C Q will become one-tenth of what it previously was, hence ten times as much current will pass through A and through B, but of this larger current only one-tenth part will pass round the coil C, and, consequently, there will still be no deflection of the needle. We can generally conclude that if one coil, C, having a resistance g ohms, of a differential galvanometer be shunted with a shunt of s ohms, no deflection will be produced when

$$\frac{\text{resistance of A}}{\text{resistance of B}} = \frac{s}{s+g}.$$

If, therefore, we have a box of resistance coils, the resistance of which can be varied from, say, 1 to 10,000 ohms, we can, by the addition of a tenth shunt to one of the coils of a differential galvanometer, measure resistances varying between 0·1 and 100,000 ohms.

108. Sliding Resistance Boxes.—The resistance box S (Fig. 67) is different from any of the forms used in the previous experiments. Fig. 68 shows this resistance box in plan, and from that it will be seen that there are two ways of altering the resistance, the one by inserting plugs into the holes between P and Q, or by removing these plugs in the manner previously described, the other by turning one, or both, of the sliding handles H h. Turning these handles can be effected without looking at the box, and hence such sliding resistance boxes are commonly employed for "*duplex telegraphy*," or the sending of two messages simultaneously, in opposite directions, along *one* telegraph wire, in connection with which the signaller requires to vary the resistance without having to take his attention off the message he is sending or receiving.

Between each pair of adjacent studs s_1, s_2, s_3, &c., in one half of the box are coils, each having the value of 40 ohms, while between each pair of adjacent studs S_1, S_2, S_3, &c., in the other half of the box are coils, each having the value of 400 ohms. Hence, with the arms in the positions shown in the figure, the current entering at the binding screw T has first to pass through as many of the coils between P and Q as are unplugged, next through eight coils, each of 40 ohms, then from the arm h to the arm H, and lastly through five coils, each of 400 ohms, and out by the terminal t. In addition, therefore, to any resistance that may be unplugged between P and Q, there is a resistance of 2,320 ohms in circuit.

Resistance boxes with sliding arms are much

cheaper to construct than plug resistance boxes, as the labour and expense of grinding the plugs into the conical holes is saved. As, however, it is very difficult to avoid an unknown small resistance being introduced at

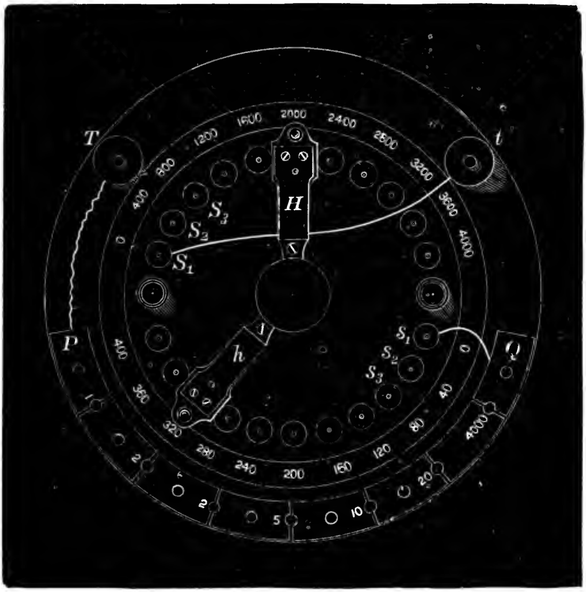

Fig. 68.

the contact of a stud and the revolving arm, well-made plug resistance boxes are far better for accurate work.

109. Measuring a Resistance during the Passage of a Strong Current.—In cases where a conductor is warmed by the passage of a strong current, and so has its resistance altered, it is not sufficient to know what the resistance of the conductor was when cold, but we must know what it is *while the current is passing through it.* This cannot, of course, be done with a Wheatstone's

bridge or a differential galvanometer, but an approximation to the true resistance can, in some cases, be made by stopping the current that was passing through the conductor, and then measuring, *as quickly as possible*, its resistance with a Wheatstone's bridge or differential galvanometer in the ordinary way (*see* § 97, page 167). This, at the best, can give but an approximation, and when the conductor cools very rapidly on the stoppage of the current, as, for example, in the case of the filament of an incandescent lamp, the result so obtained would differ very seriously from the true value. Further, this method could not be employed at all when it is desired, for example, to measure the resistance of an

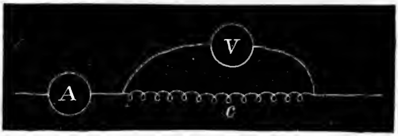

Fig. 69.

"*electric arc*," that is, the intensely heated space between the carbon points in an "*arc lamp*," because the *arc* ceases to exist immediately the current producing it is stopped.

In such a case, the following method must be employed. By means of an electrometer, or a voltmeter V (Fig. 69), measure the potential difference, in volts, at the ends of the conductor *c*, whose resistance we desire to know, and simultaneously measure with an ammeter the current A, in amperes, passing through the conductor; then, if o be the unknown resistance of *c* in ohms, we have, from the definition of resistance,

$$o = \frac{V}{A}.$$

This method can, of course, be employed in all cases, but is especially useful when a conductor has a fairly strong current passing through it, and we desire to

measure the resistance of the conductor while this strong current is passing through.

If the instrument used to measure the potential difference be an electrometer, through which no current passes, the deflection of the ammeter will measure the true current passing through the conductor only; but, on the other hand, if a voltmeter be employed, through which some current passes, then it must not be forgotten that the current passing through the ammeter is the sum of the currents passing through the conductor *c* and the voltmeter. As a rule, this will not introduce any serious practical error, as the resistance of the voltmeter being very large compared with that of *c*, the current

Fig. 70.

passing through the voltmeter is very small compared with that passing through *c*. If, however, this be not quite the case, on account of the resistance of *c* being large, then the current passing through the voltmeter must be subtracted from that measured by the deflection of the ammeter to obtain the value of A in the above formula. Or, more simply, interrupt the voltmeter circuit and now observe the ammeter reading.

If, however, the resistance of *c* be large, the making and breaking of the voltmeter shunt circuit may very possibly alter not merely the current passing through the ammeter, but even that passing through *c*, so that the reading given by the ammeter when the voltmeter shunt circuit is broken, although indicating quite accurately the current *then* passing through *c*, would not give the amount that *was* passing through *c* when the voltmeter reading was taken. Therefore, from these two observations the resistance of *c* could not be accurately deter-

mined, unless the resistance of the voltmeter were known, and the current passing through it calculated and allowed for. In such a case it is better to make the voltmeter a shunt to both the ammeter and *c*, as shown in Fig. 70; for, with this arrangement, the resistance of *c*, plus that of the ammeter, will be correctly found, and if the resistance of the ammeter be either small compared with that of *c*, or if it be correctly known, then the resistance of *c* can also be found by a simple subtraction.

The determination of the resistance of a battery by means of simultaneous readings on an ammeter and voltmeter will be found described in § 116, page 205.

110. Ohmmeter.—The necessity of observing two

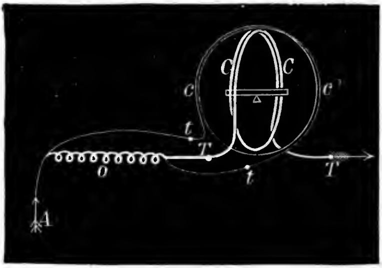

Fig. 71.

instruments at the same time is a disadvantage in the employment of the method of testing just described, and hence the following instrument, called an "*ohmmeter*," was devised by the author for measuring, by a single observation, the resistance of any part of a circuit through which a strong current is passing. The ohmmeter contains two coils acting on the same soft iron needle; one of these coils, C C (Fig. 71), attached to the terminals T T (Fig. 72), is made of a short piece of thick wire, and is placed in series with the resistance *o* to be measured; while the other, *c c* (Fig. 71), attached to the terminals *t t* (Fig. 72), is composed of very fine wire, and is put

as a shunt to the unknown resistance. Hence the main current A produces its effect by means of the thick wire coil, and the difference of potentials V at the terminals of the unknown resistance by means of the fine wire coil; these coils are placed at right angles to one another, and in consequence of this, it may be shown that the action on the needle is due to the ratio of V to A, that is, to the value of *o*. When no current is passing through

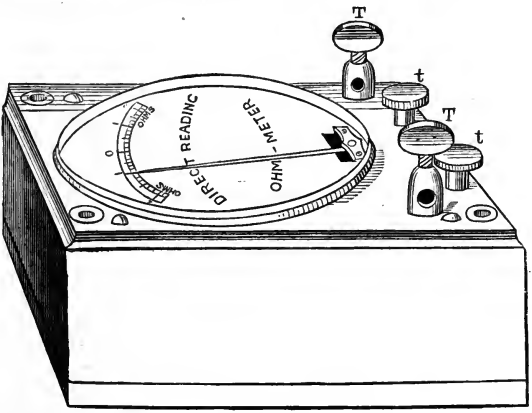

Fig. 72.

either coil the needle will rest in any position, but on sending a current through the thick coil alone the pointer at once moves to nought. And by properly proportioning the shapes of the coils, and by winding the wire on them in a definite way, it is possible to make the angular deflection of the needle from the zero position directly proportional to the resistance *o*. The thick wire coil may be always kept in the main circuit, or in any branch circuit, then on attaching the terminals *t t* to any two points on the same circuit by means of wires the needle will at once move to a number on the dial, which will indicate the resistance

in ohms at the time in question of that part of the circuit between the two points.

On account of the alteration of resistance by heating, it is very difficult, unless very thick German silver wire is employed, to construct resistance coils for use with strong currents, so that the resistance shall not be changed by the passage of the currents. But the use of an ohmmeter permits the employment of an iron wire, or even of a bit of wet rope, as a temporary resistance for experimental purposes, the resistance of the iron wire or of the wet rope being determined with the ohmmeter at the moment the experiment is being made.

HEAT GENERATED AND WORK DONE BY AN ELECTRIC CURRENT.

111. Amount of Heat Generated by an Electric Current.—When considering the effects produced by a current earlier in the book, we saw that the rise of temperature of the calorimeter in a given time was not proportional to the current strength. We will now examine this more fully, and for doing this the apparatus shown in Fig. 73 may be conveniently employed. It consists of a coil of German silver wire dipping into a small metal vessel of paraffin oil, the temperature of which can be observed by means of a delicately graduated thermometer T, the bulb of which dips into the oil. T is supported by an indiarubber stopper, through which it passes, and which itself fits into a small wooden cap, seen in the figure, which forms the top of the vessel containing the paraffin oil. This little vessel is supported in the middle of a very much larger metal vessel, seen in the figure, made with double sides, double top, and double bottom, the space between the two being filled with water. This water jacket, as it is called, is for the purpose of preventing heat passing from the body of the experimenter, or from any adjacent lamp, into the paraffin oil, which would interfere with the experiment, seeing that our

object is to measure the heat produced in the vessel of paraffin oil solely by the current passing through the coil of German silver wire immersed in it. It might, at first sight, appear that the simplest plan of avoiding this, as well as of avoiding the loss of heat from the vessel

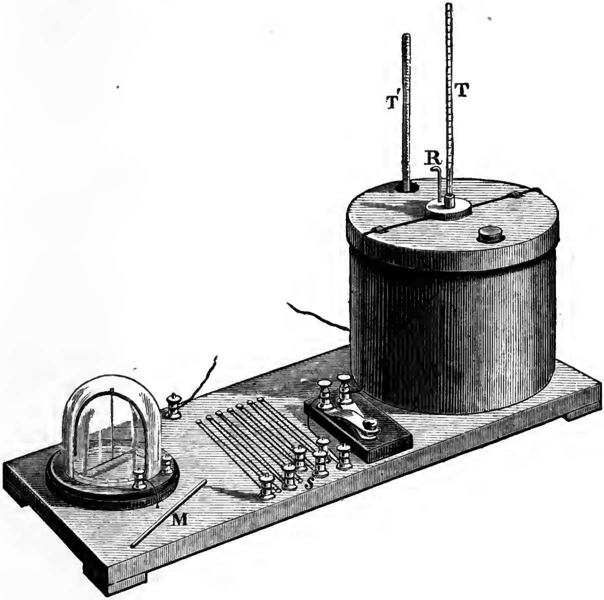

Fig. 73.

of paraffin oil, would be to surround it by a sort of coat of cotton-wool or of fur. *As, however, it is impossible to make such a coat which shall prevent all loss of heat, there being no insulators for heat* (see § 93, page 159), *and as this loss, although small, would be vague in amount, it is better to allow a greater loss provided that the loss is known in amount;* this result is obtained by using a water jacket, and by maintaining this water jacket at

constant temperature, which can be tested by means of the thermometer T′ dipping into the water of the jacket. The two wires to the left side of the figure go to the current generator, which, on pressing down the key, sends a current through the coil of wire in the calorimeter, the galvanometer, and a longer or shorter portion of the stretched wire shown at S in the figure. The length of this stretched wire put in the circuit can be regulated by a loose flexible wire, not shown in the figure, which is attached at one end to the free binding screw of the galvanometer, and at the other end to one of the binding screws S.

It will be seen that in the experiments shown in Figs. 6, 15, and 73, the current strength is varied by inserting a shorter or a longer length of wire in the circuit, whereas in the experiments shown in Figs. 17 and 20 the same result is much more simply attained by altering the distance between two zinc plates, or rods, dipping into a small quantity of a saturated solution of zinc sulphate. The reason of this is that in the first and third experiments the current must be kept *quite constant* for a minute or so while the gas is being steadily generated in the first case, and the heat is being generated in the calorimeter in the third, whereas in the last two experiments it is only necessary to keep the current constant just long enough to take a reading of the galvanometers. Now it would be somewhat difficult to maintain a current quite constant for some little while by means of plates dipping into liquid, unless some plan of fixing the plates in any particular position were employed; and even then, as will be seen later on, a liquid resistance would not be as constant in value as that produced by a given length of wire. Hence the latter plan should always be adopted when it is necessary to maintain the current constant for any length of time.

In Fig. 73, M is a controlling magnet, and, as already explained, if M be placed *near* the galvanometer,

the latter must be calibrated with M in the same position as it is in when the galvanometer is used, since changing the position of a magnet when it is near a galvanometer not only alters the absolute, but generally also the relative calibration curve.

To carry out the heating experiment, a certain current is allowed to pass through the apparatus, and the deflection on the galvanometer is observed. The current being kept constant, the *time rise of temperature* of the liquid in the calorimeter is measured by the thermometer T, the liquid being kept constantly stirred with the stirrer R to prevent its becoming hotter in one part than in another. The time rise of temperature is obtained by making a series of *simultaneous* observations of time and temperature; for example, successive observations of the thermometer may be made by one observer at times which are noted on a watch by another. A curve can now be drawn having its abscissæ, or distances measured along one line, proportional to the times measured from the instant of closing the circuit, or better, from the instant that the first observation of temperature is made after the current has become steady, and its ordinates, or distances measured along a line perpendicular to the former, proportional to the temperature at the ends of each of the periods of time. From this curve, which experiment shows to be concave to the axis along which time is reckoned, the temperature of the thermometer T at any instant of time, or the rise of temperature during any interval of time, can be seen. This time rise of temperature curve does not, however, represent the time production of heat, since, while the calorimeter is gaining heat from the coil in it through which the current is passing, *it is losing heat on account of radiation and convection.** Now it is the

* Radiation is the transference of heat from one body to another without the intervening space becoming warm, as, for example, the way in which the sun warms the earth; conduction is the transference of heat from one part of a body to another, due to all the intervening

rate of production of heat that we want to ascertain, not the time rise of temperature, which, without altering the current strength, or the coil through which it is passing, or the liquid in which the coil is placed, can be made greater or less by diminishing or increasing the facility of the liquid for cooling.

112. Cooling Correction of the Observed Rise of Temperature Curve.—Hence we must make experiments and ascertain the amount of heat that was lost while the temperature was rising, and this may be done with great accuracy by stopping the current and observing the *time fall of temperature*, since the calorimeter is now, while cooling, surrounded by the same body at the same temperature as it was while it was being warmed. From observations on the time fall of temperature, a time cooling curve, which will be found convex to the axis along which time is reckoned, can be constructed. This second curve must be used to correct the first, and so to obtain a third curve indicating what the rise of temperature would have been had there been no loss of heat. To obtain this third curve divide the time into a number of *small* equal intervals; then, starting from the lowest part of the heating curve, observe what was the observed rise of temperature in the first short interval of time; next, referring to the cooling curve, observe what was the observed loss of temperature in the same interval of time and for about the same value of a mean temperature. This observed rise and observed loss must then be added together, and the sum will measure what the rise of temperature would have been during the first interval of time had there been no loss of heat. This gives us a new point indicating what the temperature would have

portions of the body becoming necessarily warmed, as, for example, the way in which, when one end of a cold iron poker is inserted in the fire, the other end gradually becomes warmed; convection is the transference of heat from one place to another by the bodily conveyance of heated liquid and gas; as, for example, the way in which the top of a chimney over a lighted fire becomes warm.

been had there been no cooling. Do the same for the second, third, fourth, &c., intervals, and add what the rise in temperature during each interval would have been without cooling to what the temperature would have been at the commencement of that interval without cooling.

In examining this third curve we find that it is a straight line, which means that for a constant current the amount of heat produced in every second would, were there no loss of heat, be able to raise the temperature by the same amount. Hence the amount of heat actually produced in every second is a constant quantity, and this constant rate of production is measured by the tangent of the angle between this line and the line along which time is measured.

Next vary the current by changing the length of the wires (Fig. 73) in circuit, which, as stated above, is done by connecting the free terminal of the galvanometer with a different binding screw at S, and obtain the corrected curve as before. On comparing the slopes of the straight lines so obtained from the two experiments, it will be found that they are not proportional to the respective current strengths as measured by the galvanometer, but to the squares of the current strengths. And the same result will be obtained if any other two current strengths be employed in the experiment. We conclude, therefore, from this experiment, that *the heat generated in a conductor by a current in a given time is proportional to the square of the current strength, or simply to the square of the current.**

113. Measuring a Current by the Rate of Production of Heat.—This method of measuring the rise of temperature by the heat generated in a coil of wire may be conveniently employed when the currents are so strong as to raise the temperature several degrees

* In practice it is found best to take the observations on cooling at the end of the experiment, after the complete series of observations of the heating produced by the various currents has been carried out.

in a time so short that in it no appreciable loss of heat can occur, since in that case the two currents will be simply proportional to the square roots of the elevations of temperature in the same time, and no cooling experiments need be made. And the method is especially useful when we desire to measure an "*alternating current*," that is, a current the direction of which is being rapidly reversed, since in such a case neither a voltameter nor a galvanometer with a controlling permanent magnet can be used to measure the current strength. As to the voltameter, the compound gas, of which we measured the volume in the experiment described in § 3 and § 12, was, as seen from the experiment described in § 7, composed of hydrogen steadily given off at one of the platinum plates, and oxygen steadily given off at the other, so that if the current were rapidly reversed many times a minute, a little hydrogen would first be formed at one of the plates, then a little oxygen at the same plate, which, combining with the hydrogen previously formed, would re-form water, so that on the whole no gas would be produced which could be collected and measured.

If the coil of wire, having a resistance of o ohms in the vessel in Fig. 73, be replaced by another coil of wire having a different resistance of o' ohms, it will be found that for the same current and for the same quantity of liquid in the calorimeter, the corrected elevations of temperature in the two cases in the same time, which are proportional to the amounts of heat produced, will be in the ratio of o to o'. Combining this with the result previously obtained with different currents flowing through the same coil, it follows that *the heat generated in a conductor by a current in a given time is proportional to the product of the square of the current into the resistance of the conductor.*

If the *water equivalent* of the calorimeter, that is, the weight of water that is raised as much in temperature as is the calorimeter with its contents by the addi-

tion of the same amount of heat, be determined, we can measure the actual amount of heat produced per minute by a given number of amperes flowing through a given number of ohms. And experiment shows when the unit of heat is taken as the amount of heat that will raise 1 lb. of water from 0° C. to 1° C., that *A amperes flowing through r ohms produce per minute* $0{\cdot}0315\ A^2r$ *units of heat*, or, generally, if H be the number of units produced in t minutes,

$$H = 0{\cdot}0315\ A^2 r\, t.$$

If we take as the unit of heat the heat required to raise one gramme of water from 0° C. to 1° C., and if t be measured in seconds, then

$$H = 0{\cdot}239\ A^2 r\, t.$$

114. Work done in an Electric Circuit.—If two conductors at different potentials be joined by a wire, electricity will flow from the one of higher potential tc that of lower as long as any potential difference exists between them, exactly as, when a reservoir containing water at a higher level than the surrounding country has an opening made in the bottom of it, water will flow out of the reservoir from the higher to the lower level until all the water has fallen to the same level. *In order, therefore, to keep up a constant electric current we must employ a machine that will transport electricity from a place of low to a place of high potential*, just as, to keep up a steady stream of water, we must have some machine or pump, or it may be the evaporating power of the sun, to keep raising the water from a low to a high level. With any such pump a certain portion of the power expended on it would be spent, not in actually raising water, but in overcoming the friction opposed by the channels inside the pump to the passage of the water, and the portion of the power so spent would be converted into heat, so that the work the water could do in falling would be less than that spent on the pump by the

amount that had been wasted in heat in the pump. Consequently, the number of feet to which a pump could raise a pound of water would be less than the number of foot-pounds of work expended on the pump to raise the pound of water, and experiment shows that this difference would be greater the more quickly the pound of water was raised.

Now a generator of an electric current, for example, a galvanic battery, a dynamo machine, a magneto machine, or a thermopile, is employed for the same sort of object as a pump, viz., to raise electricity from a low to a high potential, *in opposition* to the tendency the electricity apparently possesses to flow from a place of high to a place of low potential. The work the electricity so raised in potential can do per second, or per minute, in the external circuit, in the form of an electric current, may be partly done in turning an electromotor, or may be partly done in decomposing the substances in a voltameter, but in both cases a portion must be done in heating the external circuit; or the whole of the work may be done in merely heating the external circuit, and since we know from the experiment described in § 113, page 198, that the heat generated by the current is proportional to the product of the square of the current into the resistance, it follows that, when all the work done in a circuit is done in the form of heat, the total work done is proportional to the square of the current into the resistance. But we also know that in this simple case the current is proportional to the potential difference at the terminals of the generator divided by the resistance of the external circuit. *Hence, in this case, the work must be proportional to the product of the current into the potential difference.*

To express this result numerically, let the current A be measured in amperes, the potential difference V in volts, the resistance r in ohms, the work W in foot-pounds, and the time t in minutes. Then, since, as we have already seen (§ 113, page 199) that H, the heat

generated, equals $0{\cdot}0315\ A^2 r\, t$, and since Joule has shown that the quantity of heat required to raise 1 lb. of water from 0° C. to 1° C. (that is our unit of heat) is equivalent to 1,400 foot-pounds of work, it follows that

$$\begin{aligned} W &= 1{,}400 \times 0{\cdot}0315\ A^2 r\, t \\ &= 44{\cdot}25\ A^2 r\, t. \end{aligned}$$

Also we know that when all the work done in the circuit is expended in producing heat,

$$A = \frac{V}{r},$$

$$\therefore\ W = 44{\cdot}25\ AV\, t.$$

And this result is true in *all* cases whenever a *steady* current flows through *any* circuit, whether it consist of merely a resistance or, in addition, of an electromotor, or of both an electromotor and a voltameter in addition to mere resistance. *Consequently, whenever a steady current of* A *amperes flows through a circuit, at the terminals of which* V *volts are maintained, the work done in foot-pounds per minute is* 44·25 AV, *and the horse-power expended on the circuit* $\frac{44{\cdot}25}{33{,}000}$ AV, or $\frac{AV}{746}$, or 0·00134 AV, since one horse-power corresponds with 33,000 foot-pounds per minute. (*See* also Chap. IX.)

Example 41.—If 4 amperes flow through a resistance of $4\frac{1}{2}$ ohms for twenty minutes, how many foot-pounds of work are done? *Answer.*—63,720.

Example 42.—If three-quarters of an ampere flows through an Edison lamp when 108 volts are maintained at its terminals, how many foot-pounds of work per minute are expended on the lamp? *Answer.*—$3{,}584\frac{1}{4}$.

Example 43.—A horse-power being 33,000 foot-pounds per minute, how many such Edison lamps as are referred to in the last question would be made to incandesce with the expenditure of $2\frac{1}{2}$ horse-power?

Answer.—23.

Example 44.—If a lamp through which half an ampere is flowing, and at the terminals of which 85 volts are maintained, emit 10 candles of light, how many candles per horse-power are being produced?

The horse-power expended equals

$$\frac{44{\cdot}25 \times \frac{1}{2} \times 85}{33{,}000};$$

therefore one candle requires an expenditure of

$$\frac{44{\cdot}25 \times \frac{1}{2} \times 85}{330{,}000} \text{ horse-power};$$

therefore one horse-power will produce

$$\frac{330{,}000}{44{\cdot}25 \times \frac{1}{2} \times 85} \text{ illumination.}$$

Answer.—175·5 candles' illumination nearly.

As already stated, it can be shown in all cases that the total work done in the external circuit equals 44·25 AV foot-pounds per minute, whether there be electromotors or voltameters or not in this circuit. If, however, there be either of these, the total work done will be more than 44·25 $A^2 r$ foot-pounds per minute where r is the total resistance of the external circuit, that is,

$$44{\cdot}25 \text{ AV.} > 44{\cdot}25 \text{ A}^2 r.$$

In fact, 44·25 $A^2 r$ foot-pounds per minute represents the portion of the energy that is turned into heat, and the difference represents the amount of electric energy, measured in foot-pounds per minute, that is transformed into some form of energy other than heat.

Electromotive Force.

115. Work done by a Current Generator. Electromotive Force.—In order that a given amount of work may be done on the external circuit, a *greater* amount of work must be done by the generator itself, on account of

the resistance of the generator against which the current has to be sent, just as a pump has to do more work than the energy stored up in the water. Consequently, if b be the resistance of the generator in ohms, $44{\cdot}25\ A^2 b$ foot-pounds per minute must be expended in sending the current through the generator itself, and, consequently, the total work done by the generator in foot-pounds per minute equals

$$44{\cdot}25\ A^2 (r + b).$$

Now if v be the potential difference, that would send the current A amperes through b ohms,

$$A = \frac{v}{b},$$

$$\text{or } A^2 b = A v.$$

Hence the total work done by the generator equals

$$44{\cdot}25\ A\ (V + v) \text{ foot-pounds per minute.}$$

Further, we know that when a current passes through a voltameter, the amount of chemical action that is produced in a given time is proportional to the current; indeed, it was the amount of chemical action per minute that gave us our original definition of current strength. And a galvanic battery is but a form of voltameter, hence we may conclude that the amount of each of the various chemical actions that take place in a battery in a given time is proportional to the current, if no action takes place when no current is passing, a condition that is approximately fulfilled in a good galvanic battery. Also we know that the amount of chemical action that takes place in a given time in a battery represents the amount of fuel burnt in that time, and therefore is proportional to the *total* amount of work done by the battery in the same time. The total work, therefore, done by the battery per minute is proportional to A; but we have seen that it is also proportional to A $(V+v)$, consequently, $V+v$ must be a *constant* for a

particular battery. This constant is called the "*electromotive force*," and is shortly represented by the letters "*E. M. F.*" If the current passing through the battery is very large its chemical constitution changes somewhat, so that the same current passing through it for the same time does not produce the same chemical decomposition as before; hence the work now done, compared with the work previously done, ceases to be in the ratio of the present value of the current to the former value, or, in other words, $V + v$ or the E. M. F. is no longer constant.

However, excluding such extreme cases, we can say that the current

$$A = \frac{V}{r};$$

$$\text{or } A = \frac{v}{b};$$

$$\text{or } A = \frac{E}{r + b}$$

$$\text{Hence } V = \frac{r}{r + b} E,$$

if E stands for the electromotive force of the battery.

116. Variation of External Resistance, Current, and Potential Difference at the Battery Terminals.—When r, the external resistance, is extremely great compared with b, and the current, as seen from the third equation above, is very small, V, the "*terminal potential difference*," is, as seen from the last equation, a maximum, and becomes equal to E. And as long as r is fairly large in comparison with b, the current remains small, and V remains nearly equal to E. When r diminishes so as to become small compared with b, A increases rapidly, until when r is nought A becomes a maximum, and equals $\frac{E}{b}$. V, then, is nought.

The preceding is all given concisely in the following table :—

r	V	A
Infinity.	E	O
Great compared with b.	Very little less than E.	Small.
p, say	$\frac{p}{p+b}$ E.	$\frac{E}{p+b}$.
Small compared with b.	Small.	Great.
O.	O.	Maximum, and equal to $\frac{E}{b}$.

The apparatus shown in Fig. 74, consisting of a battery B, a delicate ammeter A, a voltmeter V, and a variable resistance R, enables all the preceding to be tried experimentally.

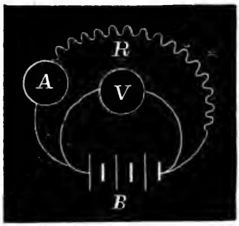

Fig. 74.

First, make R equal to infinity, then the reading on the voltmeter gives E.

Secondly, make R have any suitable value, so that the current can be easily read accurately on the ammeter; let it be A amperes, and the corresponding potential difference at the terminals of the battery V volts; then,

$$V = E - A\,b,$$

where b is the battery resistance;

$$\therefore\ b = \frac{E - V}{A} \text{ ohms.}$$

The resistance of the battery can in this way be determined without knowing r the value of R, that is, without employing resistance coils of known value, and this is the best method of measuring the resistance of a current generator when the resistance is very small, as in the case of an "*accumulator.*"

Thirdly, take various values of r, and see whether the current always equals $\frac{E}{r+b}$ amperes, and the terminal potential difference $E - A\,b$ volts.

As a rough analogy, the terminal potential difference of a battery may be likened to the force exerted by a locomotive engine in dragging the carriages, which is, of course, equal to the pull on the coupling connecting the engine with the first carriage, while the current strength may be likened to the speed of the train, and the external resistance to the mass of the carriages composing the train. If the train be long and heavy, corresponding with a great external resistance, the pull exerted by the engine is great, but the speed of the train is slow. Whereas if there be only a few carriages the pull is less but the speed is greater, and in the extreme case, when the engine is running alone the pull exerted on the coupling, which is now hanging loose, is nought, and the speed of the train is the greatest. Also the pull exerted by the engine on the first carriage is always less than the total force exerted by the engine, unless the engine is attempting to pull so heavy a train that it does not move, corresponding with infinite external resistance and current nought, because if the engine is moving at all, some of its pulling power is employed in moving itself. And so with a battery, if any current at all is flowing, the terminal potential difference must always be slightly less than the electromotive force.

Example 45.—A Daniell's cell has an E. M. F. of 1·07 volts, and an internal resistance of $2\frac{1}{2}$ ohms; what

current will it send through an external resistance of 32 ohms? *Answer.*—0·031 ampere nearly.

Example 46.—A battery having an E. M. F. of 15 volts, and an internal resistance of 25 ohms, is sending a current through an external resistance of 5 ohms; what is the potential difference at the battery terminals?

Answer.—$2\frac{1}{2}$ volts.

Example 47.—What current must the battery in the last question send so that its terminal potential difference may be 7·5 volts? *Answer.*—0·3 ampere.

Example 48.—If a battery, having an E. M. F. of 8 volts, have its terminal potential difference reduced to 2 volts on sending a current of 2 amperes, what is its internal resistance? *Answer.*—3 ohms.

Example 49.—A battery has a terminal potential difference of 15 volts when sending a current of 2 amperes, and 12 volts when sending a current of 3 amperes; what is its internal resistance?

If E be the unknown E. M. F. of the battery, and b its resistance,

$$\text{we have } 15 = \text{E} - 2\,b,$$

$$\text{also } 12 = \text{E} - 3\,b,$$

$$\text{or } \quad b = 3 \text{ ohms.}$$

Answer.—3 ohms.

CHAPTER V.

CURRENT GENERATORS.

117. Current Generators.—The *current generators* in practical use may be divided into—

1. "**Batteries.**"
2. "**Accumulators**" or "**Secondary batteries.**"
3. "**Magneto machines.**"
4. "**Dynamos.**"
5. "**Thermopiles.**"

All of these are simply contrivances for converting various forms of energy into electric energy. In thermopiles heat energy is directly transformed into electric energy, just as in a steam-engine heat energy is directly transformed into mechanical energy, or energy of visible motion. In dynamos and magneto machines there is a direct transformation of mechanical energy into electric energy, whereas in accumulators and batteries it is *stored up*, or *potential*, chemical energy that is converted into electric energy.

118. Batteries.—A "*battery*" is the name given to a collection of "*galvanic cells*," arranged so as to produce a larger current than could be obtained with a single cell under the particular circumstances. Fig. 75 shows a battery composed of five cells of the very simplest form, each cell consisting of a plate of zinc z and a plate of copper c, dipping into dilute sulphuric acid. Such a cell is frequently called a "*simple Voltaic element.*" The copper plate of one cell is joined by means of a copper wire to the zinc plate of the next, so that the cells are in series (*see* Arrangements of Cells, § 135, page 239), and on joining the two terminal copper wires marked + and — in the figure, directly together, or to the terminals of a galvanometer, voltameter, or other indicator of the direction of the current, the current is found to flow in the direction of the arrows (*see* Definition of the Direction of the Current, § 7, page 14).

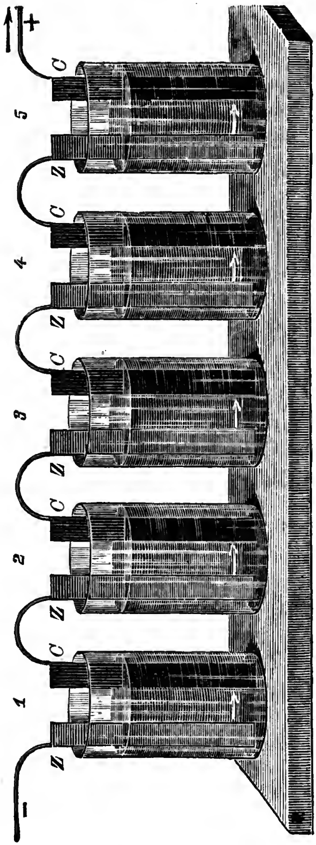

Fig. 75.

A great number of cells have been devised from time to time, but the most important are the

1. "Daniell's" cell.
2. "Grove's" cell.
3. "Bunsen's" cell.
4. "Leclanché" cell.
5. "Potash bichromate" cell.

Other cells, such as the "*Lalande Chaperon*," the "*Ross*," the "*Upward*," the "*Regent*," &c., may be used for the

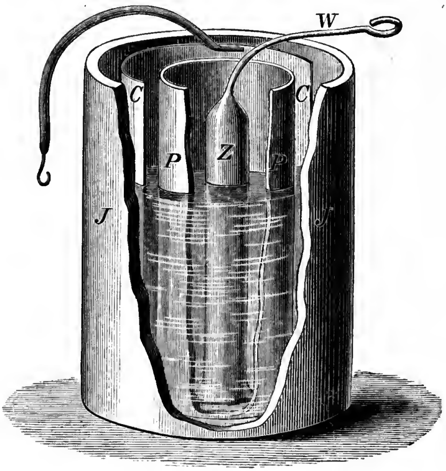

Fig. 76.

comparatively cheap production of large currents, when a dynamo is not available, but such cells cannot, as far as the author is aware, compare with the dynamo in economy.

119. Daniell's Cell.—The "*Daniell's*" cell consists of a *copper plate* C, Fig. 76, dipping into a *solution of copper sulphate* contained in a glass, or glazed, highly vitrified stoneware jar, J, and a *zinc plate, or rod*, Z, to which a copper wire, or strip, W, is soldered, dipping into either dilute sulphuric acid or a *solution of zinc sulphate*, the two solutions being separated by a *porous partition* P,

made of unglazed earthenware, and called a "*porous pot.*" The E. M. F. of a Daniell's cell, and of all its modifications, is roughly 1·1 volts, but it varies from about 1·07 volts to 1·14 volts, depending on the densities of the solutions of copper and zinc sulphate. With *equi-dense* solutions, and with plates of *pure* zinc and copper, the E. M. F. is 1·104 volts. This value is increased by increasing the density of the copper sulphate solution, and diminished by increasing the density of the zinc sulphate solution, and is scarcely at all affected by the ordinary atmospheric changes of temperature. (*See* § 215, page 411.)

The resistance of the cell varies with the area of the copper and zinc plates immersed in the liquids, the distance between the plates, and the thickness and constitution of the walls of the porous cell. With a cell about 7 inches high, of the relative dimensions shown in the above figure, the resistance may be as low as $\frac{1}{3}$ of an ohm when the solution in which the zinc plate is immersed is dilute sulphuric acid of a specific gravity of about 1·15 at 15° C. Occasionally, however, porous pot Daniell's cells, with smaller plates, are used, having a resistance of as much as 10 ohms. *The E. M. F. of the Daniell, or of any other form of cell, is quite independent of the size of the various parts of the cell, or of the cell as a whole, and depends solely on the materials employed in its construction.* (*See* § 133, page 236.)

120. Minotto's Cell.—In the "*Minotto's*"* cell the porous pot is replaced by a layer of sand or sawdust, and it is constructed as shown in Fig. 77. At the bottom of a glass, or glazed and highly vitrified stoneware jar J, there is placed a disc of sheet copper C, to which is attached one end of an insulated copper wire, which passes up through the cell. Above this plate are placed some crystals of copper sulphate C S, and on the top a piece of thin canvas *c*, separating the copper sulphate from the layer of sand or sawdust *s*, and on the top of the saw-

* Often wrongly spelt "*Menotti's.*"

dust rests the zinc plate Z, separated from the sand or sawdust by a piece of thin canvas *c*. The cell is completed by pouring in some solution of zinc sulphate, so as to cover the zinc disc, but not so much as to reach up to the brass binding screw B, cast into the top of a little column of zinc, forming part of the zinc disc. Before putting in the sand or sawdust, it should be soaked in a solution of zinc sulphate, and squeezed partially dry, because, if put into the cell quite dry, a long time must elapse before the liquid will soak through the sand or sawdust, and until this happens the cell will not come into action.

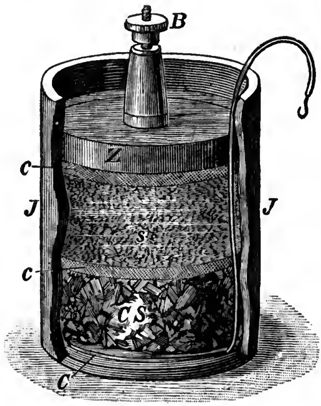

Fig. 77.

It is better to employ sand in stationary Minotto's cells, as it sinks down as the copper sulphate is consumed, but if the cells have to be moved about, then it is better to use sawdust.

121. **Gravity Daniell.**—In some types of Daniell's cells, no form of porous partition is employed, and the copper sulphate and zinc sulphate are kept separated solely by the action of gravity, the zinc sulphate solution being put at the top, as it is the lighter of the two. Such cells are called "*gravity Daniell's*," and examples of them are shown in Figs. 78, 79, and 80. Fig. 78 shows two forms of the "*Meidinger*" cell, in each of which the copper plate is put inside a small inner glass tumbler *d d*, so that the particles of zinc sulphate, which may become detached from the zinc plate, may fall clear of the copper plate, and be prevented from coming into contact with it. In the type of Meidinger shown on the left, the crystals

of copper sulphate are in a glass tube *h*, with only a small hole at the bottom, while in the type to the right the crystals are contained in an inverted flask open at the neck. In both, the zinc plate z z, which is in the form of a cylinder, is supported on a shoulder *b b*, formed by a contraction of the lower part of the outer glass vessel. The *Callaud* cell, Fig. 79, is a simplification of the Meidinger, being without the reservoir for

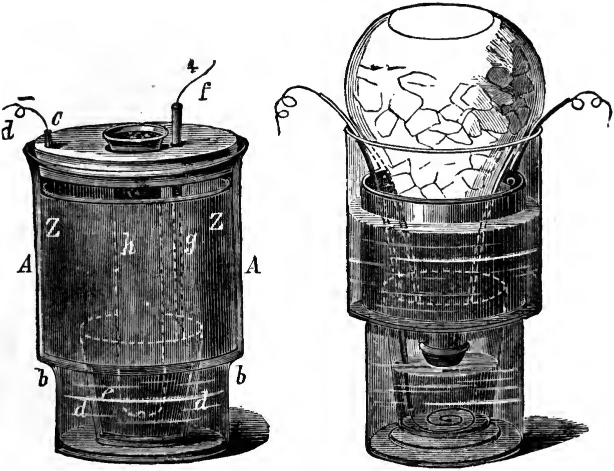

Fig. 78.

the copper sulphate crystals, and the small glass tumbler to hold the copper plate.

In the "*Lockwood*" cell, Fig. 80, the zinc plate is made like a kind of wheel with spokes, so as to expose a large surface to the liquid, and is supported by three lugs resting on the edge of the glass vessel. The copper plate is made of thick copper wire, bent into the form of a double spiral, with the crystals of copper sulphate placed between the spirals, the upper spiral

being found to retard the travelling up of the copper sulphate solution to the zinc plate if the cell be kept sending even only a weak current. For the lower spiral, a copper disc, similar to that used with the Minotto's cell (§ 120, page 211), may be substituted, and for the upper one, a perforated copper disc, without interfering with the action of the Lockwood cell. All gravity cells have the disadvantage that they cannot be moved about,

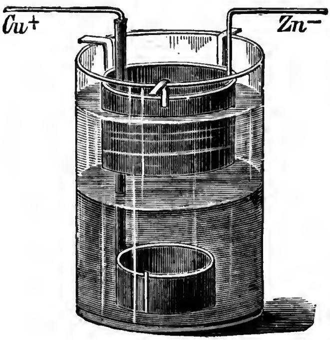

Fig. 79.

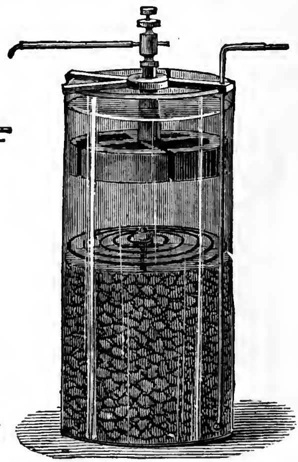
Fig. 80.

otherwise the liquids mix, and the sulphate of copper solution coming into contact with the zinc plate, deposits copper on it. This impairs the action of the cell by causing the zinc plate to act electrically like a copper one. Indeed, without any shaking, the liquids mix by diffusion, even when a porous pot is employed, and hence a Daniell's cell is found to keep in better order if it be always allowed to send a weak current when not in use, since the current uses up the copper sulphate solution instead of allowing it to diffuse.

Ayrton

122. Chemical Action in the Daniell's Cell.—*The Daniell's cell, and all its modifications, produce a current by the formation of zinc sulphate, and the using up of copper sulphate, the zinc plate being eaten up to form the zinc sulphate, and the copper plate growing by the deposit of metallic copper on it.* Chemically, the action may be represented as follows—the "*water of crystallisation*" of the copper and zinc sulphate crystals, as well as the water employed to form the solutions, being omitted for the sake of simplicity :—

	Porous Partition.	
Before sending a current $k(\mathrm{Cu})+l(\mathrm{CuSO_4})$		$m(\mathrm{ZnSO_4})+n(\mathrm{Zn})$,
after sending a current $(k+1)(\mathrm{Cu})+(l-1)(\mathrm{CuSO_4})$		$(m+1)(\mathrm{ZnSO_4})+(n-1)(\mathrm{Zn})$;

k and *n* being any arbitrary quantities of copper and zinc used in the copper and zinc plate, and *l* and *m* any arbitrary quantities of the copper sulphate and zinc sulphate employed. Substituting the "*atomic weights*" for the various substances employed, we find that for every 26 ounces of zinc that are dissolved off the zinc plate, about 100 ounces of copper sulphate crystals are decomposed, and about 25 ounces of copper added to the copper plate. If dilute sulphuric acid be employed in place of a solution of zinc sulphate, the resistance of the cell is lower, and the E. M. F. higher, but the latter is not so constant as when zinc sulphate alone is used, because, if we start with dilute sulphuric acid, zinc sulphate will be gradually formed by the action of the cell, and the increase of the amount of zinc sulphate we have already seen lowers the E. M. F. The chemical action in that case will be as follows :—

	Porous Partition.	
Before sending the current $k(\mathrm{Cu})+l(\mathrm{CuSO_4})$		$m(\mathrm{H_2SO_4})+n(\mathrm{Zn})$,
after sending the current $(k+1)\mathrm{Cu}+(l-1)(\mathrm{CuSO_4})$		$m(\mathrm{H_2SO_4})+(\mathrm{ZnSO_4})+(n-1)(\mathrm{Zn})$

When, therefore, constancy of E. M. F. is desired, a solution of zinc sulphate should be used, and not dilute sulphuric acid.

If the copper sulphate solution becomes too weak, the water is decomposed instead of the copper sulphate, and hydrogen is deposited on the copper plate. This deposition of hydrogen lowers the E. M. F., and care should therefore be taken to keep up a sufficient supply of crystals of copper sulphate. Indeed, it was for the purpose of preventing the deposition of hydrogen on the copper plate which occurs with a simple voltaic element, that Prof. Daniell was led to use copper sulphate as a "*depolariser*," and thus invent the "*two-fluid cell.*" This polarisation is easily seen by dipping two pieces of *clean* copper, C_1 and C_2, and a piece of zinc, into dilute sulphuric acid, a part of each of the three pieces being inside the liquid and a part outside, but the three pieces not touching one another, either inside or outside the liquid. If the two pieces of copper, C_1 and C_2, be first joined by wires with a delicate galvanometer, no current will be observed; but if one of them, C_1, be connected for a time with the zinc by a wire, so that a current flows from C_1 to the zinc through this wire, and from the zinc to C_1 through the liquid, it will be found on stopping this current and connecting C_1 and C_2 again with the galvanometer, that a current now flows round it from C_2 to C_1, that is, from C_1 to C_2 through the liquid. Using C_1, therefore, as the copper plate in a simple voltaic element, causes it to act subsequently as a zinc plate to a *clean* copper plate. And the longer C_1 is used as the copper plate of the simple voltaic cell, which is sending a current through a piece of wire to the zinc plate, the more like a zinc plate does C_1 become, and the weaker grows the current that C_1 with the zinc plate can send through a given external resistance, while the stronger becomes the current that C_1 and a *clean* piece of copper will send through a given resistance. This change in the behaviour of C_1 is due to a deposition of hydrogen on

it, which deposition gradually disappears when C_1 and C_2 are left connected. Both then when the "*primary current*" flows from the zinc to C_1 through the liquid, and subsequently when the "*secondary current*" flows from C_1 to C_2 also through the liquid, the hydrogen moves in the direction of the current, the result obtained with a sulphuric acid voltameter (*see* § 7, page 15).

If the solution of zinc sulphate in a Daniell's cell (Figs. 77, 80) becomes too strong by the evaporation of the water, the zinc sulphate crystallises on the sides of the cell, and the liquid passes up by capillary attraction between the film of crystals and the side of the vessel, crystallising again above. At last the film passes over the edge of the jar and forms on the outside, thus making a kind of syphon, which draws off the liquid. This action may, to a great extent, be prevented by warming the edges of the glass or stoneware jars, and of the porous pots, before the cells are made up, and dipping them while warm into some paraffin wax melted in warm oil. It is desirable also with those Daniell's cells in which the zinc is inside the porous pot, as in Fig. 76, to dip the bottom of the porous pot into the melted paraffin wax, otherwise particles of metallic copper will be gradually deposited in the pores at the bottom of the porous pot on which the zinc rests, and the cell will become "*short-circuited*," that is, a strong current will be sent through this copper, and the material in the cell will be used up rapidly, exactly as would be the case if the zinc and copper plates were permanently connected by a short piece of thick copper wire outside the cell.

123. Local Action.—Another cause of "*local action*," or the production of useless currents, is impurities, such as bits of coke, in the zinc. If a piece of coke and a piece of *pure* zinc be put into dilute sulphuric acid, then, as long as the coke and zinc do not touch one another, either in the liquid, or outside, no appreciable chemical action will take place; but if now the parts of

the coke and zinc that are in the liquid, or the parts that are outside, be touched together, a rapid evolution of hydrogen gas will take place, together with the formation of zinc sulphate. And exactly the same effect is produced when a piece of zinc containing impurities is dipped into dilute acid. This local action, however, can be prevented by coating the surface of the zinc with an "*amalgam*" of zinc and mercury, or "*amalgamating*" the zinc, as it is shortly called, this amalgam covering up the impurities. To amalgamate a piece of zinc, it should be dipped into dilute sulphuric acid, to clean the surface, when a little mercury should be rubbed over the zinc with a piece of rag tied to a stick. A plate of commercial zinc amalgamated, although much cheaper than a plate of pure zinc, does not give an E. M. F. as constant as is obtained with a pure zinc plate.

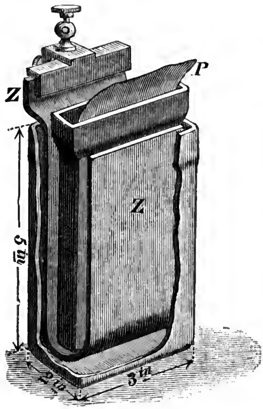

Fig. 81.

124. Grove's Cell.—In the "*Grove's*" cell the copper plate in the Daniell's cell is replaced by a sheet of *platinum*, P, Fig. 81, and the solution of copper sulphate by *strong nitric acid.* Dilute sulphuric acid, in the proportion of about one pint of acid to ten pints of water, is used in place of zinc sulphate solution, since, with the Grove's cell, we wish to obtain the highest E. M. F., and the lowest resistance rather than very great constancy. The E. M. F. is about 1·93 volts, and with good porous cells the *resistance is very low,* being only about

$$\frac{3{\cdot}6 \times d}{A} \text{ ohms,}$$

where d is the distance, in inches, between the platinum

and the zinc plates, and A the area, in square inches, of the platinum plate immersed in the nitric acid. If, as is frequently the case, the zinc plate Z Z is cast in the shape shown in Fig. 81, A must be reckoned on both sides of the platinum plate P. When the cell has the dimensions indicated in the figure, the resistance is about 0·15 ohms when the nitric acid is strong, and the dilute sulphuric acid has but little zinc sulphate in it. After a Grove's cell has been sending a current for some time, the nitric acid becomes weakened, as water is formed by the action of the cell, and a considerable quantity of zinc sulphate is also dissolved in the dilute sulphuric acid, both of which have the effect of diminishing the E. M. F., and increasing the resistance of the cell.

The chemical action is as follows :—

Before sending a current $k(\mathrm{Pt})+l(\mathrm{HNO_3})$	Porous Partition.	$m(\mathrm{H_2SO_4})+n(\mathrm{Zn})$,
after sending a current $k(\mathrm{Pt})+(l-2)(\mathrm{HNO_3})$ $+(\mathrm{N_2O_4})+2(\mathrm{H_2O})$		$(m-1)(\mathrm{H_2SO_4})+(\mathrm{ZnSO_4})$ $+(n-1)(\mathrm{Zn})$;

the water originally in the cell being omitted for simplification. Peroxide of nitrogen, N_2O_4, comes off as a dark brown gas, extremely unpleasant and unhealthy when breathed for any time; a Grove's battery should, therefore, always be placed either in the open air or under a chimney when in use.

The large E. M. F., combined with the small resistance, makes Grove's cells very valuable when a very strong current has to be produced; hence, before the perfection of the dynamo and of secondary batteries, they were largely used for the production of the electric light.

125. Bunsen's Cell.—The "*Bunsen's*" cell differs from the Grove's only in having a cylinder, or block, of *carbon* in place of the sheet of platinum, as seen in Fig. 82, which shows a common form of circular Bunsen's cell, *C* being the carbon, and *Zn* the zinc. A Bunsen's cell is

cheaper to construct than a Grove's cell, as carbon is so much less expensive than platinum ; it is, however, more cumbersome, and more nitric acid is required to fill it, as the nitric acid soaks into the pores of the carbon. The E. M. F. of a Bunsen's cell is also somewhat lower than that of a Grove's, although the chemical action in the two cells is nearly the same.

The carbons for the Bunsen's cells are either cut out

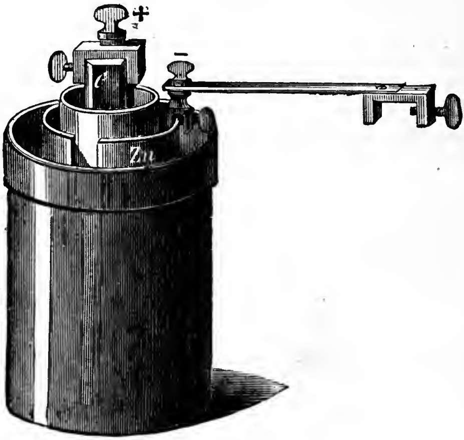

Fig. 82.

of *retort carbon*, or are made by baking in a furnace fine coke-dust and caking coal in an iron mould ; then, in accordance with a process invented by Bunsen, the baked mass is soaked repeatedly in thick syrup or gas-tar, and re-baked to impart solidity and conducting power to it.

126. Leclanché Cell.—The "*Leclanché*" cell consists, as seen in Fig. 83, of a *zinc* rod to the left of the figure, immersed in a solution of ordinary *sal ammoniac*, and a plate of carbon put inside a porous pot, and packed tightly with a mixture of the needle form of *manganese peroxide* and broken *gas-carbon*. Both the manganese

peroxide and the gas-carbon must be sifted to remove the dust, in order that as much surface as possible may be exposed to the action of the liquid. The porous pot is merely for the purpose of holding the mixture in position, and not for keeping two liquids separated, as in the cells previously described; for, in fact, there is only one liquid on both sides of the porous pot—the solution of sal ammoniac. The upper part of the porous pot is closed with pitch, in which a small hole is left, so that a little water or a little solution of sal ammoniac may be poured in to start the action.

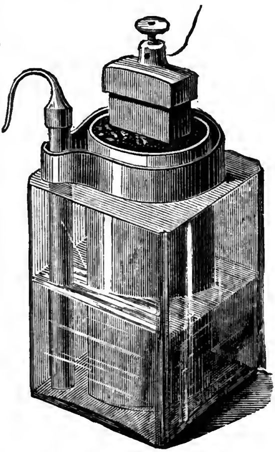

Fig. 83.

The chemical action is as follows:—

Before sending a current

$$kC+l(MnO_2)+m(NH_4Cl)+n(Zn),$$

after sending a current

$$kC+(l-2)(MnO_2)+(m-2)(NH_4Cl)+(Mn_2O_3)+2(NH_3)+(H_2O)+(ZnCl_2)+(n-1)(Zn).$$

Ammonia, NH_3, therefore, comes off from the cell, and substituting the atomic weight we see that for every 50 grains of zinc used up about 82 grains of sal ammoniac are consumed, and about 134 grains of manganese peroxide, MnO_2, are reduced to the lower, or sesqui-oxide, Mn_2O_3. If too little sal ammoniac be present, zinc oxide is formed instead of zinc chloride, and the solution becomes milky. When this happens, more sal ammoniac should be added. Connection with the carbon rod is made by means of a *lead cap* cast on it; and to prevent a salt of lead being formed between the cap and the carbon, which would introduce a high resistance, the end of the carbon rod is heated for an hour in paraffin wax, at a temperature of 110° C., before the cap

is cast on, then two quarter-inch holes are drilled sideways through the carbon, and the cap cast on, the lead which runs into these holes serving as rivets.

The E. M. F. of a Leclanché cell is 1·47 volts, but it falls rapidly when the cell is used to send a strong current. It will, however, regain its value if the cell be left for some time unused, and it does not sensibly diminish when the cell is put on one side, even for some months. *Hence, while the Leclanché cell is much inferior to the Daniell's for the purpose of sending a steady current for an hour or two, it is much superior to the Daniell for the sending of intermittent currents at any time during the course of many months—for example, such currents as are employed for the ringing of electric bells.*

127. Potash Bichromate Cell.—These cells are sometimes made without a porous cell, as seen in Fig. 84, and sometimes with, as seen in Fig. 85. The plates employed are of carbon and zinc, and in Fig. 84 the two outer plates are of carbon, and dip continuously into the liquid, while the middle plate is of zinc, and is only pushed down, by means of the handle *a*, into the liquid when it is desired that the cell shall send a current, and withdrawn as soon as the current is interrupted. The following is the best composition to give to the liquid :—

Potash bichromate	1 lb.
Strong sulphuric acid	2 lbs.
Water	12 lbs.

or, as it is inconvenient to weigh the sulphuric acid and the water, ten pints of the same composition may be made as follows :—Add with constant stirring to 0·832 pints of sulphuric acid, having a specific gravity of about 1·836, 0·955 lbs. of pulverised commercial potash bichromate, $K_2Cr_2O_7$; and when the formation of the chromic acid, CrO_3, and potash sulphate, K_2SO_4, produced by the mixture, is completed, pour in slowly 9·2 pints of cold water. The liquid will become gradually warm, and the crystalline precipitate be entirely dissolved.

The chemical action produced by this mixing may be represented as follows :—

$$K_2Cr_2O_7 + 7H_2SO_4 = 2CrO_3 + K_2SO_4 + H_2O + 6H_2SO_4,$$

and the chemical action that takes place in the cell during the passage of the current consists in the formation

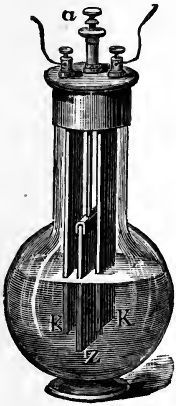

Fig. 84.

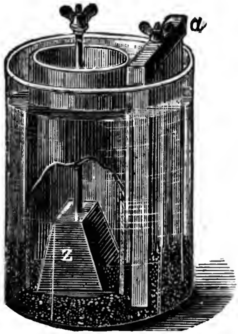

Fig. 85.

of chromium sulphate, $Cr_23(SO_4)$; zinc sulphate, $ZnSO_4$; and water, H_2O, and may be represented thus :—

$$2CrO_3 + 6H_2SO_4 + 3Zn = Cr_23(SO_4) + 3ZnSO_4 + 6H_2O.$$

This cell gives rise to no disagreeable fumes, has a high E. M. F. of something like two volts, and a low internal resistance. The E. M. F., however, rapidly falls when the cell is employed to send a strong current continuously, but recovers its original value when the cell has remained out of action for some time.

With the type of potash bichromate cell, having a porous pot, the zinc z (Fig. 85) is frequently cast, in the

form of a block, on to a stout copper wire, carrying the binding screw, and both the block and the wire are well amalgamated. In the porous pot containing the zinc, there is put a small quantity of mercury to maintain the amalgamation, and either dilute sulphuric acid, in which case the chemical action is the same as in the cell without the porous pot, or, instead, a solution of common salt, NaCl, when zinc chloride, $ZnCl_2$, is formed instead of zinc sulphate, and sodium sulphate, Na_2SO_4, in addition to the chromium sulphate. The complete chemical action is in this latter case :—

	Porous Pot.	
Before sending the current $kC+lCrO_3+3lH_2SO_4$		mNaCl$+$ n Zn,
After sending the current $kC+(l-2)CrO_3+3(l-2)H_2SO_4+Cr_2 3(SO_4)$		$(m-6)$ NaCl $+ 3Na_2SO_4 + 3ZnCl_2+6H_2O+(n-3).Zn.$

When the supply of potash bichromate becomes exhausted, the orange colour of the solution turns blue, and when this change of colour is observed, more potash bichromate should be added. If, however, the cell begins to fail when the orange colour still remains, then more sulphuric acid is needed.

As no other form of current generator than galvanic cells need be employed for any of the experiments that precede this, or, indeed, for many that follow, the description of dynamos, thermopiles, &c., will be deferred.

128. Measuring the Electromotive Force of a Current Generator. — An electrometer, or voltmeter, measures the potential difference at its terminals, and, as shown in § 116, page 204, the potential difference at the terminals of a generator of *constant* E. M. F. is equal to its E. M. F. when no current is flowing, and practically differs but very little from its E. M. F. when but an extremely small current is flowing. Hence, to measure the E. M. F. of a generator of *constant* E. M. F., we must arrange that either it shall send no current at all, or, at any rate, but a very small one. The first condition can be fulfilled when an electrometer is employed, and

the second even with a voltmeter if it has a very large resistance. In order to ascertain how large this resistance may be, we must consider the equation

$$V = \frac{r}{r + b} E;$$

and from that we see that in order that V may be practically equal to E it is necessary that r and b should be practically equal to r; that is, r must be large compared with b, and hence the battery must be sending a very small current through the voltmeter, compared with what it could produce if its terminals were joined with a short bit of thick wire. (*See* § 131, page 231, and following sections, for further details about measurement of E.M.Fs.)

129. Measuring the Resistances of Batteries.—We have already seen, in § 116, page 205, one way of determining the resistance of a battery without the aid of a resistance box, by making simultaneous measurements with an ammeter and voltmeter. This method is particularly suitable to be employed with current generators of very low resistance, such as accumulators, since such generators would send a very powerful current through any coil having a resistance comparable with their own, and this current would tend to heat such a coil, and alter its resistance, unless it were made of very thick wire. Hence, it would be very difficult to employ, with such a generator, resistance coils having perfectly constant and known resistances, unless their value, compared with the resistance of the generator, was so high that the slightest proportional error in the value of the coils would make a serious error in the determination of the resistance of the generator, just as a large error would probably be introduced if an attempt were made to weigh a few grains of some powder in a weighing machine suitable for weighing a hundredweight. Beginners are apt too frequently to forget that, although a coil of 10,000 ohms, and another of $\frac{1}{100}$th of an ohm, may be put in boxes of about the same size, there is the

same sort of difference between these resistances as between twelve pounds and one grain, or between thirty tons and one ounce, and hence that apparatus which is arranged to measure the one is totally unsuited to measure the other.

With current generators of constant E. M. F., and having higher resistances, the following methods, with which resistance coils of known value are employed, may be used.

1st. Let C and C′, as determined from the deflections on a galvanometer and reference to the relative calibration curve, be the relative strengths of the currents produced by the generator when resistances r and r' in ohms are introduced in the circuit; then, if b be the resistance in ohms of the generator, g that of the galvanometer, and E the E. M. F. in volts—which latter need not, however, be known—

$$\frac{E}{b+r+g} \div \frac{E}{b+r'+g} = \frac{C}{C'},$$

$$\text{or} \quad \frac{b+r'+g}{b+r+g} = \frac{C}{C'},$$

$$\therefore \quad b = \frac{C'(r'+g) - C(r+g)}{C - C'}.$$

If r and r' be so chosen that C is twice C′, then

$$b = r' - 2r - g.$$

2nd. Let C and C′ be the relative strengths of the currents produced: first, when the galvanometer is unshunted, and a resistance r ohms introduced in the main circuit; secondly, when the galvanometer of resistance g ohms is shunted with a shunt of s ohms, and when a resistance r' ohms is in the main circuit, then

$$\frac{E}{b+r+g} \div \frac{s}{s+g} \cdot \frac{E}{b+r'+\frac{sg}{s+g}} = \frac{C}{C'},$$

$$\therefore \frac{(s+g)(b+r')+sg}{s(b+r+g)} = \frac{C}{C'},$$

$$b = \frac{C(r+g)s - C'\{(s+g)r'+sg\}}{C'(s+g)-Cs}.$$

If s and r' be so selected by trial that C′ equals C, then we have

$$b = \frac{s(r-r')-g\,r'}{g}.$$

The objection to both these methods is that on account of the variation in the current strength, and on account of the time that each of the two currents C and C′ has to be allowed to flow until the deflection of the galvanometer needle becomes steady in each case, the E. M. F. and resistance in some types of cells is liable to undergo a change from polarisation. On this account the "*condenser method of measuring the resistance of current generators*," described in § 184, page 342, is to be preferred.

Example 50.—A Daniell's battery produces a deflection of 38° on a tangent galvanometer when a resistance of 27 ohms is inserted in the circuit, and a deflection of 46° when this resistance is reduced to 12 ohms. What is the resistance of the battery if that of the galvanometer be $2\frac{1}{2}$ ohms?

Inserting these values in the equation, we have

$$b = \frac{\tan. 38° \times (27+2\frac{1}{2}) - \tan. 46° \times (12+2\frac{1}{2})}{\tan. 46° - \tan. 38°.}$$

Answer.—$31\frac{1}{2}$ ohms about.

Example 51.—With a galvanometer having a resistance of half an ohm, and constructed so that the angular deflection is directly proportional to the current, a battery of 20 Grove's cells in series produces a deflection of 28 divisions when a resistance of two ohms is inserted, and 14 divisions when a resistance of eight ohms is inserted. What is the resistance of the battery?

If b be the resistance of the entire battery,

$$b = 8 - 2 \times 2 - \tfrac{1}{2}$$

Answer.—$3\frac{1}{2}$ ohms.

Example 52.—When four ohms are introduced into the circuit of a sine galvanometer, having 6 ohms' resistance, and a Leclanché cell, a deflection is produced corresponding with a necessary rotation of the sine galvanometer through 22°. When, however, the sine galvanometer is shunted with two ohms, the rotation required is only 8°. What is the resistance of the Leclanché cell?

Substituting the values in the equation, we have

$$b = \frac{\sin. 22° \times (4+6) \times 2 - \sin. 8° \times \{(2+6) \times 4 + 2 \times 6\}}{\sin. 8° \times (2+6) - \sin. 22° \times 2}.$$

Answer.—4 ohms about.

Example 53.—The same deflection is produced on a galvanometer of $2\frac{1}{2}$ ohms' resistance, when 8 ohms are in circuit, as when only 2 ohms are in circuit, and the galvanometer is shunted with 2 ohms. What is the resistance of the current generator?

$$b = \frac{2 \times (8-2) - 2\frac{1}{2} \times 2}{2\frac{1}{2}},$$

Answer.—$2\frac{4}{5}$ of an ohm.

In making measurements of the resistance of batteries by any of the foregoing methods, care must be taken not to introduce into the circuit resistances that are very large compared with the resistance of the battery which we desire to find, since any error in such a high resistance will probably introduce a large error into the answer. For example, suppose it be desired to use a galvanometer which happens to be so delicate that on attaching the battery directly to its terminals, so large a deflection is produced that it requires a considerable resistance to be introduced into the circuit to reduce this deflection to readable limits, then it would be better to reduce the practical sensibility in some other way than by adding resistance

in the main circuit. This may be done either by putting a magnet near the galvanometer or by shunting it. In the latter case the shunted galvanometer would take the place of the simple galvanometer in the first method given above for determining the resistance of a battery, and of the unshunted galvanometer in the second method; the second experiments referred to in the second method being performed with the galvanometer shunted with a *different* shunt.

For example, suppose we desire to determine the resistance of a battery that we know to be about one ohm, and the only galvanometer available is a very delicate one, having 1,000 ohms' resistance, how should we proceed? The deflection can be reduced to readable limits either by inserting a large resistance into the circuit, or by putting a magnet near the galvanometer, or by shunting it. As the resistance of the galvanometer is 1,000 ohms, which is large compared with that of the battery, introducing another large resistance into the circuit for the purpose of diminishing the deflection would only increase the probable error due to the large resistance in the circuit. Putting a magnet near the galvanometer would be better than this, but a still better method would be to shunt the galvanometer, because, if it be very sensitive, a suitable deflection may be obtained with a shunt perhaps of one or two ohms, and with one or two ohms in the main circuit. Suppose with a shunt of two ohms, and a resistance of three ohms in the main circuit, a deflection extending over about half the scale is obtained, then this arrangement can be well used, either for the first or for the second method of measuring the battery resistance. For carrying out the first method, we may make two tests, the first with the three ohms, and the second with, say, one-and-a-half ohms in the main circuit, the galvanometer being shunted in each case with the two ohms, and having, therefore, a combined resistance with the shunt of $\frac{2 \times 1000}{2 + 1000}$ ohms. For carrying out the second method we

might make the same first test as before, but the second might be made with an interposed resistance of perhaps one-and-a-half ohms in the main circuit, and with the galvanometer shunted with, say, one ohm instead of the two ohms previously employed.

To ascertain what is the formula to be employed in this case, let r and r' be, as before, the resistances put into the main circuit in the two tests, and s and s' the two shunts employed, then

$$\frac{s}{s+g} \cdot \frac{\text{E}}{b+r+\dfrac{sg}{s+g}} \div \frac{s}{s'+g} \cdot \frac{\text{E}}{b+r'+\dfrac{s'g}{s+g}} = \frac{\text{C}}{\text{C}'},$$

$$\text{or } \frac{s}{s'} \cdot \frac{(s'+g)(b+r')+s'g}{(s+g)(b+r)+sg} = \frac{\text{C}}{\text{C}'},$$

$$\therefore\ b = \frac{\text{C}s'\{(s+g)r+sg\} - \text{C}'s\{(s'+g)r'+s'g\}}{\text{C}'s(s'+g) - \text{C}s'(s+g)}.$$

If the battery be one that does not polarise quickly, that is, be one in which the E. M. F. does not fall rapidly when the battery sends strong currents, then the best way of carrying out the first method of measuring the resistance of the current generator with a delicate galvanometer, is to put no resistance r in the main circuit, but to shunt the galvanometer with a shunt that has a very small resistance compared with the battery, and yet is not so small but that a suitable deflection may be obtained. Now introduce such a resistance r' into the circuit that the current through the galvanometer becomes halved, then this resistance is necessarily equal to b, since b was practically the whole of the resistance in the circuit before the introduction of r'.

130. P. D.—Throughout the remainder of this book the letters "*P. D.*" will be used to stand for potential difference, in the same way as the letters E. M. F. are universally now employed to stand for electromotive

force. As these letters P. D. are here proposed as a new abbreviation, the ordinary cumbersome expression, "difference of potentials," has been used up to this point in the book, in order to familiarise the reader with the meaning of an expression that he will frequently meet with.

131. Comparing the Electromotive Forces of Batteries.—The relative electromotive forces E and E′ of the batteries, or other current generators of *constant* E. M. F., can be compared by observing the resistance through which they will send equal currents. Let b and b' be the resistances of the batteries themselves, and r and r' the resistances, including in each case that of the galvanometer, which, added to the resistances b and b' respectively, cause the currents in the two cases to be equal, then

$$\frac{\mathrm{E}}{b+r} = \frac{\mathrm{E}'}{b'+r'},$$

$$\therefore \frac{\mathrm{E}}{\mathrm{E}'} = \frac{b+r}{b'+r'}.$$

If the galvanometer is sensitive, so that r and r', which each include the resistance of the galvanometer, are large compared with b and b' respectively, then

$$\frac{\mathrm{E}}{\mathrm{E}'} = \frac{r}{r'} \text{ approximately.}$$

The preceding method of comparing E. M. Fs. has the advantage that the law of the galvanometer need not be known.

If the currents be not the same, let C and C′ be the relative current strengths obtained from the deflection of the galvanometer and reference to the calibration curve, then

$$\frac{\mathrm{E}}{b+r} \div \frac{\mathrm{E}'}{b'+r'} = \frac{\mathrm{C}}{\mathrm{C}'},$$

$$\text{or } \frac{\mathrm{E}}{\mathrm{E}'} = \frac{b+r}{b'+r'} \cdot \frac{\mathrm{C}}{\mathrm{C}'}.$$

And, as before, when r and r' are large compared with b and b' respectively,

$$\frac{E}{E'} = \frac{r}{r'} \cdot \frac{C}{C'} \text{ approximately.}$$

Another method for determining the ratio of E to E′ consists in first joining the batteries up together so that they assist one another in sending a current, and secondly in joining them up so as to oppose one another's action. Let C and C′ be the relative strength of the currents in the two cases ascertained from the deflection of the galvanometer and the relative calibration curve, then if p be the total resistance in circuit in the two cases, we have, since p remains constant,

$$\frac{E + E'}{p} \div \frac{E - E'}{p} = \frac{C}{C'},$$

$$\text{or } \frac{E + E'}{E - E'} = \frac{C}{C'},$$

$$\therefore \frac{E}{E'} = \frac{C + C'}{C - C'}.$$

This method has the advantage that *the resistances of neither of the batteries nor of the galvanometer need be known*; but it has the *disadvantage* that the sending of currents in opposite directions through the battery which has the smaller electromotive force is very likely to alter this electromotive force during the experiment.

Example 54.—Two batteries having internal resistances of 10 and 15 ohms produce the same deflection on a galvanometer of 40 ohms, when 250 and 305 ohms are respectively introduced into the circuit. What is the ratio of their E. M. Fs.?

Substituting the values in the equation, we have

$$\frac{E}{E'} = \frac{10+40+250}{15+40+305},$$

$$\therefore E' = 1{\cdot}2\,E.$$

Example 55.—The same two batteries produce the same deflection on a much more delicate galvanometer, having 120 ohms' resistance, when 5,000 and 6,031 ohms are respectively introduced into the circuit. What is the ratio of their E. M. Fs.?

Using the complete formula, we have

$$\frac{E}{E'} = \frac{10 + 120 + 5000}{15 + 120 + 6031},$$

$$\text{or } E' = 1{\cdot}2\ E \text{ as before.}$$

Using the approximate formula,

$$\frac{E}{E'} = \frac{5000}{6031},$$

$$\text{or } E' = 1{\cdot}206\ E,$$

from which we see the error made by omitting the resistances of the batteries and of the galvanometer in the calculation.

Example 56.—A magneto-electric machine running at a certain speed, and having a resistance of two ohms, produces on a tangent galvanometer a deflection of 30° when a resistance of 2,100 ohms is introduced in circuit with it and the galvanometer, which has three ohms' resistance. A Daniell's cell, on the other hand, having an E. M. F. of 1·07 volts, and one-and-a-half ohms' resistance, produces a deflection of 45° when 84 ohms is introduced in the circuit. What is the E. M. F. of the magneto machine?

If E be the E. M. F. of the machine,

$$E = 1{\cdot}07 \frac{2100}{3+1{\cdot}5+84} \times \frac{\frac{1}{\sqrt{3}}}{1} \text{ volts approximately.}$$

Answer.—14·7 volts approximately.

Example 57.—What about is the E. M. F. of a Grove's cell, if, when joined so as to assist a Daniell's

cell having an E. M. F. of 1·1 volts, a rotation of 38° of a sine galvanometer is necessary to be made to bring the needle to the fixed mark, whereas, when the Grove's cell is reversed, a rotation of about $8\frac{1}{3}°$ in the opposite direction is necessary? *Answer.*—1·83 volts.

132. Poggendorff's Method of Comparing Electromotive Forces.—With many types of cells the electromotive force is fairly constant, even for wide variations in the current passing through the cells, and in such a case any of the previous methods can be employed for comparing their electromotive forces. But with other types, a very small current passing through the cell is sufficient

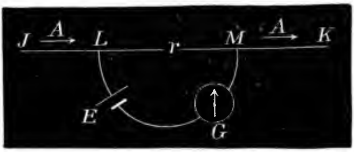

Fig. 86.

to diminish the electromotive force. In such a case the following method, due originally to Poggendorff, may be employed. From what has preceded we know that if a current of A amperes flow along a wire, J K, the potential difference, or, shortly, the P. D., in volts between any two points, L M, is equal to the product of A into the resistance r of the wire, in ohms, between the points L and M. Hence if L and M (Fig. 86) be joined by another circuit containing a cell or battery of E. M. F. equal to E and a galvanoscope, G, and if one or both of the ends of this second circuit be moved along the wire J K composing the first circuit until no current passes through the galvanoscope G, then we know that E is equal and opposite to the P. D. between L and M, or

$$E = A\,r.$$

If, now, a second battery of E. M. F. equal to E′, and a second galvanoscope, G′, be attached to two other points,

U V, of the wire J K (Fig. 87), the points U and V being so selected by trial that no current passes through this galvanoscope, and if r' be the resistance of the wire U V, then

$$E' = A\,r',$$

$$\therefore \frac{E}{E'} = \frac{r}{r'};$$

and hence the two E. M. Fs. can be compared without our knowing the value of the current flowing through the wire J K. If the generator is of such a nature as to produce a *constant* current through the wire J K, then

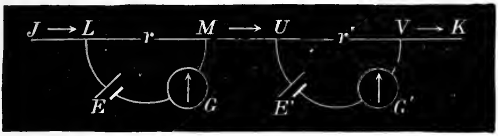

Fig. 87.

there is no occasion to use two galvanoscopes, as the points L and M can be first ascertained with the first cell, and then the points U V with the second, such that in each case no current passes through the galvanoscope. If, however, the current in J K is liable to fluctuate, then, since the essence of the test depends on the same currents flowing from L to M as from U to V, it is better to use two galvanoscopes, and make the two tests of no currents through the galvanoscopes simultaneously.

Of course, care must be taken to attach the cells or batteries whose E. M. F. we desire to compare, in such a way that their E. M. Fs. tend to oppose the potential differences between L and M and between U and V respectively, since, if either of the cells or batteries be attached in the opposite way, no two points, L and M or U and V, can, of course, be found such that the current passing through the galvanoscope attached to them is nought.

If the wire J K is everywhere uniform in material,

section, and temperature, the resistances r and r' are simply proportioned to the lengths L M and U V, so that the E. M. Fs. of the batteries are simply proportioned to the lengths of L M and U V.

The great advantage of Poggendorff's method of comparing E. M. Fs. is that the comparison is made when neither of the batteries is sending a current; hence the same result is obtained as if the comparison had been made with an electrometer, and the resistances of the cells under comparison need not be known. And, further, the sensibility of the test may be far greater than could be obtained with any electrometer, since the method is a "*null*" method, that is, we aim at obtaining a deflection *nought*, instead of measuring the deflections corresponding with the currents produced by the batteries; consequently the galvanoscope may be made as sensitive as we please.

If the galvanometers G and G′ be both sensitive, the accuracy of the method will be the greater the longer are the wires L M and U V, because any given small error in the position of one of the sliders corresponding with say a millimetre in the length of the wire, will represent a less proportional error in the length, and so r and r' can the more accurately be compared. Hence it is desirable to make the wire J K *as long as possible*, and to send through it a steady current, so weak that the P. D., at its *extreme ends*, is just equal to the larger of the two E. M. Fs. to be compared. (*See* § 215, page 413.)

133. Electromotive Force of a Cell is Independent of its Size and Shape.—The Daniell's cell (Fig. 88) is so arranged that the copper plate C, which dips into a solution of copper sulphate, may be made to approach, or recede from, the zinc plate Z, which dips into a solution of zinc sulphate contained in a porous cell. By turning the screw P, the slider, carrying the wire supporting C, can be clamped in any position, and electric connection can be made with the binding screws B B. Experiments made with this cell show that, *although the resistance of*

the cell is varied by moving the copper plate, the E. M. F. remains exactly the same. Further, if the screws s s be

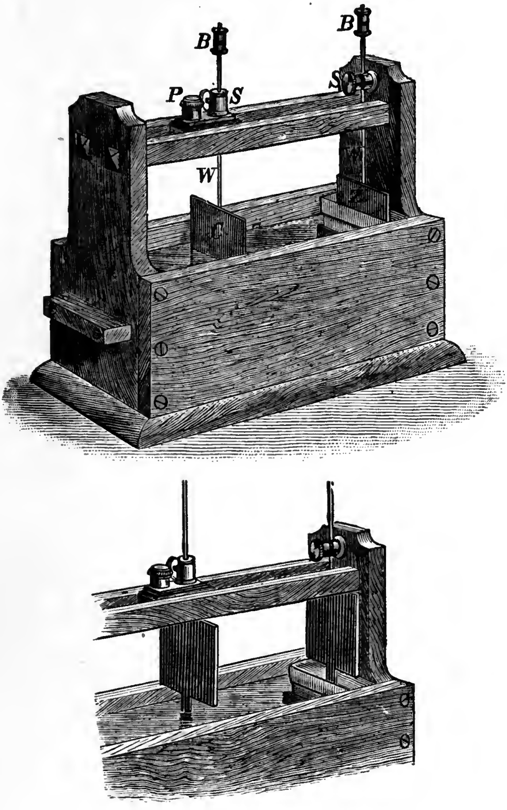

Fig. 88.

loosened, and the copper and zinc plates be raised up as shown in the lower figure, so that *only the little projections at the bottom of these plates are in contact with the*

liquids, the E. M. F. is still unaltered. This experiment may be quickly made by using Poggendorff's method to compare the E. M. F. of the cell with movable plates with that of a Daniell's cell with fixed plates, since, as already explained, Poggendorff's method is independent of the resistance of the cells compared. The condenser method of comparing E. M. Fs., described in § 183, page 341, may conveniently be used in place of Poggendorff's method.

134. Calibrating a Galvanometer by Employing Known Resistances and a Cell of Constant E. M. F. —We have seen, in § 26, page 58, that a galvanometer can be calibrated by direct comparison with a tangent galvanometer; also in § 30, page 64, that when the controlling force is that produced by a uniform magnetic field, and when also the galvanometer can be easily turned backwards and forwards round its centre, the employment of the sine principle enables us to calibrate it without the use of any other galvanometer. We have also seen, in § 96, page 164, that when we have no other galvanometer at hand that has been already calibrated, and when the galvanometer cannot be moved without interfering with its adjustment, which is generally the case when we are employing a galvanometer with fibre suspension and levelling screws, we may calibrate the galvanometer by employing known resistances, when a constant P. D. is maintained at the terminals of the circuit.

The same thing may be done without having a constant P. D. between the terminals (Fig. 61, § 96, page 165), if we have a cell of *constant* E. M. F. of E volts instead. Let b ohms be the resistance of the cell, then, if $d_1°$, $d_2°$, $d_3°$, &c., be the deflections on the galvanometer, when r_1, r_2, r_3, &c., ohms are the resistances respectively in R, we know that the currents producing these deflections are respectively

$$\frac{E}{b+g+r_1},\ \frac{E}{b+g+r_2},\ \frac{E}{b+g+r_3},\ \text{\&c., amperes,}$$

so that an absolute calibration curve can be drawn for this galvanometer.

If the E. M. F. of the cell is not known in volts, but if we are sure that it is constant, we can draw the relative calibration curve, although not the absolute one.

In order to see quickly the kind of law connecting deflection and current for any particular galvanometer, it is convenient in making this experiment to select values of R, such that $b + g + r_2$ equals $\frac{1}{2}(b + g + r_1)$, $(b + g + r_3)$ equals $\frac{1}{3}(b + g + r_1)$, &c., since in that case the second current is double the first, the third thrice the first, &c. Of course r_1 should be chosen so that the deflection corresponding with this resistance is a conveniently small one, for example, about 10° in an ordinary galvanometer having a scale reading up to 90°.

135. Arrangements of Cells.—A battery may be formed of galvanic cells, or elements, as they are some-

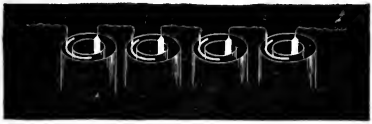

Fig. 89.

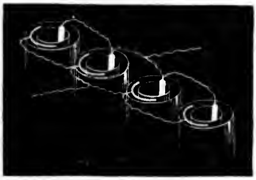

Fig. 90.

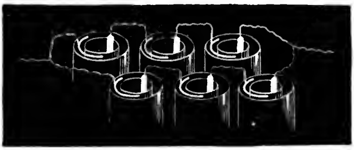

Fig. 91.

times called, in a variety of ways. All the cells may be "*in series*," as in Fig. 89, or they may be joined up all "*in parallel*," as in Fig. 90, or "*partly in series and partly in parallel*," as in Fig. 91. These three arrangements are symbolically shown in A, B, C (Fig. 92), where the long thin lines stand for the plates in the

battery from which the positive electricity flows; or, with the definition of direction of current we have already adopted, the current flows in the circuit outside the battery from the plate represented by the long thin line to that represented by the short thick line, while in the battery itself the current flows from the short thick line to the long thin one.

For example, in the *Daniell's* cell, which consists, as previously described in § 119, page 210, of a plate of copper in a solution of copper sulphate, separated by a

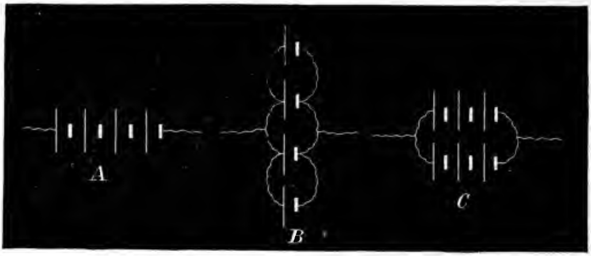

Fig. 92.

porous diaphragm of unglazed earthenware from a plate of zinc in a solution of zinc sulphate, the long thin line represents the copper plate, and the short thick one the zinc plate; the wavy line in each case stands for the copper wires attached to the copper and zinc plates respectively. In the *Grove's* cell, consisting, as we have seen in § 124, page 218, of a platinum plate in strong nitric acid, separated by a porous cell from a plate of zinc in dilute sulphuric acid, the long thin line represents the platinum plate, and the short thick line the zinc plate. In a *Bunsen's* cell, which, as explained in § 125, page 219, differs only from a Grove's in that the platinum plate is replaced by a carbon one, the long thin line stands for the carbon plate.

When all the cells are in series, the total current produced by the battery passes through each cell; there-

fore it follows, from what has preceded (§ 115, page 203), that the E. M. F. of the battery is equal to the sum of the E. M. Fs. of each of the cells. If, on the other hand, the cells are joined up all in parallel, the current divides itself between the cells; and if the cells are all made with the same materials, but not necessarily of the same size nor of the same internal resistance, the total chemical action, and therefore the total amount of fuel burnt per second, is exactly the same as if the entire current went through one of the cells. Hence the E. M. F. of the battery is simply that of any one of the component cells. The resistance, however, of the battery will be less than that of one cell, as the road for the current through the battery is made wider by putting cells in parallel; and if the cells have each the same resistance of b ohms, and if there be p of them in parallel, the resistance of the battery is $\frac{b}{p}$ ohms. If the cells be partly in series and partly in parallel, we must combine the last two sets of conclusions, so that if the E. M. F. of each cell be e volts, and if there be s cells in series, and p in parallel, the total E. M. F. of the battery E, and the total resistance B, will be given by

$$\text{E} = s\,e \text{ volts,}$$

$$\text{B} = \frac{s\,b}{p} \text{ ohms;}$$

so that if A be the current in amperes which the battery sends through an external resistance r,

$$\text{A} = \frac{s\,e}{r + \frac{s\,b}{p}}.$$

In order to experimentally test the accuracy of these results, a number of cells, freshly put together, and having their corresponding plates of the same size, the plates in the different cells at the same distance apart, and the amount of liquid in each cell the same, should be joined

up in a variety of ways, and the resistances of the combinations measured, as well as the E. M. Fs. of the batteries compared with the E. M. F. of a single cell, selected at random from the battery, by one or other of the methods of testing previously given. The cells should be of such a type that the E. M. F. of each cell is a constant, a condition very satisfactorily fulfilled with Daniell's cells, and to avoid the cell used as the standard having a higher or a lower E. M. F. than the average E. M. F. of the cells employed, different cells may be selected from the combination as the standard cell in the different experiments.

Example 58. — To find the current that twelve Daniell's cells, each having a resistance of 0·6 ohm and an E. M. F. of 1·1 volt, can send through an external resistance of 5 ohms if the cells be formed four in series and three parallel:

$$A = \frac{4 \times 1{\cdot}1}{5 + \dfrac{4 \times 0{\cdot}6}{3}}.$$

Answer.—0·76 ampere.

Example 59.—How many such Daniell's cells must be used in series to send a current of 1 ampere through an external resistance of 8 ohms, if one line of cells in series only be employed?

Let x be the required number of cells, then

$$1 = \frac{x \times 1{\cdot}1}{8 + x \times 0{\cdot}6},$$

$$\therefore x = 16.$$

Example 60.—If in the last question the current be 2 amperes instead of 1, then how many cells will be required?

$$2 = \frac{x \times 1{\cdot}1}{8 + x \times 0{\cdot}6},$$

$$\therefore x = -160.$$

Therefore no number of such cells put in one line in series could send this current. In fact, if one cell be short-circuited with a piece of thick wire, the current it will send will be $\frac{1\cdot1}{0\cdot6}$, or 1·83 amperes, and this is the maximum current one, or any number of cells, arranged simply in series, can send. For if there be n of them arranged in series, and the whole be short-circuited, the current will be $\frac{n \times 1\cdot1}{n \times 0\cdot6}$ or 1·83 amperes, or, simply, the current sent by one cell when short-circuited. Hence, if there be any external resistance, the current sent by one row of these cells in series, no matter how many there may be in the row, will be less than 1·83 amperes.

Example 61.—Forty exactly similar cells, each having an internal resistance of $\frac{3}{4}$ ohm, when joined in series send a current of 0·5 amperes through an incandescent lamp of 80 ohms' resistance: how many cells in series would be required to produce the same current through each of two such lamps arranged in parallel?

Let e be the E. M. F. of one cell in volts, then

$$\frac{40 \times e}{80 + 40 \times 0\cdot75} = \tfrac{1}{2},$$

$$\therefore\ e = 1\cdot375 \text{ volts};$$

therefore, if x be the required number of cells,

$$\frac{x \times 1\cdot375}{\frac{80}{2} + x \times 0\cdot75} = 1,$$

since the resistance of the two lamps in parallel will be $\frac{80}{2}$ ohms, and they will require together 1 ampere,

$$\therefore\ x = 64.$$

136. Arrangement of a Given Number of Cells to produce the Maximum Current through a given Ex-

ternal Resistance.—If N be the total number of cells employed in a battery, p being arranged in parallel, and s in series,

$$N = p\,s,$$

and the formulæ on page 241 may be written

$$A = \frac{s.e}{r + \dfrac{s^2 b}{N}}.$$

If, therefore, we desire to ascertain what arrangement of a definite number of cells, each having a fixed E. M. F. of e volts, and internal resistance b ohms, will give the greatest current through a *fixed external resistance of r ohms*, we must ascertain what value of s will make the last expression a maximum. But to do this by trial by calculating the value of A corresponding with each of a very large number of values of s would be extremely laborious, and a far better plan for those who are not acquainted with the differential calculus is as follows:—

Give numerical values to e, r, and $\frac{b}{N}$, let them for example be 2, 3, and 4, then the expression becomes

$$\frac{2s}{3 + 4s^2};$$

next draw a curve having the values of s for the abscissæ, and the corresponding value of the expression for the ordinates, and ascertain, from the shape of the curve, for what value of s the expression has its maximum value, then that value of s is the value required. In selecting values for s, a certain amount of practice is, of course, necessary, in order to select the best values, but one may be guided by remembering that if on taking two or three values of s we obtain practically the same value for the expression for A, it can be no use taking intermediate values of s.

The curve obtained for A has the general shape shown

in Fig. 93, the values of A being calculated on the supposition that e, r, and $\frac{b}{N}$, have the values 2, 3, and 4 re-

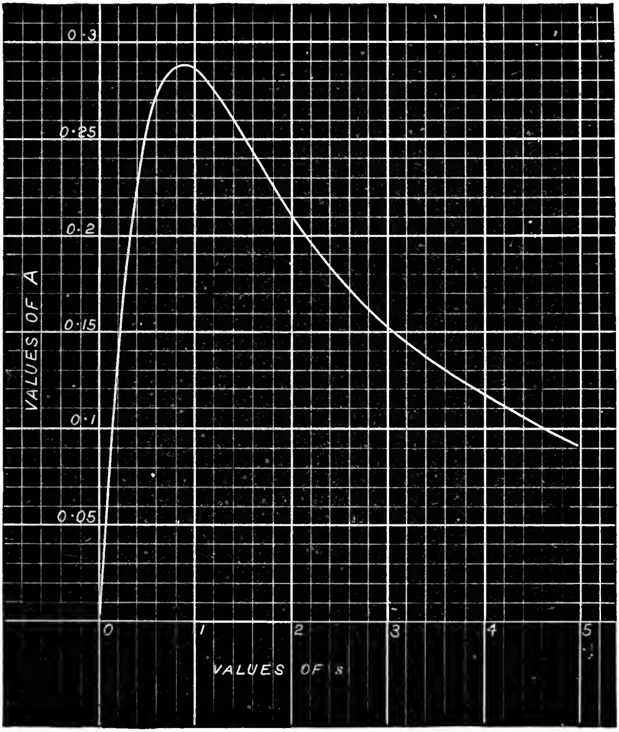

Fig. 93.

spectively, and we find that the value of s that makes A a maximum is about 0·85, and this is the value of s which makes

$$\frac{s^2 b}{N} = r,$$

or, in other words, the proper arrangement of a *given number* of cells to send the maximum current through a *given external* resistance is that which makes the resistance of the battery equal to the external resistance.

The curve falls more slowly for values of s greater than that which makes A a maximum than for values less than this, and this tells us that the current will be not so much lessened by making s too large as it will be by making it too small; hence if the number of cells and the resistance of each are such that it is impossible to arrange the battery so that its internal resistance is equal to the fixed external resistance, it is better, when the external resistance is midway between the resistances the battery has when arranged in these two ways, to select the arrangement that puts rather too many cells in series than the one that puts rather too many in parallel. For example, suppose we have twelve cells, each having a resistance of 3 ohms, and we desire to arrange them so that they send a maximum current through an external resistance of $3\frac{1}{8}$ ohms, if we arrange them three in series and four in parallel, the resistance of the battery will be

$$\frac{3 \times 3}{4} \text{ or } 2\tfrac{1}{4} \text{ ohms,}$$

on the other hand, if we put them four in series and three in parallel, the resistance will be

$$\frac{4 \times 3}{3} \text{ or } 4 \text{ ohms;}$$

and the given external resistance of $3\frac{1}{8}$ ohms is exactly half-way between $2\frac{1}{4}$ and 4. Let us consider the currents produced by these two arrangements of the cells. With the first,

$$A = \frac{3e}{3\frac{1}{8} + 2\frac{1}{4}} \text{ amperes,}$$

if e be the E. M. F. of each cell in volts. With the second arrangement,

$$A = \frac{4e}{3\frac{1}{8} + 4} \text{ amperes.}$$

The first reduces to $\frac{24}{43}e$ and the second to $\frac{32}{57}e$ ampere, and of these the second is the greater by $\frac{8}{2451}e$ of an ampere.

Example 62.—What is the least number of Grove's cells, each having an E. M. F. of 1·8 volts, and an internal resistance of 0·09 ohm, that must be arranged in series to send half an ampere through a 50 volt incandescent lamp?

This question may be solved in two ways—we may either first find the resistance of the lamp and then the number of such Grove's cells that it is necessary to put in series to send half an ampere through this external resistance—or we may consider what is the P. D. at the terminals of such a Grove's cell, when half an ampere is passing through, and hence deduce how many such cells must be put in series so that when half an ampere is passing through them, the P. D. at the terminals of the battery is 50 volts.

$$\begin{aligned} \text{1. The resistance of the lamp} &= \frac{50}{\frac{1}{2}} \text{ ohms,} \\ &= 100 \text{ ohms,} \end{aligned}$$

$\therefore$ if n be the required number of cells,

$$\frac{1}{2} = \frac{n \times 1{\cdot}8}{n \times 0{\cdot}09 + 100},$$

$$\therefore n = 28{\cdot}5.$$

Hence, 28 cells would produce rather too small a current, and 29 rather too much. We should have, therefore, to choose between using 28 cells and having the lamp not quite bright enough, or using 29 cells and having it a

little too bright, or using 29 cells and interposing a small resistance by means of a piece of wire or in any other convenient way.

2. If n be the number of cells in series, then from § 116, page 206, the P. D. maintained at the terminals of the battery equals

$$n \times 1{\cdot}8 - \tfrac{1}{2} n \times 0{\cdot}09.$$

And this is to equal 50.

Hence,

$$n \times 1{\cdot}8 - \tfrac{1}{2} n \times 0{\cdot}09 = 50,$$

which is the same equation as was used before, and therefore must lead to the same value of n.

Example 63.—If 29 cells were used in series in the last question, what must be the value of the added resistance, so that the current through the lamp may be exactly half an ampere?

Let x be the required resistance in ohms, then

$$\frac{1}{2} = \frac{29 \times 1{\cdot}8}{29 \times 0{\cdot}09 + 100 + x},$$

$$\therefore \; x = 0{\cdot}895 \text{ ohm}.$$

Example 64.—If four incandescent lamps, each requiring half an ampere, and 50 volts P. D. maintained at the terminals, are to be fed with Grove's cells, each having an E. M. F. of 1·8 volts, and an internal resistance of 0·1 ohms, what arrangement of cells and of lamps will require the least number of cells to be used?

First, let the four lamps be put in series, and let all the cells, n in number, be in series, then the P. D. at the terminals of the battery must be 4×50, and

$$n \times 1{\cdot}8 - \tfrac{1}{2} n \times 0{\cdot}1 = 4 \times 50,$$

$$\therefore \; n = 114{\cdot}3.$$

Next, let all the lamps be put in parallel, and all the

cells in series, then the total current required will be $4 \times \frac{1}{2}$ or 2 amperes, therefore

$$n \times 1{\cdot}8 - 2n \times 0{\cdot}1 = 50,$$
$$\therefore n = 31{\cdot}2;$$

hence, 32 cells, with a small resistance interposed, would give the required current, and this arrangement of all the lamps in parallel would only require about one-quarter of the number of cells necessary if all the lamps were in series.

Various other cases might be tried; for example, the lamps two in series, and two in parallel, or the cells two in parallel and half in series; but it would be found that all the cells in series, and all the lamps in parallel, is the best arrangement.

Example 65.—If 40 such lamps as are referred to in the last few questions instead of 4 had to be fed with Grove's cells, what would be the best arrangement of the cells and of the lamps?

First, let us try all the lamps in parallel, and all the cells in series, which arrangement we found was the best in the previous case, then, as the total current required will be $40 \times \frac{1}{2}$ or 20 amperes, and the P. D. at the terminals of the battery 50 volts,

$$n \times 1{\cdot}8 - 20n \times 0{\cdot}1 = 50,$$
$$\text{or} \quad n = -250,$$

a negative answer. This means that no number, no matter how great, of such Grove's cells, if the cells were arranged in series, could feed 20 such lamps if arranged in parallel; and the reason of this is clear, because, if one Grove's cell were simply short-circuited, the current that it would produce would be

$$\frac{1{\cdot}8}{0{\cdot}1} \text{ or } 18 \text{ amperes,}$$

hence, no number of such Grove's cells arranged in series can produce more than 18 amperes, even if short-cir-

cuited, and hence they can only produce less than 18 amperes if there be any external resistance, whereas we want them to produce 20 amperes.

Secondly, let us try half the lamps in parallel and two in series. In that case the total current must be 10 amperes, and the P. D. 100 volts.

Hence we have

$$n \times 1{\cdot}8 - 10\,n \times 0{\cdot}1 = 100,$$
$$\text{or } n = 125.$$

We may now try all the lamps in parallel and half the cells in series, and two in parallel. Let n be the number in series, that is, half the total number, then

$$n \times 1{\cdot}8 - 20\,\frac{n \times 0{\cdot}1}{2} = 50,$$
$$\therefore\ n = 62{\cdot}5.$$

Consequently the total number of cells required is 125. Hence, whether we put the 40 lamps two in series and 20 in parallel, and use all the cells in series, or put half the cells in series and two in parallel, and use all the lamps in parallel, exactly the same number, 125, of cells is required.

There is one other arrangement that might be tried, viz., all the lamps and all the cells in series, but from what we saw in the first part of example No. 64, we may anticipate that this will be a very bad arrangement. With this arrangement the current required will be half an ampere, the P. D. 40 × 50 volts,

$$\therefore\ n \times 1{\cdot}8 - \tfrac{1}{2}\,n \times 0{\cdot}1 = 2{,}000,$$
$$\text{or } n = 1{,}142{\cdot}9.$$

Hence 1,143 cells would be required with this arrangement.

Example 66.—How many Daniell's cells, each having an E. M. F. of 1·1 volts, and an internal resistance of 0·8 ohms, would be required to feed two Edison incandescent lamps, each requiring 0·75 of an ampere, and 110 volts at its terminals?

One such Daniell's cell, short-circuited, would produce

$$\frac{1{\cdot}1}{0{\cdot}8} \text{ or } 1{\cdot}375 \text{ amperes,}$$

hence, if we put the lamps in series, one row of Daniell's cells in series will produce sufficient current. If, however, we put the two lamps in parallel, then, since the total current must be 1·5 amperes, we must have two rows of cells.

First, let the lamps and cells be in series, then

$$n \times 1{\cdot}1 - 0{\cdot}75\, n \times 0{\cdot}8 = 220,$$
$$\text{or } n = 440.$$

Second, let the cells be half in series and two in parallel, and let n be the number in series, the lamps being still in series, then

$$n \times 1{\cdot}1 - 0{\cdot}75 \frac{n \times 0{\cdot}8}{2} = 220,$$
$$\text{or } n = 275.$$

Hence, the total number of cells necessary will be 550, or this arrangement is worse than the preceding.

Third, let the cells be half in series and two in parallel, but let the lamps be also in parallel, then

$$n \times 1{\cdot}1 - 1{\cdot}5 \frac{n \times 0{\cdot}8}{2} = 110,$$
$$\text{or } n = 220.$$

Hence, the total number of cells required is 440, or the same as in the first case.

Fourth, let the cells be three in parallel and n in series, and let the two lamps be still in parallel, then

$$n \times 1{\cdot}1 - 1{\cdot}5 \frac{n \times 0{\cdot}8}{3} = 110,$$
$$\therefore\ n = 157{\cdot}1,$$

and the total number of cells required would be 472.

Therefore, arrangements Nos. 1 and 3 require the least number of cells, but with any arrangement the number of Daniell's cells required is very large in consequence of the high resistance of the cells, and of the fact that the greater part of the energy is expended in sending the current through the cells themselves.

Example 67.—How many lamps in parallel, each requiring 80 volts, and 0·6 of an ampere, can be fed with 42 accumulators in series, each having 1·95 volts E.M.F. on discharging, and 0·005 ohms' internal resistance?

Let l be the number of lamps, then, since the total current will be $l \times 0{\cdot}6$, we have

$$42 \times 1{\cdot}95 - l \times 0{\cdot}6 \times 42 \times 0{\cdot}005 = 80.$$

Answer.—15.

Example 68.—If the number of accumulators in the last question be increased by one, by how many may the number of lamps be increased?

Answer.—The number of lamps may now be 29·8, that is, may be 30 all a trifle too dull, or 29 a trifle too bright, unless a small resistance be introduced. The addition, therefore, of one accumulator practically doubles the number of lamps that can be fed by them.

Example 69.—If there be 44 accumulators in series, and if 46 lamps be fed by them, each lamp requiring, as before, 80 volts at its terminals, and 0·6 of an ampere passing through it when properly glowing, how much per cent. will the current passing through the lamps be too great or too small?

The resistance of each lamp is $\frac{80}{0{\cdot}6}$ or 133·3 ohms, hence the resistance of all the lamps will be $\frac{133{\cdot}3}{46}$ or 2·899 ohms, consequently the current passing through them will be

$$\frac{44 \times 1{\cdot}95}{44 \times 0{\cdot}005 + 2{\cdot}899} \text{ amperes,}$$
$$\text{or } 27{\cdot}51 \quad \text{,,}$$

The current that ought to pass through the lamps is $46 \times 0{\cdot}6$, or $27{\cdot}6$ amperes. Hence the current is about $0{\cdot}3$ per cent. too small.

137. Variation produced in the Total Current by Shunting a Portion of the Circuit.—We can now calculate the *entire* effect produced on the current passing through a galvanometer of resistance g, by shunting the galvanometer with a shunt of resistance s. Let E be the E. M. F. in volts, and b the resistance in ohms, of a battery, r the resistance in ohms of the rest of the circuit, excluding the galvanometer, and g the resistance of the galvanometer; then, before shunting, the current G_1, in amperes, that passes through the galvanometer, is simply the whole current A_1, that passes through the battery, and this equals

$$\frac{E}{b + r + g} \text{ amperes.}$$

After shunting, the current A_2, now flowing through the battery, becomes

$$\frac{E}{b + r + \dfrac{s\,g}{s + g}} \text{ amperes,}$$

and the fraction $\dfrac{s}{s + g}$ of this passes through the galvanometer; therefore, if G_2 be the current now passing through the galvanometer,

$$G_2 = \frac{s}{s + g} \cdot \frac{E}{b + r + \dfrac{s\,g}{s+g}}$$

$$= \frac{s\,E}{(s + g)(b + r) + s\,g}.$$

If $b + r$ be very large compared with g, then, approximately,

$$G_2 = \frac{s}{s+g} \cdot \frac{E}{b+r},$$

$$\text{and } A_2 = \frac{E}{b+r},$$

$$\text{also } A_1 = \frac{E}{b+r},$$

$$\therefore \quad G_2 = \frac{s}{s+g} \cdot A_1,$$

that is to say, the current passing through the battery and through r is practically unchanged by shunting the galvanometer, and, therefore, after the galvanometer has been shunted, it is not merely the fraction $\frac{s}{s+g}$ of A_2, but $\frac{s}{s+g}$ of A_1, that passes through the galvanometer.

On the other hand, if $b + r$ be small compared with g, then, approximately,

$$G_2 = \frac{sE}{sg}$$

$$= \frac{E}{g},$$

$$\text{and } G_1 = A_1 = \frac{E}{g},$$

$$\therefore \quad G_2 = G_1 \text{ approximately.}$$

Hence, as long as $\frac{sg}{s+g}$ is large compared with $b + r$, that is, as long as the shunted galvanometer is the major part of the whole resistance in the circuit, shunting the galvanometer produces no diminution in the current flowing through it. And it is not until the resistance

of the shunted galvanometer is reduced to a value comparable with $b + r$, that the galvanometer deflection is seriously diminished.

Example 70.—If the resistance of a galvanometer be 1,000 ohms, what must be the resistance of a shunt to diminish the current passing through the galvanometer to one-half, first, when the resistance of the rest of the circuit is 100,000 ohms; secondly, when it is only 100 ohms?

In the first case we have

$$\frac{s\,\mathrm{E}}{(s+g)(b+r)+s g} = \frac{1}{2}\,\frac{\mathrm{E}}{b+r+g},$$

or substituting

$$\frac{s}{(s+1{,}000)\times 100{,}000 + s \times 1{,}000} = \frac{1}{2} \times \frac{1}{101{,}000},$$

$$\therefore\ s = 990{\cdot}1 \text{ ohms};$$

that is, s is only a little less than 1,000 ohms, which is the resistance of the galvanometer.

In the second case

$$\frac{s}{(s+1{,}000)\times 100 + s \times 1{,}000} = \frac{1}{2} \times \frac{1}{1{,}100},$$

$$\therefore\ s = 90{\cdot}9 \text{ ohms},$$

or not as much as one-tenth of the galvanometer resistance.

Example 71.—What must be the resistance of a galvanometer relatively to that of the rest of the circuit, so that shunting the galvanometer with a quarter of its own resistance may halve the current passing through it?

From what has preceded, we have

$$\frac{s}{(s+g)(b+r)+s g} = \frac{1}{2} \times \frac{1}{b+r+g},$$

$$\text{and since } s = \frac{g}{4},$$

$$\frac{\frac{g}{4}}{\left(\frac{g}{4}+g\right)\left(b+r\right)+\frac{g^2}{4}} = \frac{1}{2} \times \frac{1}{b+r+g};$$

$$\therefore \quad g = 3\ (b + r).$$

Example 72.—In example No. 38, given on page 180, what resistance must be added to the main circuit, so that the insertion of the shunt shall not alter the total current ?

To solve this question we must consider by how much the resistance of the circuit has been diminished by the insertion of the shunt, this diminution being, of course, equal to the difference between the resistances of the galvanometer shunted and unshunted.

The shunted galvanometer has a resistance of

$$\frac{1{,}808 \times 452}{1{,}808 + 452},$$

$$\text{or} \quad 361{\cdot}6 \text{ ohms,}$$

therefore the resistance of the circuit has been diminished by 452—361·6, or 90·4 ohms, and this resistance must be added if we wish that the total current shall be kept constant.

Example 73.—What resistances must be added to the main circuit to keep the total current constant when a galvanometer, having 1,000 ohms' resistance, is shunted with the three shunts which respectively allow $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th of the current to flow through the galvanometer ?

If s be the resistance of the shunt, and g the resistance of the galvanometer, the diminution of the resistance produced by shunting the galvanometer is

$$g - \frac{s\,g}{s+g}, \text{ or } \frac{g^2}{s+g}.$$

From what has been given in § 104, page 178, the resistances of the three shunts must be $\frac{1000}{9}$, $\frac{1000}{99}$, and $\frac{1000}{999}$ ohms respectively. Therefore, the resistances that must be added are

$$\frac{1000^2}{\frac{1000}{9}+1000} \text{ or } 900 \text{ ohms.}$$

$$\frac{1000^2}{\frac{1000}{99}+1000} \text{ or } 990 \quad ,,$$

$$\frac{1000^2}{\frac{1000}{999}+1000} \text{ or } 999 \quad ,,$$

138. Constant Total Current Shunts.—There are two ways, differing somewhat from one another, by means of which a box of shunts can be so arranged that the insertion of the shunt coil, parallel to the galvanometer, also introduces a compensating resistance in the main circuit, and so keeps the main current unaltered in strength. The first of these is due to Mr. Kempe, and the second to Mr. Rymer Jones.

Fig. 94 shows symbolically Mr. Kempe's arrangement, and it will be seen that the insertion of a plug into one of the holes A, B, C, for the purpose of introducing a shunt parallel to the galvanometer G, also adds one or more of the resistances r_1, r_2, r_3, to the main circuit, whereas, if the plug be inserted in the hole which is not lettered, the galvanometer is unshunted, and all the three coils r_1, r_2, r_3, are cut out of the circuit. A plan of the actual shunt box is seen in Fig. 95.

To determine what should be the values of these resistances, we have to remember that, if n_1, n_2, n_3 be the three multiplying

powers of the shunts, so that the three currents G_1, G_2, G_3, passing through the galvanometer are respectively equal to $\frac{C}{n_1}$, $\frac{C}{n_2}$, $\frac{C}{n_3}$, where C is the total current in each case, the resistances of the

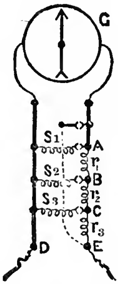

Fig. 94.

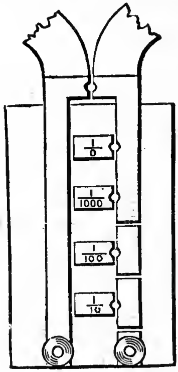

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Fig. 95.

shunted galvanometer are in the three cases $\frac{g}{n_1}$, $\frac{g}{n_2}$, $\frac{g}{n_3}$, if g be the resistance of the galvanometer itself. In order, therefore, that the total resistance in the circuit may be constant, we must have

$$r_1 + r_2 + r_3 = g - \frac{g}{n_1},$$

$$r_2 + r_3 = g - \frac{g + r_1}{n_2},$$

$$\text{and} \quad r_3 = g - \frac{g + r_1 + r_2}{n_3}.$$

From which it may be shown that

$$r_1 = \frac{n_1 - n_2}{n_1(n_2 - 1)} \times g,$$

$$r_2 = \frac{n_1 - n_3}{n_1(n_3 - 1)}\, g - r_1,$$

$$r_3 = \frac{n_1 - 1}{n_1} \times g - r_2 - r_3.$$

Also that
$$s_1 = \frac{g}{n_1 - 1},$$

$$s_2 = \frac{n_2(n_1 - 1)}{n_1(n_2 - 1)} \times g,$$

$$s_3 = \frac{n_3(n_1 - 1)}{n_1(n_3 - 1)} \times g.$$

Example 74.—If the galvanometer have a resistance of 5,000 ohms, and if we wish either the $\frac{1}{10}$th, or the $\frac{1}{100}$th, or the $\frac{1}{1000}$th

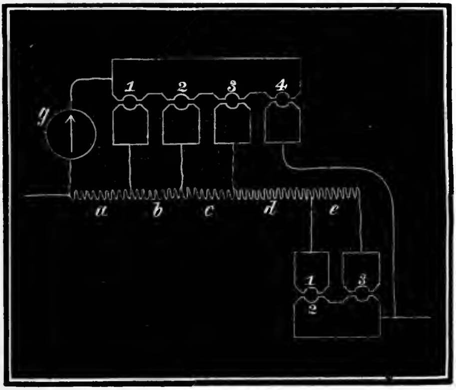

Fig. 96.

of the total current to pass through the galvanometer, what must be the resistances of s_1, s_2, s_3, r_1, r_2, and r_3?

Answer.—$s_1 = 5{\cdot}006$, $s_2 = 50{\cdot}964$, $s_3 = 616{\cdot}667$, $r_1 = 45{\cdot}455$, $r_2 = 504{\cdot}545$, and $r_3 = 4{,}445{\cdot}000$ ohms.

Fig. 96 shows, symbolically, Mr. Rymer Jones's arrangement. To use it *two* plugs have *always* to be inserted in the holes marked with the corresponding figures. If the plugs be inserted in the two holes marked 1, we have a shunt equal to a and a resistance added to the main circuit equal to $b + c + d$. If the plugs be inserted in the two holes marked 2, then we have a shunt equal to $a + b$ and a resistance $c + d$ added to the main circuit, &c. Hence, it follows that

$$a = \frac{g}{n_1 - 1},$$

$$a + b = \frac{g}{n_2 - 1},$$

$$a + b + c = \frac{g}{n_3 - 1},$$

$$b + c + d = \frac{n_1 - 1}{n_1} \times g,$$

$$c + d = \frac{n_2 - 1}{n_2} \times g,$$

$$d + e = \frac{n_3 - 1}{n_3} \times g,$$

from which a, b, c, d, e can be easily calculated for any particular values of $g, n_1, n_2,$ and n_3.

If one of the plugs be inserted in the hole marked 4, the circuit will be completed through the galvanometer unshunted.

Example 75.—If the galvanometer have a resistance of 5,000 ohms, and we wish either the $\frac{1}{10}$th, or the $\frac{1}{100}$th, or the $\frac{1}{1000}$th of the total current to pass through the galvanometer, what must be the resistances of $a, b, c, d,$ and e?

Answer.—$a = 5{\cdot}005$, $b = 45{\cdot}500$, $c = 505{\cdot}060$, $d = 4{,}444{\cdot}94$, and $e = 55{\cdot}06$ ohms.

Fewer coils are, therefore, required with this second arrangement, but it has the slight disadvantage that it requires two plugs to be inserted instead of only one as with Mr. Kempe's arrangement.

139. Independence of the Currents in Various Circuits in Parallel.—From what has preceded it follows that if a, c, d, &c. (Fig. 97) be circuits in parallel with the battery b, the currents A, C, D, &c., passing through the circuits respectively, will be each independent of the stoppage, or variation, of any, or of all, of the other currents, as long as the combined resistance of the circuits, that is,

$$\frac{1}{\frac{1}{a} + \frac{1}{c} + \frac{1}{d} + \text{\&c.}}$$

is large compared with the resistance of the battery, *b*. Because the current through any one of the circuits simply depends on the potential difference at the terminals of the battery, and on the resistance of the particular circuit. The latter is, of course, not altered by altering the resistance of any or of all the other circuits, and the potential difference at the terminals of the battery remains constant when the above relationship of resistance is fulfilled.

Practically, therefore, in all cases where a generator of *very small internal resistance* is employed, the currents in various parallel circuits fed by it are all independent of one another. And this is one of the great advantages of the very small resistance of "*accumulators*," or "*secondary batteries*," or "*storage cells*," as they are differently called, for electric lighting, in that any one of a number of lights fed in parallel by these cells can be turned on or off without materially altering the intensity of the light given off by any one of the remainder.

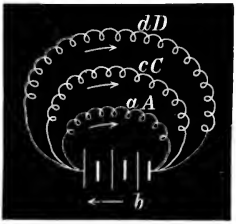

Fig. 97.

It also explains why Grove's cells, which, as stated in § 124, page 219, have a small resistance compared with Daniell's, Minotto's, and other well-known cells, were used in the early days in telegraph offices, when the *different* messages used to be sent along *several* telegraph wires *with one battery*. The trouble and expense, however, involved in keeping the Grove's cells in order caused the plan of working several telegraph wires with one battery to be abandoned in favour of having a separate battery of much higher resistance to work each line *independently*. But the invention of accumulators by Planté, and the improvements that have been effected in them by Faure, Swan, Sellon, Volckmar, and others, during the last few years, are leading to a return to the

old plan of several telegraph wires being worked with one current generator.

Example 76.—If three telegraph wires, having resistances of 200, 250, and 300 ohms respectively, including in each case the resistance of the "*receiving instrument*," or the instrument by means of which the messages are received, be worked by *one* battery having a resistance of 20 ohms, by how much per cent. will the current passing along the first line, when no current is passing along either the second or the third lines, be altered: 1st, by a current being sent along the second also; 2nd, by a current being sent along both the second and the third lines, in addition to the one sent along the first?

If E be the E. M. F. of the battery in volts, then the current C_1, flowing along the first line when no current is flowing along either the second or the third, is

$$\frac{E}{20 + 200} \text{ amperes.}$$

If a current is also being sent along the second wire, the total current flowing through the battery is

$$\frac{E}{20 + \dfrac{200 \times 250}{200 + 250}} \text{ amperes,}$$

and of this the current C_2, flowing along the first line, is

$$\frac{250}{200 + 250} \times \frac{E}{20 + \dfrac{200 \times 250}{200 + 250}},$$

$$\text{or} \quad \frac{250\,E}{20\,(200 + 250) + 200 \times 250} \text{ amperes.}$$

Similarly, if a current is also being sent along the third line, the current C_3, flowing along the first line, is

$$\frac{\frac{1}{200}}{\frac{1}{200}+\frac{1}{250}+\frac{1}{300}} \times \frac{E}{20+\frac{1}{\frac{1}{200}+\frac{1}{250}+\frac{1}{300}}} \text{ amperes.}$$

Therefore, $C_1 = \frac{1}{220}$ amperes,

$C_2 = \frac{1}{236}$ „

and $C_3 = \frac{1}{249 \cdot 4}$ „

Hence, C_1 is diminished by about 6·8 per cent. by allowing a current to flow along the second line, and by about 11·7 per cent. by allowing a current to flow along both the second and the third lines.

Example 77.—If two telegraph lines each have a resistance of 500 ohms, including the resistances of the receiving instruments, what may be the greatest resistance of the battery employed to send the current along both, so that the current flowing along either shall not be diminished by more than 1 per cent. by sending a current also along the other?

Let E be the E. M. F. in volts, and b the resistance of the battery in ohms, then the current flowing along either line, when no current is being sent along the other, is

$$\frac{E}{b+500} \text{ amperes;}$$

and the current flowing along either line, when a current is also being sent along the other, is

$$\frac{1}{2} \times \frac{E}{b+250} \text{ amperes.}$$

Now we want b to be of such a value that

$$\frac{E}{b+500} - \frac{1}{2} \times \frac{E}{b+250} \text{ is not greater than } \frac{1}{100} \times \frac{E}{b+500}.$$

Consequently, the largest permissible value of b will be found by making

$$\frac{E}{b+500} - \frac{1}{2} \times \frac{E}{b+250} = \frac{1}{100} \times \frac{E}{b+500},$$

$$\text{or} \quad \frac{99}{100} \times \frac{1}{b+500} = \frac{1}{2} \times \frac{1}{b+250}.$$

Answer.—5·1 ohms.

Example 78.—There are two telegraph lines, one having a resistance of 400 ohms, and the other of 500 ohms, including the resistance of the receiving instruments. The receiving instrument on the first line is so arranged that it will work without adjustment, with currents varying between 5 and 5·2 thousandths of an ampere. What must be the E. M. F. of, and resistance of, the common battery, for the two lines, so that the current flowing along the first line may be always between these limits, whether or not a current is being sent along the second line?

If E be the E. M. F. in volts, and b the resistance in ohms of the battery, the maximum current flowing along the first line will be

$$\frac{E}{b+400} \text{ amperes,}$$

and the minimum current

$$\frac{500}{400+500} \times \frac{E}{b + \dfrac{400 \times 500}{400+500}} \text{ amperes,}$$

$$\text{or} \quad \frac{500\,E}{900\,b + 200{,}000}.$$

The first current must not exceed 5·2 thousandths of an ampere, and the second must not be less than 5 thousandths of an ampere. Taking, therefore, the limiting values, we may say that

$$\frac{E}{b + 400} = \frac{52}{10{,}000},$$

$$\text{and} \quad \frac{5\,E}{9\,b + 2{,}000} = \frac{5}{1{,}000}.$$

Solving these two equations for E and b, we find that

$$E = 2{\cdot}19 \text{ volts about,}$$

$$\text{and} \quad b = 21 \text{ ohms} \quad ,,$$

In practice, larger E. M. Fs. than this must be used to allow for leakage along the line, in consequence of which only a portion of the current that leaves the sending or signalling end arrives at the receiving end.

Example 79.—If 10 of the 30 lamps in example 68, page 252, be turned out, what will be the P. D. at the terminals of the remaining 20?

Answer.—81·27 volts.

Example 80.—If 50 or more incandescent lamps in parallel, each requiring 0·8 amperes and 100 volts to glow properly, be fed with 55 accumulators in series, each having an E. M. F. of 1·98 volts when discharging, what must be the resistance of each accumulator, and what is the maximum number of lamps that can be lighted, so that the P. D. at their terminals never exceeds 101, and is never less than 99 volts?

The resistance of each lamp may be taken as $\frac{100}{0{\cdot}8}$ or 125 ohms. Hence, considering the case of the least number of lamps, 50, which will correspond with the highest number of volts, 101, we have, if b be the resistance of one accumulator,

$$\frac{55 \times 1{\cdot}98}{55\,b + \dfrac{125}{50}} = \frac{101}{\dfrac{125}{50}},$$

from which it follows that $b = 0{\cdot}003555$ ohms.

Next, considering the case of the largest number of lamps n, which will correspond with the lowest number of volts allowed, viz. 99, we have

$$\frac{55 \times 1{\cdot}9}{55\,b + \dfrac{125}{n}} = \frac{99}{\dfrac{125}{n}}.$$

Substituting in this the value previously found for b, and solving for n, we find that

$$n = 63{\cdot}92.$$

Hence, 64 lights would be practically the largest number.

CHAPTER VI.

INSULATION.

140. Surface Leakage, and Leakage through the Mass.—There are two ways in which electricity may pass from one body to another; it may either creep along

a layer of dirt and moisture on the surface of an insulating rod, or it may pass through the mass of the insulating material. The former may be called "*surface leakage*"; and the latter, "*leakage through the mass.*" In the case of a charged body supported on a rod of glass or ebonite, *surface leakage* is the main thing to guard against; whereas, with a *long* submarine cable, consisting of a copper conductor surrounded with guttapercha or with indiarubber, and immersed in the sea, the main loss of electricity is *through* the guttapercha or indiarubber. If, however, the piece of insulated cable be very short, then the surface leakage at the ends, arising from the electricity creeping from the ends of the copper conductor over the ends of the guttapercha covering to the water or the iron sheathing which is outside the guttapercha, may be the cause of the most important part of the loss. Hence, when it is desired to test the actual passage of the electricity from the conductor *through* the insulating material, it is usual, in order to diminish the surface leakage to a minimum, to cut the end of the core like a pencil, as shown in Fig. 98, so as to expose a long *freshly bared, clean, dry* surface of guttapercha or indiarubber. The insulation of the end can be still further improved by coating the surface with a thin layer of clean paraffin wax, which has been first melted by heating, to a temperature not however much above that of boiling water, otherwise the wax would be partially decomposed, and its resistance diminished.*

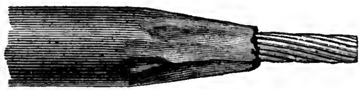

Fig. 98.

141. Coating Insulating Stems with Paraffin Wax or Shell-lac Varnish.—Coating the surface of any insulating stem which is exposed to the air with paraffin wax

* To avoid the paraffin wax being overheated, it is well to warm the vessel containing it by means of a *water bath* in the same way that glue is usually heated in an ordinary glue-pot.

has not only the advantage that it renders the surface much less "*hygroscopic*," or attractive of moisture, but it enables the wax to be easily partially scraped off at any time, and a new clean dry surface exposed. Shell-lac varnish, made by dissolving shell-lac in alcohol, may be employed in the place of paraffin wax, but, in many cases, it is not as good, partly because shell-lac, being hard and brittle, cannot be easily scraped so as to expose a new clean surface, and partly because, at the present day, it is very difficult to buy really good shell-lac, the material of commerce being much adulterated.* *If, however, a glass rod can be kept free from dust, and artificially dried, then it is better to put neither paraffin wax nor any kind of varnish on it.*

142. Sealing up One End of a Cable when under Test.—The insulation of a cable may be tested by measuring with a very delicate galvanometer the current that a battery of high E. M. F. can send through the indiarubber, guttapercha, or other insulating material used in its construction. To do this it is only necessary to have one end of the copper conductor bare, hence it is desirable after pointing the guttapercha at the other end, as shown in the last figure, to seal it up altogether by dipping it into paraffin wax two or three times, so as to cause a lump of paraffin wax to adhere to it, which can be best done when the paraffin wax has cooled until it is approaching the temperature of solidification.

143. Construction of an Insulating Stand.—In Fig. 29 the plate A, and in Fig. 40 the pot P, are supported on a special form of insulating stand, in which

* Dr. A. Muirhead, who has had great experience in the use of shell-lac in the construction of condensers, recommends the following process for obtaining good insulating varnish. Obtain "*button*" lac, pick out the cleanest lumps, and dissolve them in *absolute* alcohol. Allow the solution to stand for some time, and use only the *upper* part of the solution. When the highest insulation is required, first dissolve the button lac in ordinary alcohol, and precipitate it by allowing the solution to trickle into distilled water, then dissolve the precipitate in absolute alcohol.

the glass rod is kept *free from dust* and *artificially dried.* This device for obtaining high insulation is far superior to the old-fashioned plan of using a simple rod of glass or ebonite, since such a rod, whether it was coated with varnish or not, required perpetual cleaning and drying to prevent the electricity leaking down its surface. The special arrangement shown in these figures, and which has been designed by the author for experiments on statical electricity, consists of a glass vessel made of any convenient kind of glass, and having at its bottom a tubulure of glass attached vertically at the centre. This tubulure, or collar, of glass is ground inside like the inside of the neck of a glass-stoppered bottle, and into this tubulure the ground end of a rod of *highly insulating* glass fits, much in the same way as a glass stopper does into a bottle. On to the top of this glass rod anything can be fixed; for example, the plate A (Fig. 29), and the pot P (Fig. 40), are supported in position by a little collar of metal, which is soldered to the bottom of A and of P, and which slips fairly tightly over the top of the glass rod. Before the glass rod is inserted a little strong sulphuric acid is poured in, and rests on the expanded bottom of the glass vessel, exposing a large surface of acid for absorbing the moisture contained in the air in the vessel. When the instrument is not in use a split indiarubber stopper I, seen in Fig. 40 resting on the base of the instrument, is inserted to close up the neck of the glass vessel, which is contracted at the top, partly for this purpose, and partly to avoid a too rapid interchange of air between the inside and the outside of the glass vessel when the instrument is in use.

The advantages of this insulating stand are :—

1. The rod can be easily taken out and cleaned. To clean such a rod hold it by the end, and wash it by means of a clean brush with soda and warm water to remove the grease; then rub it with another brush while a stream of warm ordinary water flows over it, to remove the soda; and, lastly, let a stream of distilled water flow

over it to remove the trace of salt which is dissolved in ordinary water. The rod should be dried before a fire; or, better, by being hung up under a glass shade, or in some confined space free from dust, in which there is a vessel containing a little strong sulphuric acid. *On no account dry the glass rod by rubbing it with a cloth, nor touch it with the fingers except at the extreme end.*

2. The rod may be made of dense flint glass which insulates *well*, while the vessel may be made of any kind of glass that can be easily, and, therefore, cheaply blown, without reference to its insulating qualities.

3. As the rod is easily taken out, the sulphuric acid can be put into the vessel without splashing the rod; or the old acid, after it has become weak by absorbing water-vapour, may be emptied out, and fresh acid put in without fear of dirtying the rod. This it would be difficult to do, even with another opening in the vessel, if the rod were immovable.

144. Laws of Surface Leakage, and of Leakage through the Mass.—The film of dirt and moisture on a rod acts like an exceedingly thin layer of conducting matter, therefore for stems equally damp and dirty (and the cleanest glass stem rapidly becomes damp and dirty when exposed to the air), the surface resistance or insulation

$$\propto \frac{l}{d},$$

where l is the length, and d the diameter of the stem, since resistance is directly proportional to the length, and inversely as the sectional area of the conducting layer. The stem also conducts through its mass, and its resisttance in ohms is

$$g \times \frac{l}{\frac{\pi}{4} \times d^2},$$

where g is the resistance in ohms between the opposite faces of a cubic unit of the glass or other material, of

which the insulating stem is made, l its length, and d its diameter. If l and d be in centimetres, g must be the resistance of a cubic centimetre; or, if l and d be in inches, g must be the resistance of a cubic inch.

The approximate values of g in ohms per cubic centimetre, for some good insulators, are given in Table No. V. The resistance of an insulator increases up to a certain limit with the time the current is kept on, or with the time of "*electrification*," as it is shortly called, so that the values in the table, which have been obtained after several minutes' electrification, represent approximately this maximum value. The resistance of insulators also varies with the temperature, but while the resistance of conductors increases with elevation of temperature, *the resistance of insulators diminishes with elevation of temperature.*

TABLE No. V.

Substance.	Temperature — Centigrade.	Approximate Resistance in ohms per cubic centimetre after several minutes' electrification.	Authority.
Mica	20°	84×10^{12}	Author.
Guttapercha .	24°	450×10^{12}	Standard adopted by Mr. Latimer Clark.
Shell-lac. . .	28°	$9,000 \times 10^{12}$	Author.
Hooper's Vulcanised India-rubber . .	24°	$15,000 \times 10^{12}$	Tests of Cables.
Ebonite . . .	46°	$28,000 \times 10^{12}$	Author.
Paraffin Wax .	46°	$34,000 \times 10^{12}$	,,

The resistance of dense flint glass has not, as far as the author is aware, been measured at as low a temperature as 40° C. after a long period of electrification. At 100° C., Mr. Thomas Gray found that it was about 206×10^{12} ohms per cubic centimetre, at 60° C. about $1,020 \times 10^{12}$, and that it increased very rapidly as the temperature diminished. Some experiments made by

the author showed that, after several hours' electrification, the resistance per cubic centimetre at ordinary temperatures had a far greater value than this.

In the above formulæ for the surface resistance and resistance of the mass of a rod, the more l is increased, that is to say, the longer the stem is made, the larger both the surface and the mass insulation become; while, on the other hand, the larger the value of d, the smaller are both the surface and the mass insulation, the latter, however, diminishing much more rapidly than the former, as d is increased. Consequently, while for a long thin rod of fairly good insulating material the main loss of electricity will be over the surface, for a very short thick rod, for a sheet, in fact, of insulating material (for that is what a rod ultimately becomes, as it is made shorter and thicker), the main leakage will be *through* the material if the electricity is conveyed to the different parts at each side of the sheet by means of a piece of tin-foil, stuck on both sides of the sheet of insulating material, and if sufficient of the surface of the insulating material near the edges of the sheet be left uncovered to prevent surface leakage. (*See* construction of condensers, § 173, page 318.)

145. Corrugating the Sides of Ebonite Pillars.—In order to increase the value of l in the case of an insulating stem without making it very tall and weak, it may be made with *corrugations*, as shown in Fig. 99. These rings have not only the advantage that l is increased, but the thin edges may be very easily wiped with a clean cloth, and the insulation thereby improved. Further, although these edges may be dirtied if the rod be touched or taken hold of, the cavities between them will probably be left clean, and hence a continuous line of dirt will not be formed from the top to the bottom of the pillar, as would probably be the case if the surface of the pillar were smooth without corrugations.

146. Common Fault made in Constructing Ebonite Pillars.—A common fault made in constructing insulating stems of ebonite, and which should be most care-

fully guarded against, consists in drilling a hole right through the stem, and then inserting into the top of this hole the screw which holds on the terminal, and into the bottom the screw which holds the pillar to the

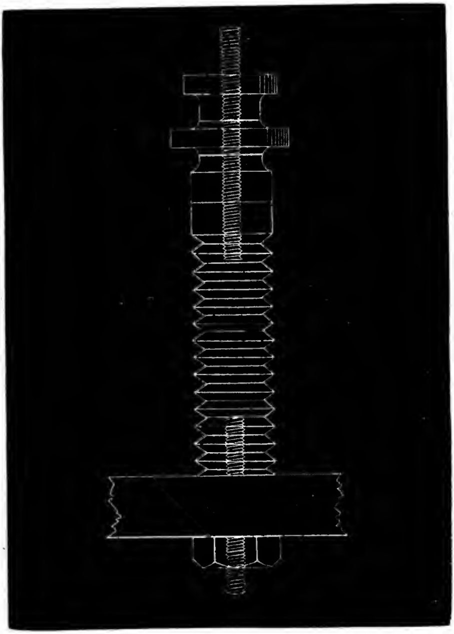

Fig. 99.

base. This continuous hole makes it impossible by any amount of cleaning and paraffining of the outside of the stem to obtain good insulation, for even if the sides of this hole between the ends of the screws were quite clean, the length of ebonite surface separating the ends of the screws would be small compared with the length of the pillar outside, and so the leakage from

screw to screw inside the ebonite pillar would be greater than along the outside; but when in addition the sides of this hole are, as is frequently the case, dirty, the insulation of the pillar is immensely diminished by the hole being bored right through. The hole should, therefore, on no account be drilled through; and in the case of any old apparatus in which this mistake has been made, the screws should be taken out, and the sides of the hole carefully cleaned with a small brush, such as is sold for cleaning glass tubes, using first soda and warm water, then warm water without soda, and, lastly, allowing a stream of distilled water to flow through the hole; finally, when the sides of the hole are quite dry, melted paraffin wax should be poured in, so that there is a little block of paraffin wax filling up the hole between the ends of the screws.

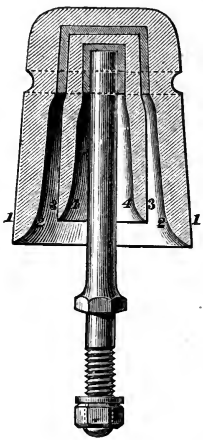

Fig. 100.

147. Telegraph Insulators.—In the case of the earthenware, or porcelain, insulators used to support telegraph wires, length of surface, combined with small periphery of a transverse section, is obtained by means of the "*double cup insulator*" (Fig. 100). This form of insulator, which was originally proposed by Mr. Latimer Clark, has also the advantage that the inner surface 2, 2 of the outer cup, as well as the inner 4, 4, and outer surface 3, 3 of the inner cup, are kept tolerably clean and dry. Before the electricity escaping from the wire, which is bound in the groove at the upper part of the insulator, can reach the iron stalk, by means of which the insulator is attached to the wooden or iron bracket on the telegraph post, it must leak down the outside of the outer cup 1, 1, then up the inside of the

outer cup 2, 2, then down the outside of the inner cup 3, 3, and, lastly, up the inside of the inner cup 4, 4.

The porcelain, or earthenware, cups should, as originally suggested by the late Mr. Cromwell Varley, be moulded separately, and cemented together after they are baked, in order that a possible flaw in the one may not be accompanied by a flaw in the other, which would probably be the case if they were moulded in one and then baked. The lips of the cups should be shaped as shown in the figure, for, with this shape, Mr. Varley found that the drops of water hanging on the lip during, or after, rain, were simply blown a little way up inside the cups, instead of being broken and the moisture scattered all over the inside of the insulator, moistening all parts.

148. Testing Insulators during Manufacture.—In order to test the quality of insulators, a hundred of them are placed, *inverted*, so that they can hold water, in a shallow metal-lined trough, containing sufficient water to come to within half an inch of their lips, and water having been poured into both the cups so as to reach to about the same height, the insulators are left in the water for at least forty-eight hours, to give time for the water to soak into any cracks in the earthenware or porcelain. The metal stalks of all the insulators are fastened together with copper wire, and the resistance between this copper wire and the water in the trough, or, what is electrically the same thing, the metallic lining of the trough, will measure the parallel resistance to leakage *through* the earthenware or porcelain of which the cups are made, and *over* the surface of the lips of the cups. To diminish the surface leakage as much as possible, the lips are dried, just before the test is made, by large red-hot rollers being rapidly rolled backwards and forwards over the troughs along iron rails fastened on the tops of the sides of the troughs, this operation being performed so quickly that the lips of the insulators are dried before any appreciable quantity of the water in the trough or in the

insulator cups is evaporated, and the air in the neighbourhood of the cups thus rendered steamy. Then, before the lips have had time to cool, and, therefore, before any fresh moisture can settle on them, the parallel resistance is measured.

The resistance of one double cup insulator made of porcelain, and tested in this manner, varies from five hundred thousand million to four million million ohms, depending on the size of the cups, and the quality of the clay of which the cups are made. Taking two million "*megohms*," that is two million million ohms, as the average resistance of each of a batch of 100, the 100 should have a parallel resistance of twenty thousand *megohms*. If a set of 100 are found to have a parallel resistance much below the other sets of 100 of the same type, it is either due to faulty drying of the lips, or to the presence of one or more cracked porcelain cups in the batch, or to one or more of the porcelain cups having been badly baked. Under these circumstances a red-hot iron roller should be again rolled backwards and forwards over the trough, when, if the same low resistance is again obtained, the wire should be unwound from the iron stalks, and each insulator should be tested roughly and *quickly*, by touching the stalk with one of the copper wires connected with the measuring apparatus, the other wire coming from the measuring apparatus being still attached to the metallic lining of the trough. In touching the stalk with the wire, care must be taken to hold the india-rubber or guttapercha covering at some little distance from the end, and the insulating coating must be cut like a pencil, as shown in Fig. 98, page 267; otherwise the leakage to earth along the outer surface of the insulated wire will be mistaken for leakage through the porcelain of an insulator. In this way the defective insulator or insulators may be detected and removed from the batch.

This rough method of picking out defective insulators may with advantage be employed before the stalks of the

insulators are wired together, and the parallel resistance of the batch of 100 tested accurately. For supposing one million megohms were taken as the "*specified*" or *contract* minimum resistance of each insulator, then, if ninety-nine of them happened to be each of them better than the specified standard, having, say, each three million megohms, whereas one of them was much below the standard, and had only, say, twenty thousand megohms, the parallel resistance of the 100 would be 12,048 megohms. But as this would be more than the *specified* resistance of a good hundred, which would be ten thousand megohms, it follows that, although the batch contained an insulator having only the $\frac{1}{150}$th of the resistance of each of the remaining ninety-nine, the batch would be allowed to pass if the insulators were only tested in hundreds, and were not subjected individually to any test. But such an insulator, which had only the $\frac{1}{150}$th of the resistance of each of the rest, should certainly be rejected, since, although the defect at present is only a small one, it is extremely probable that this defect will go on increasing, so that if it be put up with others on a telegraph line, more electricity will eventually leak through this insulator to the ground than will escape over the surface of all the insulators which support several miles of the telegraph wire.

149. Measuring High Resistances.—With an ordinary Wheatstone's bridge we can test resistances up to 1·11 million ohms, but not above that, consequently resistances of thousands of megohms are usually tested in quite a different way, by measuring the current that a known P. D. will send through them. As, however, the galvanometer must be extremely sensitive to enable such small currents to be measured by means of it, and as the absolute value of the deflection of such a very delicate or sensitive galvanometer is liable to vary from day to day, we do not attempt to calibrate the galvanometer absolutely in amperes, or rather in millionths of an ampere. Further, it is not necessary to know the value in

volts of the P. D. employed, since, if we compare the current sent by this P. D. through the unknown resistance with that sent by the same P. D., or by a known portion of it, through a known resistance, the value of the unknown resistance can be ascertained.

150. Subdividing a P. D. into Known Fractions.—The simplest arrangement for obtaining a known fraction of a P. D. is to cause a *steady* current, by means of a battery B (Fig. 101), to flow through a very high resistance L M; then the P. D. between any two points S T, bears to the P. D. between any other two points L M, the ratio that the resistance q of the part S T bears to the resistance p of the whole L M. The P. D. between the points L M may be employed to send a current through the unknown resistance x, and the P. D. between the points S T, through a known resistance r.

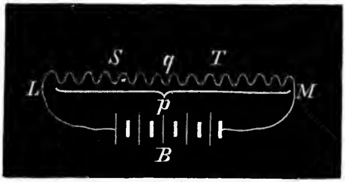

Fig. 101.

It is not, of course, necessary that *both* the points S and T should be distinct from L and M; one of them, for example, S, may be the same as L.

151. Constant of a Galvanometer.—If the unknown resistance x be very large, the galvanometer must be very sensitive; hence either the known resistance r must be also very large, or q must be very small compared with p, or, lastly, the galvanometer must be shunted in taking what is called "*the constant of the galvanometer.*" If the resistance L M be *very accurately subdivided*, then there is no objection to taking q as small as we like; indeed, taking q very small has in such a case an advantage over shunting the galvanometer, arising from the fact that the smaller q is, and the higher the resistance of the galvanometer circuit (the coils of which are attached to the two points S and T), the more accurately is the *parallel* resistance between

S and T equal simply to q. If, on the other hand, the resistance L M be not very accurately divided, then it is not advisable to take the points S and T too near together, since a very small absolute error in the value of q will make a very large error in the ratio of q to p when q is very small. In that case, shunting the galvanometer is a better method of diminishing the galvanometer deflection.

Let C and C′ be the relative strengths of the currents passing through the galvanometer when, first, the P. D. between L and M is employed in sending a current through x with the galvanometer unshunted (Fig. 102),

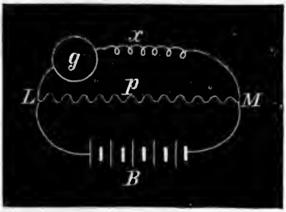

Fig. 102.

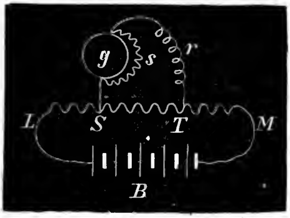

Fig. 103.

and when, second, the P. D. between S and T is sending a current through r, the galvanometer of resistance g, being shunted with a resistance s (Fig. 103), then

$$\frac{p}{x+g} \div \frac{s}{s+g} \times \frac{q}{r+\dfrac{sg}{s+g}} = \frac{\mathrm{C}}{\mathrm{C}'},$$

$$\therefore\ x = \frac{p}{q} \times \frac{\mathrm{C}'}{\mathrm{C}} \times \frac{s+g}{s}\left(r + \frac{sg}{s+g}\right) - g.$$

Generally $\dfrac{sg}{s+g}$ may be neglected in comparison with r, and g in comparison with x. In which case very approximately we have

$$x = \frac{p}{q} \times \frac{\mathrm{C}'}{\mathrm{C}} \times \frac{s+g}{s} \times r.$$

If we have not a large subdivided resistance, L M (Figs. 102, 103), then we must employ a battery of many cells in series when sending the current through the high resistance x, and a small battery, one cell perhaps, when sending the current through the known resistance r. In such a case the ratio of the electromotive forces of the large number of cells to that of the small number will be approximately proportional to the numbers of cells employed, but it may be more accurately ascertained by one of the methods already described (§§ 131, 132, pages 231, 234) for comparing electromotive forces. Let N be the ratio of the electromotive forces, and let b and b' be the resistances, in ohms, of the two batteries, then if C and C′ be the relative strengths of the current, as before,

$$\frac{\mathrm{N}}{x + b + g} \div \frac{s}{s + g} \times \frac{1}{r + b' + \dfrac{s\,g}{s+g}} = \frac{\mathrm{C}}{\mathrm{C'}},$$

$$\therefore x = \mathrm{N} \times \frac{\mathrm{C'}}{\mathrm{C}} \times \frac{s + g}{s}\left(r + b' + \frac{s\,g}{s + g}\right) - (b + g).$$

Or, as usually $b' + \dfrac{s\,g}{s+g}$ is small compared with r, and as $b + g$ is also small compared with x, we have approximately

$$x = \mathrm{N} \times \frac{\mathrm{C'}}{\mathrm{C}} \times \frac{s + g}{s} \times r.$$

Example 81.—Using a galvanometer, the deflection of which is directly proportional to the current passing through it, and having a resistance of 7,500 ohms, a deflection of 220 divisions on the scale is produced when p is 10,000 ohms, and the current is sent through the unknown resistance. On the other hand, when q is 100 ohms, and the current is sent through a known resistance of 10,000 ohms, a deflection of 300 scale divisions is obtained with the galvanometer shunted with 7·508 ohms. What is the value of the unknown resistance?

Using the complete formula we find that the un-

known resistance is 1,364,561,591, while the approximate formula gives as the result 1,363,636,364. For all practical purposes it would be sufficient to know that the resistance was 1,364 megohms, which result would be obtained quite as accurately from the second answer as from the first.

Example 82.—With 100 cells and the unknown resistance a deflection of 192 scale divisions is obtained, whereas with one cell and a known resistance of 25,000 ohms in circuit a deflection of 243 scale divisions is produced when the galvanometer is shunted with the one-hundredth shunt. What is the value of the unknown resistance? *Answer.*—316 megohms approximately.

Example 83.—If one cell give a deflection of 100 scale divisions when 10,000 ohms are in circuit, and the galvanometer is shunted with the one-thousandth shunt, how many cells must be used to test a resistance of 10,000 megohms if a deflection of not less than 50 scale divisions is to be obtained?

Answer.—500 cells approximately.

Example 84.—If one cell give a deflection of 127 scale divisions when 12,000 ohms are in circuit, and the galvanometer is shunted with the one-thousandth shunt, through what resistance would one cell give a deflection of one scale division if the galvanometer were unshunted?

Answer.—1,524 megohms approximately.

152. Very Delicate Galvanometers.—For measuring accurately the current that 100 Daniell's cells will send through, say, 20,000 megohms, which is only the one two-hundred-millionth part of an ampere, we must employ a galvanometer which is far more sensitive than anything that has hitherto been described in this book. To obtain this high degree of delicacy three conditions must be fulfilled:—

1. The number of turns of wire on the galvanometer bobbin must be very large. (*See* § 217, page 418.)

2. The suspended magnetic needle must be strongly magnetised.

3. The controlling force must be very weak.

In order to fulfil condition No. 1, and, at the same time, to keep all the turns of wire close to the suspended magnet, very fine wire must be used in winding the bobbin. No. 2 is fulfilled by making the needle of hard steel; a piece of watch spring heated to redness and cooled suddenly by being dipped in water answers well. By the proper adjustment of an auxiliary magnet the controlling force due to the earth or other controlling magnet may be rendered very weak for any one position of the suspended needle of the galvanometer, but unless the controlling magnet be very large and far away it is difficult to obtain a sufficiently uniform field for the controlling force acting on the suspended magnet to be weak throughout the whole range of motion of the suspended magnet. A better plan is to make the suspended arrangement of two magnets N S, N′ S′ rigidly fastened, with their *poles reversed*, to a stiff vertical wire (Fig. 104). If these two magnets N S, N′ S′ be of exactly the same length and strength, and if their poles be in exactly the same vertical plane, the earth's magnetism will have no effect on the arrangement, hence it will rest indifferently in any position about a vertical axis as far as the earth's attraction is concerned.* But if one of these magnets be

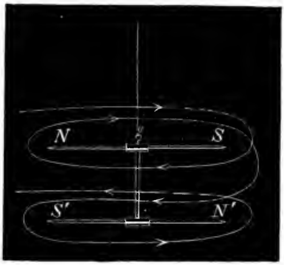

Fig. 104.

* As it is extremely difficult to fix the magnetic needles to the vertical wire so that their magnetic axes are in the same vertical plane, the practical test for the needles being equally strong is not that the arrangement will rest indifferently in any position when it is acted on by the earth's magnetism alone, but that the needles place themselves *east and west*, since this is the only position in which the

inside one coil of wire, and if the other be inside another, and if the current flow in *opposite directions* round these coils, "*the moment of the deflecting couple*"* acting on the combination will be the sum of the moments of the couples acting on the two needles separately, and hence may be made as large as we please. Such an arrangement is called an "*astatic combination*" of magnets, and with it a galvanometer of great delicacy, called an "*astatic galvanometer*," may be made.

In practice a small directive force is produced partly by one of the needles being a slightly stronger magnet than the other, and partly by a controlling magnet M (Fig. 108) being placed nearer one of the needles than the other, and so acting more strongly on that one.

153. Thomson's Astatic Galvanometers.—Usually in Sir William Thomson's astatic galvanometers the

forces acting on the arrangement due to the earth's magnetism balance one another. Actually the needles place themselves so that their axes are equally inclined to the east and west line, but the inclination is so slight that they appear to lie east and west. In Fig. 105 the equilibrium

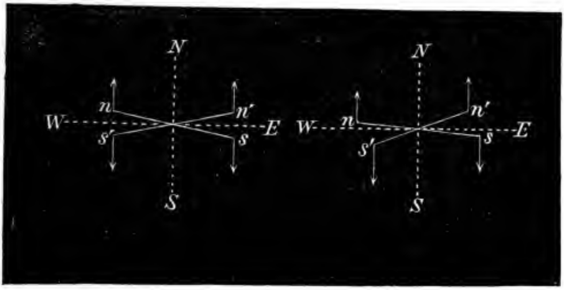

Fig. 105. Fig. 106.

position is shown, the needles being seen in plan, and their axes, for the purpose of clearness, being drawn more inclined to one another than they would be in practice. Fig. 106 shows the arrangement slightly turned round, when it is seen that equilibrium cannot exist.

* When two equal forces opposite in direction and parallel to one another, but not in the same line, act on a body, they constitute a "*couple*" whose "*moment*" is the product of either force into the perpendicular distance between them.

mirror is fastened to one of the magnets, and an *aluminium* vane to the other, to produce "*damping*," or resistance to *quick* vibrations of the needle, in consequence of which it is rapidly brought to rest when deflected; and the mirror and the vane are attached to a vertical wire—made, like the vane, of aluminium for the sake of lightness—suspended by a fibre of unspun silk. This arrangement, however, has two disadvantages: the one that, as the mirror and the vane are much larger than the magnet, the inner windings of the wire in the coils cannot be brought close to the little magnets; the other that, in order to allow the reflected ray (*see* Fig. 38, page 107) to emerge from the coil when the mirror is deflected, the hole in the coil must be enlarged at the front, that is, made trumpet-shaped, which causes the wire to be still farther removed from the suspended magnet. A better plan is to dispense with the aluminium vane and attach the mirror and the magnets to a vertical strip of mica s s (Fig. 107), as such a strip produces sufficient damping to render the galvanometer dead beat. Further, by attaching the mirror o to the part of the vertical strip that is between the coils, as shown in the figure, the space inside the coils which is not wound with wire need only be large enough to allow sufficient clearance for the free motion of the magnets when they are deflected, so that the convolutions of wire can be brought close to the magnet and the instrument made very delicate. Also the arrangement enables a larger mirror to be employed and a brighter image obtained on the scale.

The astatic combination shown in Fig. 107 consists of four small magnets *m*, in the centre of one pair of coils, with their marked poles, say, all turned to the right, and four similar small magnets *m*, in the centre of the other coil, with their marked poles all turned to the left. The strip of mica s s, to which these two sets of magnets are fastened, hangs by a fibre of unspun silk from a small hook at the end of a screw, which can be raised or lowered by turning the nut *n*. To prevent the

screw also turning and twisting the fibre when the nut n is turned, there is a small vertical groove cut in the side of the screw, in which runs a small pin attached to the framework of the galvanometer.

In order to insert the astatic combination of magnetic needles in the instrument, two of the coils must be removed. This is much facilitated if the coils be mounted

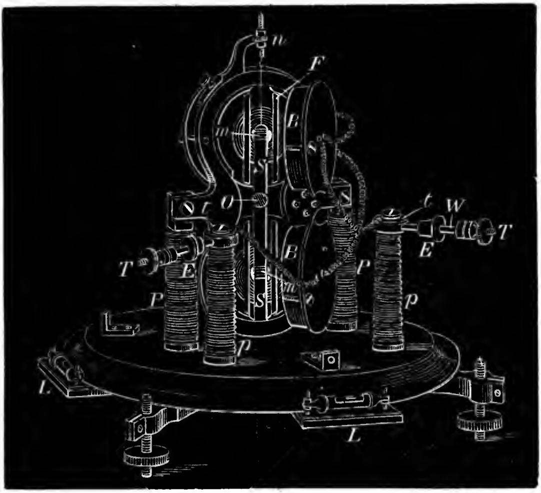

Fig. 107.

in hollow boxes B B, attached by hinges to the framework of the galvanometer, as seen in Fig. 107, which shows two of these boxes containing the coils turned back so that the interior of the galvanometer may be seen. To prevent the coils touching the suspension when the boxes are closed, strips of paraffin wax or guttapercha, F, are inserted.

All reflecting galvanometers which have not an adjustment for centring the fibre should be provided with two *adjustable* spirit-levels L L, attached, at right angles

to one another, to the base of the galvanometer. When the instrument is made, the levelling screws, on which the galvanometer rests, should be adjusted until the suspended needles hang quite freely inside the coils, then the levels should be adjusted until the bubble of air is in the middle of each tube. On all future occasions when the instrument is used, the levelling screws should be turned round until the bubbles are in the centres of the tubes, and then we may be sure that the needles are hanging freely inside the coils. If the whole apparatus could be made perfectly true, the mere levelling of the base with an ordinary carpenter's level when the galvanometer was about to be used would be sufficient to insure perfect freedom of the needles; but if the aluminium wire be not perfectly straight, or if the coils be not perfectly symmetrical, from the wire perhaps having bulged, the mere levelling of the base would not suffice.

154. Importance of the Galvanometer being Well Insulated.—In many cases when a high resistance has to be measured it is the resistance between some insulated body and the earth; for example, the resistance of the layer of guttapercha between the copper conductor of a cable and the water. It is impossible, of course, to insert the galvanometer between the guttapercha and the water, hence it must be placed between the battery and the insulated body. The currents, therefore, that will pass through the galvanometer will be the sum of the current that passes through the resistance that we desire to measure, and the current that will leak to earth from the terminal of the galvanometer that is attached to the insulated body, if this terminal be not well insulated. The value of this leakage current can be ascertained by disconnecting the galvanometer from the body whose insulation we desire to test, and testing the insulation of the galvanometer alone; but a better plan is to endeavour to render these leakage currents practically nought by having all parts of the galvanometer well insulated, as well as the wire connecting the

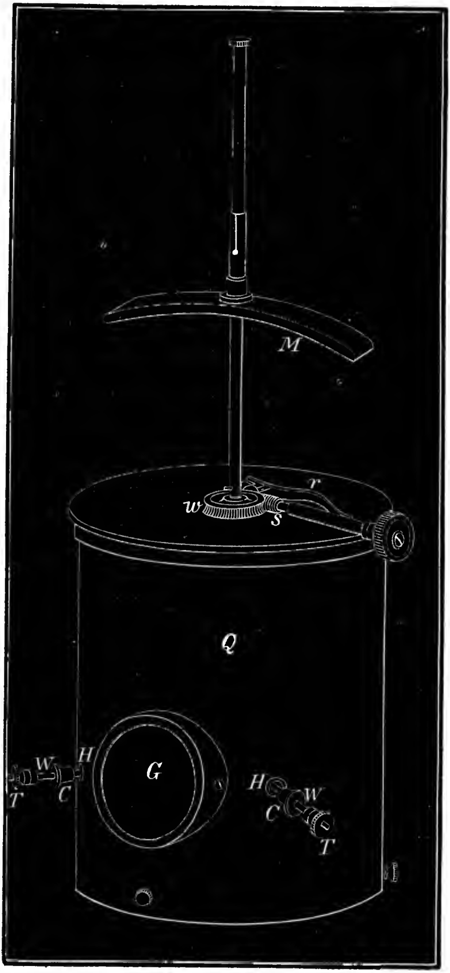

Fig. 108.

galvanometer with the insulated body. To insulate the coils of the galvanometer from the earth the hollow boxes B B (Fig. 107) in which the coils are held, as well as the pillars P P, are, in the best galvanometers, made of ebonite. The ends of the coils should be fastened to ebonite pillars *p p*, *inside* the outer brass case of the instrument, and the wires employed to connect the galvanometer with other apparatus can be attached to the terminals at the top of these pillars either by passing the wires through openings in the brass case, which openings may be closed by little doors when the galvanometer is not in use, or, better still, the flexible wires may be attached to terminals T T, at the ends of horizontal stiff brass wires W W, the other ends of which are screwed into the terminals *t t*, at the tops of the ebonite pillars *p p*, as seen in Fig. 107. These stiff brass wires pass through holes H H, in the brass cover Q, which is shown removed from the galvanometer in Fig. 108, without touching it, and by pushing in the ebonite collars E E, which slide on the wires W W, the holes H H can be closed up, either when the galvanometer is not in use, or when it is employed for experiments not requiring the highest insulation of the terminals. When it is desired to remove the cover, the wires W W are first unscrewed from the terminals *t t* and withdrawn, then the small screws at the bottom of the cover (Fig. 108), which screw into the brass lugs at the base of the galvanometer (Fig. 107), are loosened.

G (Fig. 108) is a window let into the cover for the light to pass through on its passage to and from the mirror; *s* is a screw held against the worm-wheel *w* by a spring *r*, and by turning the handle the controlling magnet M can be turned round, and the spot of light brought to the centre of the scale. By raising or lowering M the sensibility of the galvanometer is increased or diminished.

In some cases the unknown resistance is so large—when it is, for example, the insulation resistance of a

short bit of good cable—that even the method of testing described in § 151, page 279, is not sensitive enough to give its value; in such a case the "*leakage method of measuring resistance*" described in § 185, page 344, must be resorted to.

CHAPTER VII.

QUANTITY AND CAPACITY.

155. Coulomb.—A "*coulomb*" is the unit of electric quantity, and it is defined as *the quantity of electricity that flows per second past a cross section of a conductor conveying an ampere.* In the case of a stream of water

through a pipe we can measure the current by putting a bucket under the end of the pipe, and actually measuring the number of cubic feet or gallons of water that flow out per minute, but in the case of an electric current there is no end to the pipe or conductor, since the electric circuit is necessarily a closed one, and if we attempted to cut the wire for the purpose of inserting some apparatus in order to catch, so to say, the electricity, we should stop the current. What we have, therefore, to do in order to measure a quantity of electricity is to discharge the body containing it through the coil of a galvanometer, and observe the current produced during the discharge. This discharge of electricity, and the current produced by it, last a very short time, and, further, the current changes in value rapidly during the discharge. For example, suppose that an insulated conductor containing K coulombs of electricity, and charged to a potential of V volts, be discharged by being connected with the ground through the coil of a galvanometer; then, as the electricity flows out, the potential of the conductor will fall, hence the P. D. between it and the ground, and consequently the current, will rapidly grow less, until, when the discharge is nearly completed, and the potential is nearly reduced to that of the earth, the current will be extremely small. The effect, therefore, of sending such a discharge of electricity through a galvanometer coil is to cause the needle of the galvanometer to be suddenly deflected, after which it returns through the zero position, at which it finally stays at rest after a few swings. Although the current during the discharge is rapidly growing less and less, and although, therefore, the impulses given to the needle during successive equal short intervals of time during the discharge become feebler and feebler, it is possible, *when the whole discharge is completed before the needle begins to move,* to sum up the effects of all these impulses, and so to estimate the number of coulombs of electricity that pass during the discharge from the instantaneous

deflection or "*elongation*," or "*throw*" of the needle, as it is sometimes called. The magnitude of this first angular deflection of the needle k° depends—

1. On K the number of coulombs that pass.

2. On the moment of inertia of the needle and pointer, or other indicating arrangement.

3. On the moment of the controlling forces, that is, the forces which resist the needle moving away from the zero position, and which tend to pull it back to that position.

4. On the moment of the forces that "*damp*" the vibrations, that is, the forces, due to air or "*magnetic friction*," that simply resist the motion of the needle (*see* § 156, page 294).

5. On the moment of the deflecting forces exerted on the needle by a given constant current flowing through the coil.

Increasing either 1 or 5 will increase the magnitude of the first swing, which, on the other hand, will be diminished by increasing either 2, 3, or 4. If the needle be set swinging when no current is flowing, the quickness of the vibration will depend on the largeness of 3, and on the smallness of 2 and 4, so that if P be the "*periodic time of vibration*" of the needle in seconds, that is, the *number of seconds that intervene between the moment when the needle passes any position and the moment when it next passes the same position swinging in the same direction*, P will be increased by diminishing 3, or by increasing 2 or 4. On the other hand, if a° be the angular deflection produced when a steady current of A amperes flows through the coil, a° will be increased by increasing 5, or by diminishing 3, but will be unaffected by altering 2 or 4.

Taking all these effects into consideration, it can be shown that, when both k° and a° are *small*, and when the damping is *very small*,

$$K = \frac{P}{\pi} \times A \times \frac{\sin. \frac{k^\circ}{2}}{\tan. a^\circ}.$$

If a reflecting galvanometer be employed, k° and a° will necessarily be both small, because, with a scale say two feet long, put four feet away from the mirror, the spot of light will be deflected from the centre to the end of the scale by the mirror turning through an angle of only 7°. Indeed, with a reflecting galvanometer, as explained in § 56, page 108, we may, with considerable accuracy, replace the *angular* deflections by the number of divisions on the scale through which the spot of light is deflected. Let these be k and a respectively, then

$$K = \frac{P}{\pi} \times \frac{A}{2} \times \frac{k}{a} \text{ very approximately.}$$

In order that we may employ this formula without error to measure a quantity of electricity directly in coulombs, it is necessary to employ a "*ballistic galvanometer.*"

156. Ballistic Galvanometer.—In order to employ an ordinary reflecting galvanometer as a *ballistic galvanometer*, the "*air vane*" should be removed to diminish the damping as much as possible, or if the support for the mirror and the magnets be the air vane as in S S (Fig. 107), it should be replaced by a vertical aluminium wire; and, in addition, the *needle should be weighted*, as this not only still further diminishes the damping action, but makes the vibrations much slower, and so enables the periodic time P to be accurately determined. Also this increase in the periodic time *tends to prevent the needle starting before the discharge has been completed*, which is the fundamental condition that must be fulfilled in order that this formula may be true. A very suitable form of galvanometer to be used as a ballistic galvanometer is shown in Fig. 109, in which R, R are the coils, and inside which is suspended a bell-shaped magnet, devised by Messrs. Siemens and Halske, seen in elevation in M, and in plan in *n s*, to the left of Fig. 109. By means of an aluminium wire the magnet is attached to a mirror S, and the whole suspended by a long fibre of unspun silk, hanging inside a glass tube *r*. The fibre can be raised or lowered

by means of the vertical pin at the top of the tube, and it can be centred by means of the three horizontal screws (two only of which are seen in the figure) which hold in position the outer brass collar covering the vertical pin.

In the case of a galvanometer provided with a centring arrangement, such as is shown in Fig. 109, it is not necessary to have *adjustable* levels, as seen in Fig. 107, because, when the instrument is constructed, the base can be levelled with an ordinary level, and the needle then centred by means of the three adjusting screws at the top of the tube. On all future occasions when it

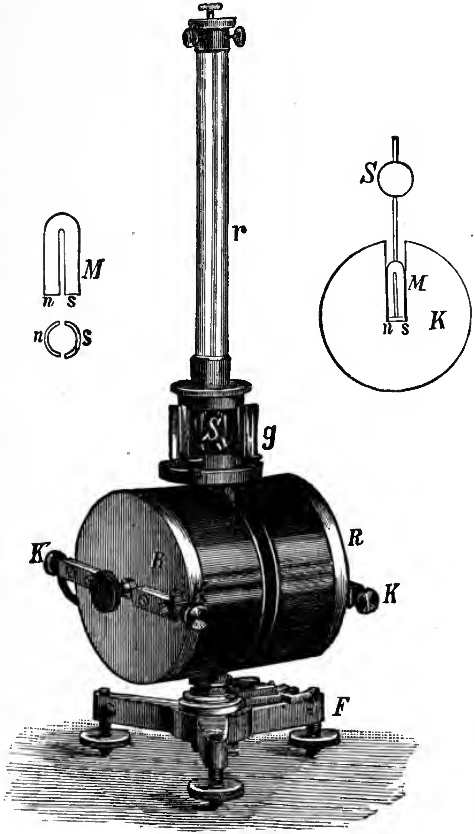

Fig. 109.

is desired to use the galvanometer, all that need be done is to level the base, since when this is done we are sure that the needle is properly centred.

This galvanometer, as usually constructed, contains a large copper ball inside the coils, which is shown in section in K, at the upper right hand of the figure; but this ball, which is introduced for the purpose of damping the vibrations, must, of course, be removed when it is desired to use the instrument as a ballistic galvanometer. The copper ball damps by the *magnetic friction* produced by the attraction between the moving magnet and the electric currents induced in the copper by the motion.

When making experiments with a ballistic galvanometer, great care must be taken that the needle is absolutely at rest when the discharge test is made, otherwise the appreciable momentum, which is possessed by the needle of large moment of inertia, even when moving slowly, will be added to, or subtracted from, that given to it by the current, and will introduce an error. This necessity of waiting for the undamped needle to come absolutely to rest makes observations with a ballistic galvanometer most tedious, and it is well to place, at some convenient spot outside the galvanometer, a small independent coil of wire, in circuit with a cell and a reversing key, by means of which small impulses may be given to the needle to stop it when it is swinging.

Example 85.—With a galvanometer, the needle of which executes 11 complete swings in $6\frac{1}{2}$ seconds 1 Daniell's cell, having an E. M. F. of 1·07 volts, and an internal resistance of 3 ohms, produces a deflection of 127 scale divisions when there is a resistance of 10,000 ohms in the circuit, excluding the galvanometer which has a resistance of 7,560, and which is shunted with the one-thousandth shunt. What number of coulombs is discharged through the galvanometer when an instantaneous deflection of 230 scale divisions is produced?

The current producing the steady deflection of 127 scale divisions, is

$$\frac{1}{1{,}000} \times \frac{1 \cdot 07}{3 + 10{,}000 + \dfrac{7{,}560}{1{,}000}} \text{ amperes,}$$

$$\text{or } \frac{1 \cdot 07}{10{,}000{,}000} \text{ amperes approximately,}$$

$$\therefore K = \frac{6 \cdot 5}{11 \times \pi} \times \frac{1 \cdot 07}{2 \times 10{,}000{,}000} \times \frac{230}{127} \text{ coulombs approximately.}$$

Answer.—0·01822 microcoulombs approximately.

Example 86.—What alteration could be made in the galvanometer referred to in the last example other than altering the coils, so that one-tenth of a microcoulomb should produce an instantaneous deflection of 100 scale divisions?

Answer.—Either the sensibility of the galvanometer must, by slightly approaching the controlling magnet, be diminished in the ratio of 0·01822 × 100 to 0·1 × 230, or the needle must be weighted so that the periodic time is increased in the ratio of 0·1 × 230 to 0·01822 × 100.

Example 87.—Which galvanometer would be the more sensitive for the measurement of quantity, one whose needle made 9 complete vibrations in 3 seconds, and with which a deflection of 200 scale divisions was produced by 1 Daniell's cell when 10,000 ohms were in circuit, and the galvanometer was shunted with the one-hundredth shunt, or one whose needle made 11 vibrations in 7 seconds, and with which a deflection of 85 scale divisions was produced by the same Daniell's cell when 6,000 ohms were in circuit, and the galvanometer was shunted with the one-thousandth shunt?

In order to produce an instantaneous deflection of 100 scale divisions, there will be required with the twc galvanometers respectively,

$$\frac{3}{9\pi} \times \frac{1}{2 \times 100} \times \frac{E}{10{,}000} \times \frac{100}{200},$$

$$\text{and } \frac{7}{11\,\pi} \times \frac{1}{2 \times 1{,}000} \times \frac{E}{6{,}000} \times \frac{100}{85} \text{ coulombs,}$$

if E be the E. M. F. in volts of the Daniell's cell,

$$\text{or } \frac{0{\cdot}08333\text{ E}}{\pi} \text{ and } \frac{0{\cdot}06238\text{ E}}{\pi} \text{ microcoulombs.}$$

Consequently the sensibility of the second galvanometer for measuring quantity bears to that of the first the ratio of 0·08333 to 0·06238, or 1·336 to 1, hence the second is rather more than one-third more sensitive than the first.

157. Correction for Damping.—If it is not possible to remove the vane of a galvanometer so as to diminish the damping to a very small value, or if it is desired to make very accurate experiments, in which case the damping, however small, ought to be allowed for, the following formula should be employed :—

$$K = \frac{A}{2} \times \frac{P}{\pi} \times \frac{k}{a}\left(1 + \frac{l}{2}\right),$$

where l is what is known as the "*Napierian logarithmic decrement.*" This formula is correct when the damping is too great to be entirely neglected, but still not exceedingly large, in which case the formula is much more complicated.

158. Logarithmic Decrement.—When there is damping, the amplitude of the oscillations of the needle will grow gradually less and less, and the "*decrement*" is the name given to the *ratio of the amplitude of one oscillation to the amplitude of the succeeding one*, and this ratio experiment shows is the same for any two successive vibrations. The *Napierian logarithmic decrement* is the logarithm of this ratio to the base ϵ, or 2·71828, and this again equals the logarithm of this ratio to the base 10, divided by the logarithm of ϵ to the base 10, that is,

$$\log._{\epsilon} \text{ ratio} = \frac{\log._{10} \text{ ratio}}{\log._{10}\epsilon},$$

$$= \frac{\log._{10} \text{ ratio}}{0{\cdot}4343}.$$

If not merely the value of $\log._{10}$ ratio be found in a table of logarithms, but if the value of the fraction be also calculated by

using logarithms, care must be taken to employ $\log._{10} \log._{10}$ ratio, that is, to extract the logarithm twice over, because

$$\log._{10} \log._{\epsilon} \text{ratio} = \log._{10} \log._{10} \text{ratio} - \log._{10} 0{\cdot}4343.$$

159. Determining the Logarithmic Decrement when the Damping is Very Slight.—If the damping is very slight, it will be very difficult to detect any difference between the amplitudes of two succeeding vibrations, so that the ratio or decrement will appear to be unity, and its logarithm nought. The decrement can, however, be determined as follows:—Since the ratio of the amplitude of the first oscillation to the amplitude of the second equals the ratio of the amplitude of the second to the amplitude of the third, &c., each ratio being equal to the decrement, it follows that the ratio of the amplitude of the first oscillation to the amplitude of the nth oscillation after it, that is the $(n+1)$th oscillation, equals the nth power of the decrement, or generally the ratio of the amplitude of any oscillation to the amplitude of the nth oscillation after it equals the nth power of the decrement.

Consequently,

$$\log._{\epsilon} \frac{\text{amplitude of any oscillation}}{\text{,, \quad ,, the } n\text{th ,, \quad after it}} = n \times l,$$

$$\therefore\ l = \frac{1}{n} \times \log._{\epsilon} \frac{\text{amplitude of any oscillation}}{\text{,, \quad ,, the } n\text{th ,, after it}}.$$

Now, although it may be difficult to distinguish the decrement from unity, it is comparatively easy to measure the ratio of the amplitude of an oscillation to the amplitude of the nth after it, since n may be taken so large that the ratio differs considerably from unity.

Example 88.—If, on causing the needle of a galvanometer to vibrate, the readings on the scale, at which the spot of light stops, be $+130$, -120, $+105$, -97, $+85$, &c., the $+$ and $-$ indicating deflections to the opposite side of the zero, what is the value of the factor $1 + \frac{l}{2}$, the correction for damping?

Answer.—The amplitude of the first oscillation is $130 + 120$, of the second $120 + 105$, of the third $105 + 97$, &c. Hence, the decrement equals

$$\frac{250}{225} \text{ or } \frac{225}{202}, \text{ \&c., or about } 1{\cdot}111.$$

$$\text{And } \log._{\epsilon} 1{\cdot}111 = \frac{\log._{10} 1{\cdot}111}{0{\cdot}4343},$$

$$= \frac{0{\cdot}0457}{0{\cdot}4343},$$
$$= 0{\cdot}1052.$$

$$\text{Hence, } 1 + \frac{l}{2} = 1{\cdot}0526.$$

Example 89.—What amount of damping is allowable so that the omission of the factor employed to correct for damping shall not make an error of more than $\frac{1}{2}$ per cent.?

Answer.—l must equal 0·01, consequently if d be the decrement

$$\log._\epsilon d = 0{\cdot}01,$$
$$\log._{10} d = 0{\cdot}01 \times 0{\cdot}4343,$$
$$\therefore\ d = 1{\cdot}010,$$

or the ratio of the amplitude of one vibration to the amplitude of the next must not exceed 1·01, or the amplitude of one vibration must not exceed that of the next by more than 1 per cent.

Example 90.—With the value of the decrement given in the last answer, what will be the ratio of the amplitudes of the 1st and the 15th vibrations?

Answer.— $$0{\cdot}01 = \tfrac{1}{14} \log._\epsilon \frac{\text{amplitude of 1st vibration}}{\text{,, ,, 15th}}.$$

$$\frac{\text{amplitude of 1st vibration}}{\text{,, ,, 15th ,,}} = \epsilon^{0{\cdot}14}$$

or more simply, thus:—

$$\frac{\text{amplitude of 1st vibration}}{\text{,, ,, 15th ,,}} = 1{\cdot}010^{14}$$
$$= 1{\cdot}150.$$

Example 91.—If the ratio of the amplitude of the 1st vibration to that of the 21st is 1·2, what is the value of the decrement?

Answer.— $$l = \tfrac{1}{20} \log._\epsilon 1{\cdot}2,$$
$$\therefore\ l = 0{\cdot}00912,$$
$$\text{and } d = 1{\cdot}009;$$

or we may say at once,
$$d = 1{\cdot}2^{\frac{1}{20}}$$
$$\therefore\ d = 1{\cdot}009.$$

From this and the previous examples we see that the error in neglecting the damping will be about $\frac{1}{2}$ per cent. when the amplitude of any vibration exceeds the amplitude of the nth vibration after it by n per cent. of the latter.

160. Comparing Quantities of Electricity.—If two quantities of electricity K and K′ coulombs are to be compared with one another, it is not necessary to determine P nor a since, if k and k' be the number of divisions on the scale over which the spot of light swings in the two cases, we have from the complete formula in § 157, page 296,

$$\frac{\mathrm{K}}{\mathrm{K}'} = \frac{k}{k'}.$$

The correction for damping has also disappeared, hence when simply comparing two quantities of electricity our galvanometer may conveniently, and without in the least complicating the calculation, have a certain *small* amount of damping.

A simple, convenient, and *cheap* reflecting galvanometer, to be used for the simple comparison of quantities of electricity, has been arranged by Mr. Mather, and is shown in Fig. 110. It consists of two coils, C C′, supported in position by fitting into channels formed on the base, and a vertical narrow strip of mica, S S, suspended by a fibre of unspun silk, F, carrying the mirror M, and three sets of magnets, m_1, m_2, and m_3, the first and third of which form an astatic combination with the middle set, m_2, which is inside the coils : m_1 and m_3, although not surrounded with wire, are nevertheless deflected by the current passing round the adjacent convolutions of the coil

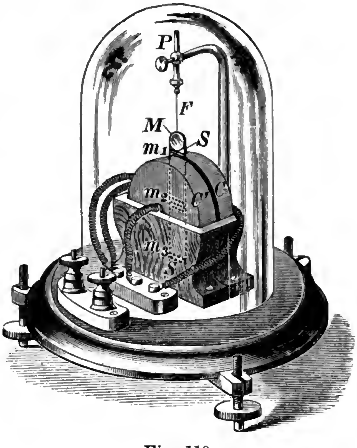

Fig. 110.

in the same direction as m_2, which is inside the coil, so that the magnetic forces acting on all three sets of magnets conjoin in their effects. The damping arising from the resistance of the air to the motion of the mirror will be sufficient for very accurate capacity experiments, and the strip S S may be replaced by an aluminium wire. If, however, rather greater damping be desired it can easily be produced by using the narrow strip of mica to support the needles and mirror, as in the galvanometer shown in Fig. 110. The magnets may be raised or lowered by the pin P, and to avoid torsion

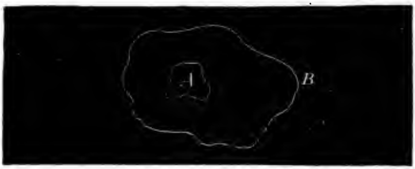

Fig. 111.

being given to the fibre by the head of the pin being turned round in an unknown way, there is a vertical line drawn on the pin, and a mark made on the collar in which this pin slides, and by keeping the line on the pin always opposite the mark on the collar when the pin is raised or lowered, all turning of the pin can be avoided. This contrivance is, of course, cheaper than the simplest mechanical arrangement for preventing rotation of the pin when it is raised or lowered.

161. Capacity.—When one conductor is completely surrounded by another, the "*capacity*" of the inner one is *the number of coulombs required to be given to the inner to produce 1 volt P. D. between the two.* For example, the *capacity* of A (Fig. 111), is the number of coulombs on A when there is 1 volt P. D. between A and B.

The capacity of a conductor, therefore, depends on its external shape, and on its position relatively to the

conductor surrounding it, since, as seen in §§ 66, 67, page 119, the potential of a conductor relatively to another can be varied without altering the quantity of electricity on the former, by varying either its external shape or its position relatively to the latter. If a metallic plate A (Fig. 112) be surrounded with a flat metallic box B, the top and bottom of the box being parallel to A, and *very near* A, then the capacity of A will be very large, since it will require a very large charge of

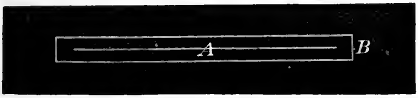

Fig. 112.

electricity to be given to A in order to raise the P. D. between A and B to 1 volt.

162. Condenser.—An arrangement of conductors such as is shown in the last figure is called a "*condenser*," so that a *condenser* may be defined as *two conductors separated by an insulator, and so placed relatively to one another that the capacity of the arrangement is large compared with the size of the conductors.*

A condenser having a large capacity does not, of course, mean that it would hold a large charge without its insulation breaking down, but that it would hold a large charge for the P. D. between its coatings. As far as power to hold a charge from the non-breaking down of the insulation is concerned, a condenser of small capacity may be able to hold a larger charge than a condenser of much larger capacity.

If A (Fig. 112) be charged with positive electricity, there will be a charge of negative electricity on the inside of B, whereas if A's charge be negative, then the charge on the inside of B will be positive. We have further seen (§ 60, page 113) that the quantity of electricity on A is exactly equal in amount to the charge of the opposite kind of electricity on the inside of B. We

may, therefore, define the *capacity of the condenser* either as *the number of coulombs necessary to be given to* A, *or the number of coulombs on the inner surface of* B *when the P. D. between them is* 1 *volt.*

If we desire to make a condenser with a very large capacity, we may either make the plates very large, or the distance between them very small. There are

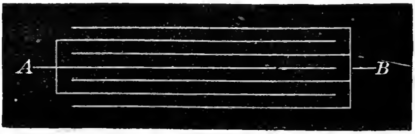

Fig. 113.

obviously practical difficulties in making the distance separating the plates very small, as the insulation is liable to be insufficient, either from particles of dust passing rapidly backwards and forwards between the charged plates, and so discharging them, or from actual sparks passing when the P. D. between the plates is high. On the other hand, if the plate A and the box B

Fig. 114.

(Fig. 112) be very large in area the apparatus becomes cumbersome. This difficulty, however, may be overcome by making both A and B consist of a series of plates (shown in section in Fig. 113), and a condenser is usually symbolically represented in this way, or, still more simply, by two lines drawn parallel to one another, as in Fig. 114, and the sets of plates, A and B, are called the "*coatings*" of the condenser.

163. Capacity of a Condenser is Constant.—By charging a condenser with different P. Ds., and measuring with a galvanometer the quantity of electricity that

enters one of the coatings, or the quantity that leaves this coating when the condenser is discharged, it can be experimentally proved that this quantity is directly proportional to the P. D. The capacity of a condenser may, therefore, be defined as *the ratio of the number of coulombs in one coating to the P. D. in volts between the coatings,* this ratio being a *constant* for a given condenser.

Unless the galvanometer employed be very sensitive, it is better when making the experiment just referred to, for testing the constancy of the capacity of a condenser, to use a condenser of *large* capacity of the type described in § 173, page 317.

164. Variation of the Capacity of a Condenser with the Area of its Coatings.—That *the capacity of a condenser is directly proportional to the effective area of either of the coatings* hardly needs proof, because a condenser with coatings of large area may be regarded as being made up of two or more smaller condensers, such that the sum of the areas of one set of coatings of the smaller condensers is equal to the area of one of the coatings of the larger, the distance between the coatings in the large condenser and in each of the smaller ones being the same, and it is clear that the capacity of the set of smaller condensers is the sum of their capacities.

165. Variation of the Capacity of a Condenser with the Distance between the Coatings.—If we had a condenser of large capacity, and the distance between the coatings of which could be varied at will, an examination of the variation of the capacity, with the distance between the coatings, might be made by fixing the coatings at various distances from one another, and measuring the number of coulombs, or the fraction of a coulomb, required to charge the condenser in the different cases with the same P. D. But practically it is found that any condenser, the size of whose coatings is not so large but that the distance between them can be conveniently adjusted, has so small a capacity that when charged with even a large battery of galvanic cells in series, its charge cannot be

measured with even a very delicate galvanometer. Hence we are compelled to use some *statical* method for investigating the variation of the capacity of a condenser with the distance between its coatings. One plan would be to give the condenser a charge, and then, on varying the distance between the coatings without discharging it, to measure the variation of P. D. between the coatings by means of a suitable electrometer. From this the variation of the capacity could be at once determined, since, with a constant charge in the condenser, the capacity must be inversely proportional to the P. D. between the coatings.

The following method, devised by the author, however, enables us to ascertain the law of variation of the capacity with the distance between the coatings, without making measurements either of the various distances between the coatings, or of the various P. Ds. corresponding with these distances. B B, B′ B′, Fig. 115, are wooden boards (one of which B′ B′ in the figure is shown removed from the apparatus, in order that the interior may be seen) with their surfaces opposed to one another, carefully planed so as to be parallel, and coated with tinfoil, so as to make them conducting. These surfaces together form the outer coating of a condenser corresponding with B (Fig. 112). The inner coating consists of the two sheets of tinfoil, T T, T′ T′, which are parallel to the surfaces of B B and B′ B′. This tinfoil is stuck on thin cloth to give it strength, as it has to roll over the small rollers R R′, when the rod *n*, to which one of the edges of each of the sheets of tinfoil is attached, is pulled down by the thin silk cord C C, or when, on this cord being slackened, the weight W W, to which the opposite edges of the two sheets of tinfoil are attached, pulls T T and T′ T′ down, and the rod *n* up. The rollers R R′, which are made of steel, are only about one-tenth of an inch thick, and are placed close together, so that the surface of the tinfoil wrapped round them may be as small as possible, and so that there may be no inductive action between the tinfoil on

the vertical wooden boards and the inner surfaces of the sheets T T and T′ T′. The rollers are pointed at their

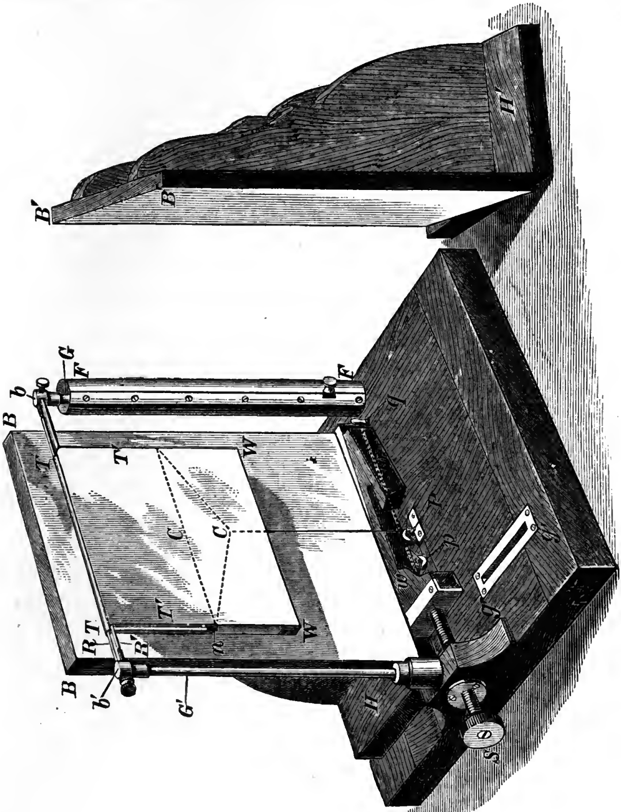

Fig. 115

ends, where they are supported by the brass pieces b b, which are firmly cemented to the tops of the glass rods G G′. The two sheets of tinfoil are, therefore, insulated

from the ground. To keep the glass rods dry they are each surrounded with a tube F F, inside which is placed dry flannel which absorbs moisture. The tubes are hinged down their sides, so that they can easily be opened and removed, and in the figure the one belonging to the rod G′ has been removed. Swaying of the weight W W sideways, as well as side attraction of the suspended sheets of tinfoil T T, T′ T′, are prevented by the weight being guided by the cord C C passing through it.

The boards B B and B′ B′, which are, as seen in the figure, strongly stayed at the back to prevent warping, can be made to recede from one another by pushing in the wedge *w w*, by means of the screw S, or to approach one another by turning the screw in the opposite direction, when the wedge is withdrawn, and a spring pressing against each plate pushes them together. In addition to the horizontal boards H H′, carrying B B and B′ B′, being always pressed by these springs against the side of the wedge *w w*, a pin on the underside of each board slides in a groove, the groove *g′ g′* seen in the figure being that in which the pin attached to H′ slides. B B and B′ B′, therefore, move parallel to themselves, so that in all positions the opposed surfaces are parallel. The cord C C first passes under a little pulley P attached to the base of the instrument, then under a second pulley *p*, moving with the wedge, and its end is attached to the pin *q* (the wedge in the figure being cut away to show the pulleys). Hence on turning the screw S, so as to push in the wedge and separate B B and B′ B′, the cord C C is slackened, and consequently the rod *n* rises, and the weight W W descends, causing the area of the surface of the tinfoil T T and T′ T′ opposed to B B and B′ B′ to increase, and by selecting a proper angle for the wedge *w w*, and a proper pitch for the screw, the area of the two surfaces of the tinfoil T T and T′ T′ can be made to increase, so as to be exactly proportional to the distance separating them from the surfaces of B B and B′ B′.

Under these conditions, if the inner coating of the

condenser be connected with the gold-leaves of an electroscope, and the outer coating of the condenser be connected with the outside of the electroscope, and if a potential difference be set up between the coatings, it will be found that no alteration of the divergence of the gold-leaves will be produced by approaching or separating B B and B′ B′. Now the quantity of electricity on the outer surfaces of T T and T′ T′ is a constant, since there is no electricity inside a conductor (§ 64, page 118). Consequently this experiment tells us that if the ratio of the area of the inner coating to the distance between the coatings is kept constant, the capacity of the condenser is constant. But we have seen (§ 164, page 303) that the capacity of a condenser is directly proportional to the effective area of either of the coatings, hence it follows that *the capacity of a condenser with plane parallel plates is inversely proportional to the distance between the coatings.*

166. **Farad.**—A "*farad*" is the unit of capacity, and a condenser has *a capacity of one farad when a P. D. of 1 volt between its two sets of plates charges each of them with 1 coulomb.*

If A be the area in square centimetres of the entire surface of either of the two sets of opposed parallel plates of an air condenser, and t be the distance in centimetres separating them, and if F be the capacity of the condenser in farads,

$$F = \frac{A}{1 \cdot 131 \times 10^{13} \times t}.$$

If A be reckoned in square inches, and t in inches,

$$F = \frac{A}{4 \cdot 452 \times 10^{12} \times t}.$$

A farad is rather a large unit of capacity for ordinary purposes, hence, one-millionth of a farad, or a "*micro-farad,*" is more commonly employed. If M be the

capacity in *microfarads* of the air condenser, and A and t be in square centimetres and centimetres respectively,

$$M = \frac{A}{1{\cdot}131 \times 10^7 \times t};$$

whereas, if A and t be in square inches and inches respectively,

$$M = \frac{A}{4{\cdot}452 \times 10^6 \times t}.$$

In order that the preceding formulæ may be strictly correct, the linear dimensions of the plates must be large compared with the distance between them. It can, however, be made rigorously true even when this is not the case if a guard-ring, described in § 44, page 89, be employed with one of the plates, and be at the same potential as this plate. In that case A is the area of the smaller plate, not including the area of the guard-ring, and F, or M, is the capacity of this plate, not including the capacity of the guard-ring itself.

167. Charge in Terms of Capacity.—If K be the charge in coulombs in an air condenser, having a capacity of F farads, when there is a P. D. between the coatings of V volts, it follows from the definition of capacity, that

$$K = F \times V,$$

also if M be the capacity in microfarads that

$$K = \frac{M \times V}{10^6}.$$

168. Capacity of a Cylindrical Condenser.—If the two coatings of an air condenser consist of two concentric cylinders A B, C D (Fig. 116), of length l centimetres, and of radii or diameters, R and r respectively, the capacity F in farads

$$= \frac{2{\cdot}413}{10^{13}} \times \frac{l}{\log_{\cdot 10} R - \log_{\cdot 10} r}.$$

As $\log_{\cdot 10} R - \log_{\cdot 10} r$ equals $\log_{\cdot 10} \frac{R}{r}$, it is obvious that it

is quite immaterial what units of length are employed in measuring R and r, provided that the same unit is employed in each case.

If M be the capacity in microfarads,

$$M = \frac{2 \cdot 413}{10^7} \times \frac{l}{\log._{10} R - \log._{10} r}.$$

A common example of a condenser having its coatings concentric cylinders is a submarine cable (*see* Fig. 98, § 140, page 267), the outer coating being the water or the iron sheathing in contact with the insulating core, and the inner coating, the surface of the copper conductor. Consequently, if R be the radius of the core, and r the radius of the conductor, and if n be the length of the cable in knots, the capacity in microfarads

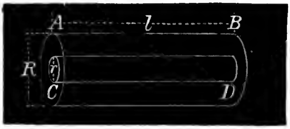

Fig. 116.

$$M = \frac{2 \cdot 413 \times 2029 \times 91 \cdot 44}{10^7} \times \frac{n}{\log._{10} R - \log._{10} r},$$

$$= \frac{4 \cdot 476}{10^2} \times \frac{n}{\log._{10} R - \log._{10} r}.$$

169. Specific Inductive Capacity.—The capacity of a condenser can be still further increased by using, instead of air for the insulator, glass, guttapercha, indiarubber, paraffin oil, or some other solid or liquid insulator.

If K be the number of coulombs of positive electricity required to be given to A, and of negative electricity to B, so as to produce 1 volt P. D. between them when they are separated by air, then if the air be replaced by some other substance, and no other change be made in the condenser, the number of coulombs now required to produce 1 volt P. D. between A and B, will be

K × "*the specific inductive capacity.*"

Hence the *specific inductive capacity of a substance is the ratio of the capacity of a condenser when its plates are separated by this substance to the capacity of the same condenser when its plates are separated by air.*

The following table gives a list of the *specific inductive*

capacities of some important substances as determined by various experimenters, whose names are given in the third column :—

TABLE No. VI.

Specific Inductive Capacity.

Substance.	Specific Inductive Capacity.	Authority.
Vacuum, air at about 0·001 millimetre pressure	0·94 about.	Author.
Vacuum, air at about 5 millimetres' pressure	0·9985	Author.
	0·99941	Boltzmann.
Hydrogen at about 760 millimetres' pressure	0·9997	Boltzmann.
	0·9998	Author.
Air at about 760 millimetres' pressure	1	Taken as the standard.
Carbonic Dioxide at about 760 millimetres' pressure	1·000356	Boltzmann.
	1·0008	Author.
Olefiant Gas at about 760 millimetres' pressure	1·000722	Boltzmann.
Sulphur Dioxide at about 760 millimetres' pressure	1·0037	Author.
Paraffin Wax, Clear	1·92	Schiller.
	1·96	Wüllner.
	1·977	Gibson and Barclay.
	2·32	Boltzmann.
Paraffin Wax, Milky	2·47	Schiller.
Indiarubber, Pure	2·34	Schiller.
„ „ Vulcanised	2·94	Schiller.
Resin	2·55	Boltzmann.
Ebonite	2·56	Wüllner.
	2·76	Schiller.
	3·15	Boltzmann.
Sulphur	2·88 to 3·21	Wüllner.
	3·84	Boltzmann.
Shell-lac	2·95 to 3·73	Wüllner.
Guttapercha	4·2	
Mica	5	
Flint Glass, Very light	6·57	J. Hopkinson.
„ „ Light	6·85	J. Hopkinson.
„ „ Dense	7·4	J. Hopkinson.
„ „ Double extra dense	10·1	J. Hopkinson.

Not merely is the capacity of a condenser increased by using, say glass instead of air, as the "*dielectric*" or *insulating material through which the induction takes place*, but the resistance to loss of charge by sparking is immensely increased; hence, with a glass condenser far greater P. Ds. can be used than with an air condenser of the same size. *The resistance to sparking does not depend on the insulating quality of the substance, but on its rigidity and the resistance it in consequence opposes to rupture.*

If, instead of air, a substance having a specific inductive capacity i be employed, in a condenser made of parallel plates,

$$F = i \times \frac{A}{1{\cdot}131 \times 10^{13} \times t},$$

$$\text{and } M = i \times \frac{A}{1{\cdot}131 \times 10^{7} \times t}$$

if A and t are reckoned in square centimetres and centimetres respectively; and

$$F = i \times \frac{A}{4{\cdot}452 \times 10^{12} \times t}$$

$$\text{and } M = i \times \frac{A}{4{\cdot}452 \times 10^{6} \times t}$$

if A and t are reckoned in square inches and inches respectively.

Similarly the logarithmic formulæ given in § 168, page 308, for the capacity of a cylindrical condenser, must be multiplied by i, the specific inductive capacity of the dielectric when this is paraffin wax, glass, &c., or when, as in the case of a submarine cable, guttapercha or indiarubber fills up the space between the two conductors.

Example 92.—If the distance between the plates in an air condenser be 1 millimetre, what must be the area

of each set of plates in order that the capacity may be 1 microfarad? *Answer.*—About 1,131,000 sq. cent.

Example 93.—How many plates about 1 foot square would be necessary to produce the area required in the last answer, and what would be the exact size of each plate?

If we assume that the plates were each 1 square foot, then, since the area on both sides of each plate is utilised, it follows that the number of plates required would be $\frac{1,131,000}{1,858 \cdot 02}$ or 608·7. We could, therefore, either use 608 plates, each a little larger than 1 square foot, or 609 plates, each a little smaller. The latter will be nearer in size to the square foot, and using this number, it is easy to calculate that each plate must be 0·9994 square feet, or 11·99 inches square. For the other coating B (Fig. 113, page 302), there must be, of course, 610 plates, since one surface of each of the outer plates of B will have no action as a condenser.

Example 94.—If the insulating material in a condenser be paraffined paper, and if we assume that the specific inductive capacity of the paraffined paper is the same as that of paraffin wax, 1·977, what must be the thickness of the paper in order that the condenser may have one-third of a microfarad capacity when the area of each set of plates is 205 square feet? *Answer.*—0·03933 of an inch.

Example 95.—A cylindrical glass jar one-tenth of an inch thick, and 3 inches in diameter, is coated inside and outside with tinfoil on the bottom, and on the sides for a height of 3 inches. If the glass be extra dense flint, what must be the P. D. between the tinfoil coatings so that the charge may be one-millionth of a coulomb?

The glass being very thin, the formulæ for a condenser formed of plane parallel plates may be used. The area of tinfoil at the bottom is $\frac{\pi \times 3^2}{4}$ sq. inches, that on the

sides $\pi \times 3 \times 3$ sq. inches. If, therefore, V be the unknown P. D. in volts,

$$\frac{1}{10^6} = 10{\cdot}1 \times \frac{\dfrac{\pi \times 3^2}{4} + \pi \times 3 \times 3}{4{\cdot}452 \times 10^{12} \times \frac{1}{10}} \text{ V}.$$

$\therefore$ V = 1247 *Answer.*—1247 volts.

Example 96.—What is the capacity of the glass condenser referred to in the last question?

If F be the capacity in farads,

$$F = \frac{1}{10^6 \times 1{,}247},$$

hence the capacity is 0·0008021 microfarads.

Example 97.—The diameter of the copper conductor of the Direct United States cable being 0·16 of an inch, the diameter of the guttapercha core 0·446 of an inch, and its length 2,443 knots, what is its capacity?

From the formulæ in § 168, page 309, we have

$$M = 4{\cdot}2 \times \frac{4{\cdot}476}{10^2} \times \frac{2443}{\log_{\cdot 10} \dfrac{446}{160}}$$

$= 1031.$ *Answer.*—1031 microfarads.

The actual capacity determined by experiment is 1000·4 microfarads.

170. Condensers for Large P. Ds.—The charge in a condenser, K coulombs, equals, as we have already seen,

$$F \times V,$$

hence this charge can be made great by making one or other, or both of the factors, F and V large. For experiments with the old form of "*frictional electrical machines,*" or with the more modern form of "*influence machines*" (*see* § 196, page 371), it is V that is always made large, whereas when galvanic batteries are used as the source of the P. D., it is F that is usually made large. In the recent experiments, however, made by Drs. De La Rue

and Hugo Müller, with their large silver chloride battery, consisting of some 20,000 cells, the condensers have been made to stand the high P. D. produced by this battery as well as to have a large capacity. When thousands of volts are to be employed, a large resistance to sparking is therefore quite as important as high specific inductive capacity, and, as already stated, requires that the dielectric should be rigid. (*See* the note to § 192, page 358.)

171. Leyden Jar.—Some kind of glass is usually employed in the construction of condensers that are to be charged with a very large P. D., and the condenser takes the form of a "*Leyden jar*," a type of which is seen in Fig. 117. The name is derived from the town of Leyden, at which the property of electric capacity was accidentally discovered in 1746, by Musschenbroek, and his pupil Cuneus. Desiring to collect the supposed electric fluid, they used a bottle partly filled with water, into which dipped a nail, passing through the cork, to carry the fluid from the electric machine to the water, and on Cuneus touching the nail with one hand, the bottle being held in the other, he received a shock.

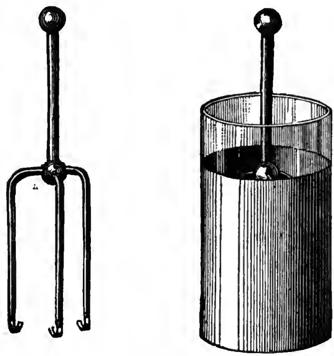

Fig. 117.

In the ordinary Leyden jar, such as is seen in Fig. 117, the tin coatings are sheets of tinfoil, one pasted inside the jar, and the other outside. Electric connection is made with the inside coating either by a metal rod or rods resting on the bottom, or more commonly, by a chain or a flexible bit of wire hanging from a brass rod, which, in this case, is supported by a wooden

cover to the jar to which the rod is fixed. But such a Leyden jar, even when the surface of the glass, which is not covered with tinfoil, is coated with shell-lac or other varnish, has but a poor insulation in damp weather, and requires the glass to be constantly held in front of the fire to be dried. For with the wooden cover in contact with both the metal rod and with the edge of the jar, in accordance with the unscientific form of construction usually adopted, the interior of the glass helps but little towards holding the charge, seeing that if the outside of the wooden cover and of the jar be dirty and moist, there is a direct road for the electricity to leak from the rod to the tinfoil outside, without passing at all over the glass on the interior. Hence, that portion of the glass which it is most easy to keep dry and clean, is rendered useless by the presence of the wooden cover in contact with the rod. On this account the form of Leyden jar shown in Fig. 118, and originally employed by Sir William Thomson, is much to be preferred. The outer coating consists of tinfoil T T, as in the ordinary Leyden jar, but the interior is formed of strong sulphuric acid S S, into which dips a leaden rod L, expanded at the lower part into a sort of foot so as to stand firmly on the bottom of the glass jar. Both rod and foot are made of lead so as not to be acted upon by the acid, but the upper part I of the rod, which does not dip into the acid, may be conveniently made of iron, being less liable to bend than lead. The mouth of the jar is partially closed with a wooden cover W, to keep out dust, and retard a too rapid interchange of the air between the inside and outside, which would prevent the sulphuric acid being able to keep the interior surface of the glass dry. A cork C, sliding on the rod I, is pressed down when the jar is not in use, but is raised up to prevent electric contact between the rod and the cover W W, when the jar is to be charged.

In Fig. 118 there is seen carried by the iron rod a metallic cone. This may be used for making experiments

in density with the proof plane (*see* § 63, page 118), and the advantage of attaching the charged cone, or other conductor (the distribution of density over whose surface we desire to measure), to another conductor of large capacity, is that the amount of electricity removed by the proof plane, each time we touch the surface of the cone, does not sensibly diminish the potential or the total charge possessed by the cone. Without the use of the Leyden jar, the effect of touching any point A on the cone with the proof plane, and removing the proof plane, is not merely to remove the amount of electricity that was on the surface of the cone touched by the proof plane, but to slightly diminish the density of every other part of the surface of the cone, since electricity has to flow from the rest of the body to recharge the part touched by the proof plane. Hence, if the cone be first charged to a given potential, and then the relative densities at any points A and B be determined by touching them successively with the proof plane, slightly different results will be obtained, according to the order in which these two points are touched. The use of a well-insulated Leyden jar removes the difficulty, which may also, to a certain extent, be overcome by first touching A, and measuring the charge q_1, taken away by the proof plane, then touching B, and measuring the charge q_2, removed, and thirdly, touching A again, and measuring the charge q_3, removed by the proof plane on touching A a

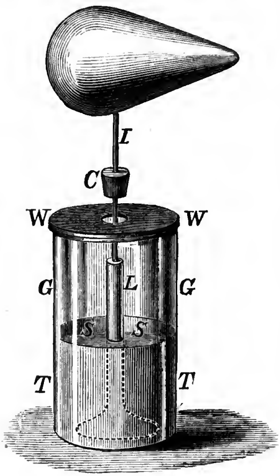

Fig. 118.

second time, because the density at B will be to the density at A approximately, as

$$q_2 \text{ to } \frac{q_1 + q_3}{2}.$$

A glass jar, with a contracted neck, as shown in Fig. 119, would have a much higher insulation as long as the interior of the neck was clean, but there would be greater difficulty in introducing the acid without splashing the neck, and in cleaning the inside of the neck when it became dirty, even if we took out the metal rod which fits into a tubulure at the bottom of the vessel, as does the glass rod in the insulating stand, Fig. 40, page 112.

Fig. 119.

172. Battery of Leyden Jars.—If a greater capacity is desired than can be obtained with one such Leyden jar, when the glass is made as thin and as large as is practicable, then a "*battery of Leyden jars,*" that is, a number of sulphuric acid Leyden jars in parallel, should be employed.

173. Constructing Condensers of Very Large Capacity.—When a very large capacity is required the dielectric employed consists usually of *sheets of paper or of mica,* which have been soaked in *melted paraffin wax or in a solution of shell-lac in alcohol.*

The sheets of tin-foil are shaped as shown in *a* (Fig. 120), one corner being cut off, and the sheets of insulating material *b* are made about two inches wider and two inches longer, and have two corners cut off. On a sheet of insulating material there is first laid a sheet of tinfoil, as in *c*, then a sheet of insulating material is laid on the top, then a second sheet of tinfoil with its uncut corner turning the other way, and so on, so that finally there are a number of alternate sheets of tinfoil with their corners projecting over the sheets of insulating

material to the right, and the other set of alternate sheets of tinfoil, with their uncut corners projecting over to the left. Each of the exposed sets of corners is soldered together, and forms an electrode or terminal of the condenser.

When paraffined paper is employed as the insulating material, the paper is first very carefully examined by holding it, sheet by sheet, up to the light, so that the existence of any small holes may be detected, and any sheet possessing such holes is discarded. The good sheets are then placed in a bath of melted paraffin wax

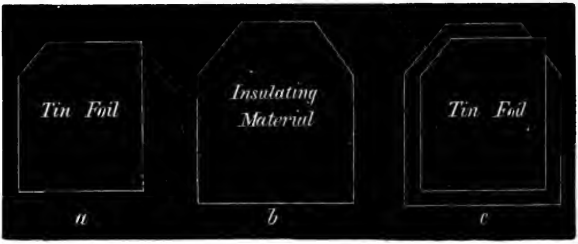

Fig. 120.

warmed by steam to about 110°C., or a little above the boiling point, so that all water may be driven off. On a horizontal slab of cast iron, also warmed by steam to about the same temperature, the sheets of paraffined paper and tinfoil are laid in the way just described, the sheets being carefully smoothed with a flat strip of wood as they are laid on. Two sheets of paper are placed between each pair of sheets of tinfoil to avoid the possibility of a hole in the paper causing leakage, it being, of course, most improbable, even if there were a minute hole in each sheet, that the holes would come exactly opposite one another. After the condenser has been built up in this way it is placed between two warm metal plates, and pressed with a weight of about eight cwts. while it is cooling, in order that the surplus paraffin wax may be squeezed out and the whole consolidated.

To avoid the paraffin wax being wasted, it is desirable to have a kind of gutter all round the cast iron plate, on which the condenser is built up, for the paraffin wax to run into.

It is not desirable to use the paraffin wax in the baths more than once, since even when the temperature is not raised more than about 110° C. or 120° C., slight decomposition of the wax may occur, which diminishes its high specific resistance. The paraffin employed for making condensers is highly purified, and the residue in the baths is sold to be used for making candles.

174. Comparing Capacities.—The capacities of two condensers can be easily compared by successively charging each condenser with the same P. D., and observing, by means of a suitable galvanometer, the amounts of electricity that rush into the condenser to charge them, or by charging them with the same P. D., and then discharging them successively through a suitable galvanometer, the instantaneous deflection produced in either case being directly proportional to the capacity.

If the condensers differ much in capacity, so that when the galvanometer is properly adjusted and a suitable P. D. selected to obtain a convenient deflection on the galvanometer with the smaller condenser, the deflection obtained with the larger would be much too great, or conversely if the sensibility of the galvanometer were arranged, and the P. D. selected with reference to the larger condenser, the deflection obtained with the smaller condenser would be much too small; hence, in order to make the comparison of the capacities, either the galvanometer must have different sensibilities, or the P. D. employed must be different in the two cases.

The only easy way of altering the sensibility of a galvanometer by a definite amount is by shunting it, and even this method, as was first pointed out by Mr. Latimer Clark, introduces a certain vagueness when we are dealing with instantaneous deflections and "*transient currents*," or currents only lasting for a very short time. (*See*

§ 188, page 349.) Hence, it is better to use different P. Ds. in the two cases, and the simplest method of obtaining two P. Ds. of a known ratio to one another is that described in § 150, page 278.

Let F and F′ be the capacities of the two condensers, V and V′ the P. Ds. employed in charging them, and k and k' the instantaneous deflections produced either on charging or on discharging, then

$$\frac{F}{F'} = \frac{V'}{V} \times \frac{k}{k'}.$$

175. Charge and Discharge Key.—If it be merely desired to observe the instantaneous deflection on charging a condenser, any simple key for closing the circuit

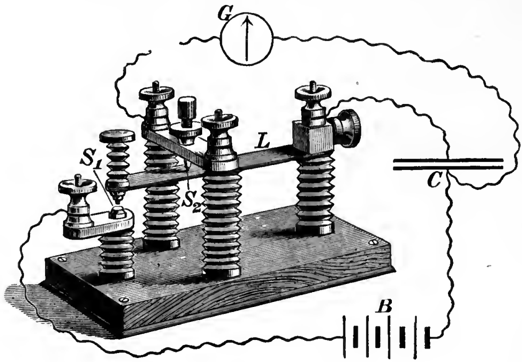

Fig. 121.

may be employed, but if we desire to observe the discharge *immediately* after removing the battery, and therefore, before the condenser has lost any of its charge by leakage from one coating to the other, some special form of key must be employed, and that shown in Fig. 121 will be found convenient, and has very high insulation. If joined up as shown in this figure, it will be seen that, on depressing the lever L, contact will be

made between the lever and the lower stop S_1, while that between L and the upper stop S_2 will be broken. This will enable the battery B to charge the condenser C without deflecting the galvanometer G. If, now, the pressure on the bent lever L be withdrawn, it will fly up, breaking the contact at S_1, and so disconnecting the battery, while *immediately* afterwards the contact at S_2 will be made and the condenser discharged through the galvanometer.

If the key be joined up as shown in Fig. 122, the galvanometer will measure the charge put into the con-

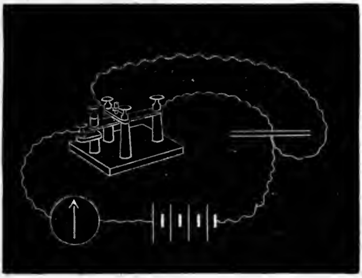

Fig. 122.

denser on depressing the key, but not the discharge that will take place on liberating the key. If the insulation of the condenser be slightly defective, so that there is a small leakage from one coating to the other, the swing of the galvanometer needle on charging the condenser (Fig. 122) will be larger than it would be were there no leakage, while, on the other hand, the swing on discharging will be smaller; the mean of the two swings may be taken as a measure of the true charge of the condenser, independently of the leakage, if the effect due to the leakage is small.

When the apparatus is arranged as in Fig. 123, both the charge and discharge will be measured, producing deflections on opposite sides of the zero, and therefore

producing practically no effect at all, if made to follow one another fairly rapidly. The effect of either on the galvanometer can be prevented by short-circuiting it during either the charge or discharge by means of the short-circuit plug P.

176. Condensers are Stores of Electric Energy, not of Electricity.—If a suitable galvanometer be inserted in each of the wires connecting the two coatings of the condenser C with the two ends of the battery B (Fig. 124), it will be found on completing the circuit by closing a key

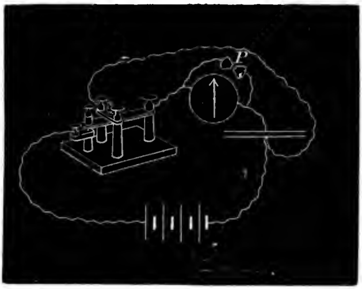

Fig. 123.

at K, that the first swings on the two galvanometers are such as indicate equal quantities of electricity passing through them. And if when the condenser is charged the battery be removed, and the condenser be discharged by connecting together the wires P and Q coming from the galvanometer, then the first swings of the galvanometer needles will again be such as to indicate that equal quantities of electricity pass through them, but in this case in the opposite direction to that in which the electricity passed during the charge. *Hence, both on charging and on discharging a condenser as much electricity passes into one coating as passes out of the other*, and there is no storing, or accumulating, of electricity. In fact, as far as the galvanometer deflections during the

charge show, we could not say whether there was a condenser at C or a resistance, the value of which was, from some cause, rapidly increased, to practically infinity, on completing the circuit. The sudden deflections, however, produced on the galvanometer when the wires P and Q are joined together after removing the battery, could not be produced if C were a resistance, since no alteration of the value of a resistance can, by itself, and without any current generator, produce a current. When the condenser has a large capacity, and when the P. D. employed in charging it is large, the current obtained on

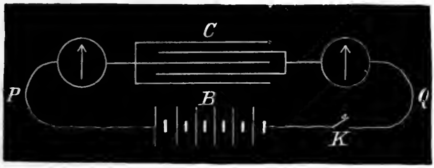

Fig. 124.

discharging it may produce very powerful effects. Hence, we are led to conclude that, *although a charged condenser contains no store of electricity, it contains a store of electric energy*, and it can be shown that, if the capacity of the condenser be F farads, and if it be charged with a P. D. of V volts, the store of electric energy, or the work this store can do when the condenser is discharged, equals

$$\frac{F \times V^2}{2{\cdot}712} \text{ foot lbs.}$$

Example 98.—How many times per second would a condenser of 10 microfarads have to be charged with 86 volts, and discharged so that it would give out about one-thousandth of a horse-power? *Answer.*—About 20.

Example 99.—If a battery having an E. M. F. equal to E volts be used to charge a condenser of F farads, how many foot lbs. of work are wasted in the charging?

Let K be the charge in coulombs held by the condenser when the P. D. between its coatings is E, then

$$K = F E,$$

and the store of energy equals, from § 176, page 323,

$$\frac{F E^2}{2 \cdot 712} \text{ foot lbs.}$$

The work done in t minutes by a battery of E. M. F. equal to E volts, when a current of A amperes flows through it, equals, from § 115, page 203,

$$44 \cdot 25 \text{ A E } t \text{ foot lbs.,}$$

$\therefore$ the work done in t seconds is

$$\frac{44 \cdot 25}{60} \text{ A E } t \text{ foot lbs.}$$

Now A t equals the number of coulombs that flow through the battery in the time t, whether t be short or long, and although when charging the condenser the current will at first be very strong, and then will gradually diminish until it becomes nought, we may consider it to remain constant during a small fraction of a second. Hence, if k be the number of coulombs that pass during a time so short that the current may be regarded as remaining constant during this time, the work done by the battery during this time equals

$$\frac{44 \cdot 25}{60} k \text{ E foot lbs.,}$$

and this is true for each short time during the charging Hence, the total work done by the battery equals

$$\frac{44 \cdot 25}{60} \text{ K E foot lbs.,}$$

$$\text{or } \frac{44 \cdot 25}{60} \text{ F E}^2 \text{ foot lbs.}$$

Hence, the waste of energy during the charging equals

$$\left(\frac{44{\cdot}25}{60} - \frac{1}{2{\cdot}712}\right) \text{F E}^2 \text{ foot lbs.,}$$

$$\text{or} \quad \tfrac{1}{2}\,\frac{44{\cdot}25}{60} \text{ F E}^2 \text{ foot lbs.,}$$

or half the energy expended by the battery is wasted, no matter what be its resistance, or the resistance of the rest of the circuit.

Example 100.—If, instead of employing a battery having an E. M. F. of E volts to charge the condenser, we first charge it with a battery of $\frac{\text{E}}{n}$ volts; then increase the E. M. F. of the battery to $\frac{2\,\text{E}}{n}$ and further charge the condenser; next increase the E. M. F. of the battery to $\frac{3\,\text{E}}{n}$, and still further charge the condenser, and so on, what will be the total waste of energy?

The number of coulombs put into the condenser in the first charge equals

$$\frac{\text{F E}}{n},$$

and the work done by the first battery equals

$$\frac{44{\cdot}25}{60} \times \frac{\text{F E}^2}{n^2} \text{ foot lbs.}$$

The number of coulombs put into the condenser in the second charge equals

$$\frac{\text{F E}}{n},$$

and the work done by the battery equals,

$$\frac{44{\cdot}25}{60} \times \frac{\text{F E}}{n} \times \frac{2\,\text{E}}{n} \text{ foot lbs.,}$$

$$\text{or} \quad \frac{44\cdot25}{60} \, 2\,F \, \frac{E^2}{n^2} \text{ foot lbs., \&c.}$$

So that the total work done in charging the condenser equals

$$\frac{44\cdot25}{60} \times \frac{F\,E^2}{n^2}\left(1 + 2 + 3 + \ldots . + n\right)$$

$$= \frac{44\cdot25}{60} \times \frac{F\,E^2}{n^2}(1+n)\frac{n}{2}$$

$$= \frac{44\cdot25}{60} F\,E^2 \left(\frac{1}{2\,n} + \frac{1}{2}\right) \text{ foot lbs.}$$

The store of energy in the condenser equals, as before,

$$\frac{F\,E^2}{2\cdot712} \text{ foot lbs.,}$$

quite independently of the way in which the condenser has been charged. Hence, the waste equals

$$\frac{44\cdot25}{60} F\,E^2 \frac{1}{2\,n} \text{ foot lbs.,}$$

which becomes the same as before if n is unity, but on the other hand becomes as small as we please if n be made larger and larger. In fact, the more nearly we make the rate of increase of the E. M. F. in charging equal to the rate of the decrease of the P. D. between the coatings of the condenser in discharging, the less will be the waste in charging.

Example 101.—If an air condenser be formed of two parallel metallic plates, each two square feet in area, placed $\frac{1}{30}$th of an inch apart, and charged with a P. D. of 250 volts, what amount of work must be done in separating the plates, so that the distance between them is increased to $\frac{1}{10}$th of an inch, if the wires used in charging the condenser be removed before the plates are

separated, so that the charge in the condenser remains unaltered during the separation?

The capacity before separation equals from § 166, page 307,

$$\frac{288}{4{\cdot}452 \times 10^{12} \times \frac{1}{30}} \text{ farads,}$$

$$\text{or } 1{\cdot}940 \times 10^{-9} \quad ,,$$

and after separation,

$$\frac{288}{4{\cdot}452 \times 10^{12} \times \frac{1}{10}} \quad ,,$$

$$\text{or } 6{\cdot}467 \times 10^{-10} \quad ,,$$

therefore if K be the charge in coulombs in the condenser, and V the P. D. after separation in volts,

$$K = 1{\cdot}940 \times 10^{-9} \times 250$$

$$= 6{\cdot}467 \times 10^{-10} \times V,$$

$$\therefore \quad V = 750 \text{ volts.}$$

The store of energy, in the condenser before separation equals

$$\frac{1{\cdot}940 \times 10^{-9} \times 250^2}{2{\cdot}712} \text{ foot lbs.,}$$

$$\text{or } 4{\cdot}471 \times 10^{-5} \quad ,,$$

and the store of energy after separation equals

$$\frac{6{\cdot}467 \times 10^{-10} \times 750^2}{2{\cdot}712} \text{ foot lbs.,}$$

$$\text{or } 1{\cdot}341 \times 10^{-4} \quad ,,$$

hence the work done in the separation equals

$$8{\cdot}939 \times 10^{-5} \text{ foot lbs.}$$

177. Absolute Measurement of a Capacity.—The absolute capacity of a condenser can be determined in

farads by using a battery, whose E. M. F. we know in volts, to charge it, when there is in the circuit a galvanometer which has been calibrated so that the number of coulombs or fraction of a coulomb that causes any particular instantaneous swing is known. But this absolute measurement of a capacity can more easily be effected as follows, the only thing that is required to be previously known being the value of a resistance in ohms.

Let B (right hand, Fig. 125) be a battery of unknown E. M. F. and resistance, but of such a large

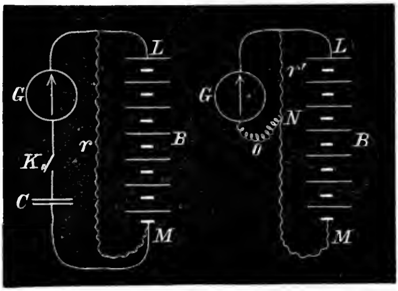

Fig. 125.

number of cells, that when it is used to charge the condenser C, F farads in capacity, a suitable instantaneous deflection is obtained on a reflecting galvanometer G. In order that we may use two P. Ds., whose ratio is known, shunt the battery with a large resistance r, then if a portion r' (right hand, Fig. 125) of this resistance bears to the whole r, a ratio equal to R, it follows, without our knowing either r or r' in ohms, that V′, the P. D. between L and N, the terminals of r', bears to V, the P. D. between L and M, the terminals of r, the same ratio R.

Charge the condenser with the battery thus shunted, by depressing the key K (left hand, Fig. 125), and let the instantaneous deflection be d_1. Next using V′

(right hand, Fig. 125), send a steady current through the galvanometer in series with a large resistance coil, and let the value of the resistance of these two be o ohms. Let d_2 be the steady deflection so obtained, then

$$F = \frac{P}{2\pi} \times \frac{R}{o} \times \frac{d_1}{d_2}.$$

For if K be the unknown number of coulombs required to charge the condenser to the unknown P. D. of V volts,

$$K = F \times V,$$

also from § 155, page 292, we know that

$$K = \frac{P}{\pi} \times \frac{A}{2} \times \frac{d_1}{a},$$

where a is the steady deflection that is produced by A amperes. But since the deflection is proportional to the current, and since the deflection d_2 is produced by a current of $\frac{V'}{o}$ amperes,

$$\frac{a}{d_2} = A \div \frac{V'}{o}$$

$$= \frac{A \times o}{R \times V},$$

$$\therefore K = \frac{P}{\pi} \times \frac{R \times V}{2 \times o} \times \frac{d_1}{d_2},$$

$$\therefore F = \frac{P}{2\pi} \times \frac{R}{o} \times \frac{d_1}{d_2}.$$

If the vibrations of the needle be damped, then the above must be multiplied by $1 + \frac{l}{2}$, where l is the Napierian logarithmic decrement (*see* § 157, page 296), in order to obtain the correct value of F.

This method was employed by the late Professor Fleeming Jenkin, in 1867, in making the first absolute measurements of the capacity of a condenser.

178. Statical Method of Comparing Capacities.— Let F and F′ be the capacities of the two condensers that are to be compared. By means of the arrangement shown to the left (Fig. 126), charge the two condensers with the P. Ds. between the points L and C, and C and M respectively. Let these P. Ds. be called V and V′ volts, the numerical value of which it is not necessary to know. Now, without discharging the condensers, separate the coating A of the one and the coating B′ of the other from the resistance coil, and join these coatings together as shown to the right (Fig. 126), the other coatings B and A′ being joined together as before. Let V_1 be the resultant P. D. in volts between A B′ and B A′, the numerical value of which also need not be known, let K and $-K'$ be respectively the numbers of coulombs on the plates A and B′ before discharge, then

$$K = F V$$

$$\text{and } K' = -F' V',$$

also we know that $K - K'$ is the charge in the compound plate A B′ of the joint condenser to the right (Fig. 126), of capacity $F + F'$,

$$\therefore \quad K - K' = (F + F') V_1.$$

Substituting, we have $$F V - F' V' = (F + F') V_1,$$

$$\therefore \quad \frac{F}{F'} = \frac{V' + V_1}{V - V_1}.$$

In order to compare V, V′, and V_1, observe the deflection produced by V_1 on a suitable electrometer, and, without altering the arrangement of the battery and resistance coil shown to the right (Fig. 126), let two points, separated by a resistance r_1 be found, by trial, such that the P. D. between them produces the same deflection on the electrometer, then

$$V : V' : V_1 :: r : r' : r_1.$$

Consequently, $$\frac{F}{F'} = \frac{r' + r_1}{r - r_1}.$$

If r' and r be so selected by shifting the connection C (Fig. 126), in one direction or other, that K equals K′, or V_1 is nought, then

$$\frac{F}{F'} = \frac{r'}{r}.$$

This method of discharging one condenser into another, and measuring the resultant P. D., may be employed not only when the condensers are small, but when one or both of them are long

lengths of submarine cable, in which case, owing to the "*retardation*," or time taken in charging or discharging the cable, the simple galvanometer method would give erroneous results unless the period of the needle were made most inconveniently long so as to insure the charge or discharge being completed before the needle began to move.

If, however, the method just described of discharging one condenser into the other, and measuring the resultant effect be employed, not on account of the smallness of the capacities of the condensers under comparison, but because one or both of them have considerable retardation, then a galvanometer can be used to measure approximately the resultant P. D., the test giving perfectly accurate results when the point c is so selected, by trial, that the discharge of the compound condenser through the galvanometer is nought.

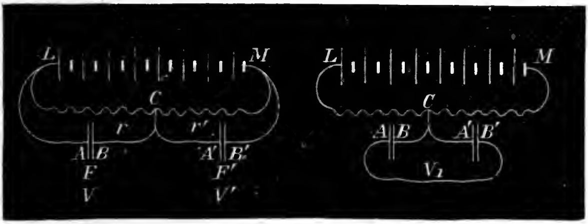

Fig. 126.

If the resultant charge be not absolutely nought, we can, instead of making a great number of tests to find the point c, for which it would be absolutely nought, and which may occupy more time than is at our disposal, correct approximately for a small resultant discharge as follows:—

Let d be the resultant deflection, and let d' be the deflection obtained on charging the compound condenser with the P. D. between two points in the resistance coil, separated by a small resistance r_2; then, if, as before, r_1 be the resistance between two points in the coil having a P. D. between them equal to V_1, but which we cannot now find directly, as we are not using an electrometer, it follows, disregarding the retardation, that

$$\frac{d}{d'}=\frac{r_1}{r_2},$$

Hence,
$$\frac{F}{F'}=\frac{r'+\frac{d}{d'}r_2}{r-\frac{d}{d'}r_2},$$

$$\text{or} \quad \frac{F}{F'} = \frac{d' r + d r_2}{d' r - d r_2}.$$

179. Measuring Specific Inductive Capacity.—If we know the area A of each of the coatings of a condenser in square centimetres, and t the thickness of the dielectric in centimetres, then, from § 169, page 311, it follows that i, its specific inductive capacity,

$$= \frac{F \times 1{\cdot}131 \times 10^{13} \times t}{A},$$

where F is the capacity of the condenser in farads, which, if large enough, can be measured either absolutely by the method described in § 177, page 328, or relatively by comparison with another condenser, whose capacity is known in farads, using the method described in § 174, page 319.

Frequently, however, we desire to measure the specific inductive capacity of a comparatively small specimen of an insulating material, too small to be employed in making a condenser of large capacity, unless the dielectric were made so thin that it would be extremely difficult to determine its thickness accurately. In such a case we may employ the *statical* method described in § 178, page 330, of comparing the capacity of a condenser made with the specimen of insulating material with the capacity of a condenser of somewhat similar dimensions, but having air for the dielectric. To use this method, however, we must have an electrometer of considerable sensibility, with its quarter cylinders far better insulated from one another and from the outside of the instrument than are those in the instrument illustrated in Figs. 47 and 48, § 75, page 131. We also must have a charge and discharge key of high insulation, and *enclosed in a metallic box*, so as to be shielded from induction (*see* § 51, page 99). This statical method, therefore, of comparing the capacities of two condensers, each of small capacity, although susceptible of giving extremely accurate

results when carried out with the various precautions that would be adopted by a skilled experimenter, is altogether unsuitable to be employed by a beginner.

The following method, however, based on a plan of experimenting originally suggested by Dr. Sauty, has been used by the author with good results. C and C′ (Fig. 127) are the two condensers of small capacity, M and M′, that we desire to compare; *a* and *b* are two adjustable resistances wound double in the ordinary manner employed in constructing resistance coils (*see* Fig. 7, § 12,

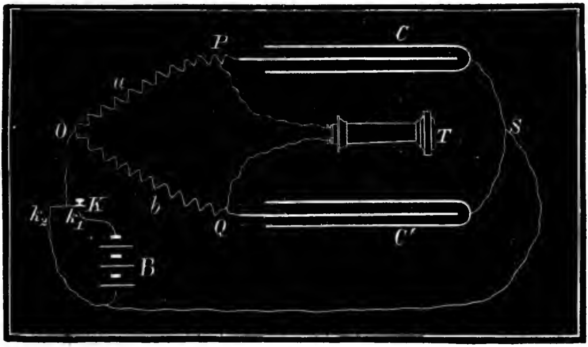

Fig. 127.

page 28), K is a key, turning about its centre and making contact either at k_1 or at k_2, so that by moving the handle down and up the two condensers can be charged by the battery B or discharged, and T is an ordinary Bell telephone connecting the points P and Q, and which is an extremely delicate instrument for detecting small rapid fluctuations in the strength of a current passing through it. If the key K be alternately moved up and down there will be a succession of currents in *opposite* directions through the telephone, unless the potentials at P and Q always remain equal to one another, and in order that the P. D. between these two points may be

always nought, the rise or fall of potential at each of these points must be the same in the same time. This condition will be fulfilled when the quantities of electricity that flow into, or out of, the two condensers in the *same time*, are directly proportional to their capacities, and when there is no sensible retardation. Further, if the potentials at P and Q are equal to one another, the quantities of electricity that flow through the two wires, O P and O Q respectively, must be inversely proportional to their resistances a and b. Hence, combining these two conditions, no sound will be heard in the telephone if a and b are adjusted until

$$\frac{M}{M'}=\frac{b}{a}.$$

The substance of which we desire to measure the specific inductive capacity, as, for example, a sheet of glass or a sheet of guttapercha, should have pasted on each side of it sheets of tinfoil of equal size, and about one inch smaller all round than the sheet of dielectric, so as to secure little surface leakage. If the sheet of dielectric be itself small, the space left uncovered with tinfoil must be less than one inch in width, but in that case the uncovered portion should be *carefully cleaned and dried.* It is also desirable for the purpose of diminishing this surface leakage to rest the condenser on a block B, as shown in Fig. 128, so as to keep the underneath portion of the sheet of dielectric D that is not covered with a sheet of tinfoil, corresponding with T above, from touching anything.

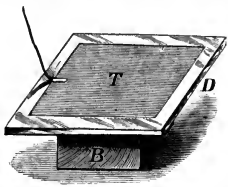

Fig. 128.

180. Standard Air Condenser.—The standard air condenser may be conveniently constructed, as shown in Fig. 129, of thin slabs of plate glass about one-eighth of an inch thick, coated on both sides with tinfoil. These sheets of glass *do not act as the dielectric,* but

merely form convenient supports, with very plane surfaces, for the sheets of tinfoil, hence the two sheets of tinfoil on the two sides of any one of the slabs of glass must be electrically connected. With every alternate slab 1, 3, 5, &c., the sheets of tinfoil are pasted over the

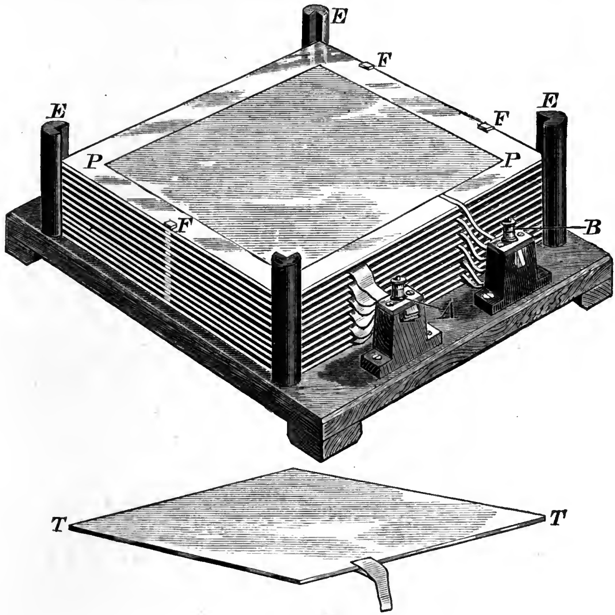

Fig. 129.

whole surface of the glass, and may be each about one square foot in area, while in the case of the other set 2, 4, 6, &c., there is one inch left all round the glass not coated with tinfoil, as seen on the top plate P P of the condenser in the figure. This is in reality the top plate but one, the top plate T T, which is wholly covered with tinfoil, having been removed to enable the plate P P to be seen. The first set form together the outer coating, and

their terminal A is connected with S (Fig. 127), while all the smaller sheets of foil form the inner coating, and their terminal B, mounted on a block of ebonite, is connected with P. The glass slabs are piled one on the top of the other, but separated by fragments of glass F F, all of the same thickness, conveniently about one-tenth of an inch; and there is one more of the slabs with the larger sheets of tinfoil on it than of the others, so that there is one of the former both at the bottom and at the top of the condenser when it is thus built up. The glass plates are prevented from sliding over one another when the condenser is moved, by their corners fitting into grooves in the four ebonite pillars E, E, E.

The capacity of the standard condenser, in farads, equals

$$\frac{A}{4{\cdot}452 \times 10^{12} \times t},$$

where A is the sum of the areas, reckoned in inches, of all the smaller sheets of tinfoil, and t is the thickness of one of the little glass fragments.

The capacity of the experimental condenser equals

$$i \times \frac{A'}{4{\cdot}452 \times 10^{12} \times t'},$$

where A′ is the area of one of the tinfoil coatings, t' the thickness of the sheet of dielectric under test, and i its specific inductive capacity. Hence, if the resistances a and b (Fig. 127) are so adjusted that no sound is heard in the telephone,

$$i = \frac{a}{b} \times \frac{A}{A'} \times \frac{t'}{t}.$$

The construction of the Bell's telephone, such as may be used in the previous experiment, is shown in Figs. 130 and 131, where m is a permanent magnet, terminated at the right-hand end (Fig. 130) by a piece of soft iron of

the same thickness. Round this piece of iron is a coil of wire *b b*, the ends of which *d d* are led to the terminals V V. Close to the end of the piece of soft iron, but not touching it, is a *thin* plate of ferrotype iron *c e*. The

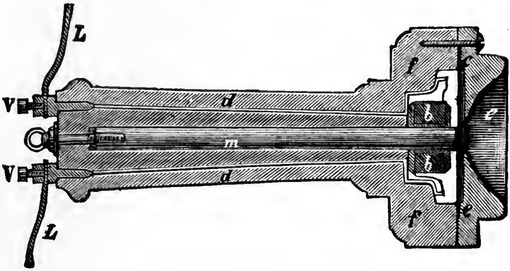

Fig. 130.

piece of soft iron is magnetised by the permanent magnet *m*, and thus attracts the centre of the thin plate of iron, and the amount of this attraction is varied by any current that passes round the coil *b*. Hence, if there be rapid fluctuations in the strength of the current passing round this coil, and still more, if there be rapid alterna-

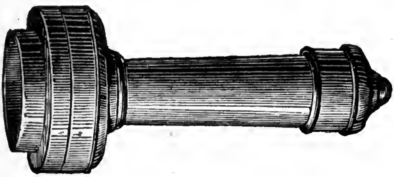

Fig. 131.

tions in the direction of the current passing round this coil, the thin iron plate will be set in rapid vibration, and a sound will be emitted. If the telephone be well made, and if the ear be placed near the opening shown at the right hand in Fig. 130, and at the left hand in Fig. 131, the sound produced by even *extremely small*

alterations in the current strength, can be heard, if they follow one another with *sufficient rapidity.*

181. Every Charged Body forms One Coating of a Condenser.—In practice, as already explained, a condenser is the name given to two sets of sheets of metal so arranged that the one set has a large capacity relatively to the other; but, in reality, every charged body forms a condenser with some other body; it may be with the walls of the room, or the ceiling, or the table, or the body of the experimenter, or with all of them; hence we see that the statement made at the foot of page 109, that when one conducting body A is entirely surrounded by another conducting body B, the quantity of electricity on A is directly proportional to the P. D. between A and B as long as the position of A, relatively to B, is absolutely fixed, is only another way of saying that the capacity of A relatively to B is constant as long as their relative positions are unchanged.

In § 67, page 120, it was explained that the potential of the charged metal plate P could be diminished by bringing near it the metal plate M, connected with the earth. We now understand that this arises from the capacity of P relatively to M being increased by approaching them, in consequence of which the potential of P, corresponding with a given charge on it, is diminished (*see* § 167, page 308).

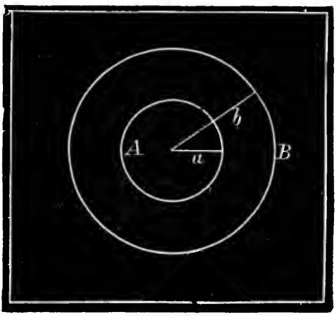

Fig. 132.

182. Capacity of a Spherical Condenser.—If a metallic sphere A (Fig. 132) of a centimetres' radius be insulated concentrically inside another hollow metallic sphere B of b centimetres' radius,

and if the dielectric separating them be air, the capacity of A, relatively to B, can be proved to be

$$\frac{a\,b}{9 \times 10^{11}\,(b-a)} \text{ farads.}$$

This last expression can be written in the form

$$\frac{a}{9 \times 10^{11}\left(1 - \frac{a}{b}\right)},$$

from which we see that as b grows greater and greater, the capacity of A grows smaller and smaller. Consequently, although we have no experience of a single charged body insulated alone in space, we can see what is the limit to which the capacity of A approaches, as b becomes larger and larger. The value of this limit is obtained by making b equal to infinity, when the capacity of A becomes

$$\frac{a}{9 \times 10^{11}} \text{ farads,}$$

and this is practically the capacity of a sphere when, as in the case of A, Fig. 43, page 121, it is so far away from other bodies as to be *practically* beyond the range of their inductive action.

But because we can calculate the capacity of a body when it is so far away from other bodies as to be *practically* beyond the range of their inductive action, it must not be imagined that we can have a charged body existing *alone* in space. Indeed, as seen in § 60, page 115, we cannot produce only a single quantity of electricity, since *equal* and *opposite* quantities are produced simultaneously, therefore it is impossible to have one body charged positively or negatively without some other body existing with an equal and opposite charge on it.

And just as we have no experience of a single

charged body existing by itself, so it is equally impossible to obtain two bodies charged with the same kind of electricity without a third one oppositely charged. Although, therefore, we are accustomed to speak of two positively or of two negatively electrified bodies repelling one another as if this action could take place without the presence of any third body, we must not allow this very convenient form of expression to cause us to forget that all our experience of the action of electrified bodies is derived from experiments made inside a room, the walls, ceiling, and floor of which are more or less good conductors, and which form condensers with the electrified bodies inside the room. For example, if A and C (Fig. 133) be two spheres electrified positively, and placed inside a conducting room B B, the distribution of the density will be roughly as in the figure, the density being greatest where the plus or minus signs are nearest together. If A and C be free to move, then, as is well known, they will separate from one another, and approach the sides of the room. This action is usually regarded as being caused, partly by the repulsion of the positive electricities on A and C, and partly by the attraction of the positive electricity on each of the bodies by the negative electricity on the side of the wall adjacent to the two bodies respectively. But as we have no experimental evidence of what would happen if A and C

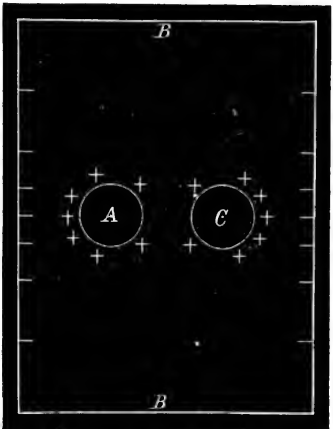

Fig. 133.

could exist with their positive charges apart from B B, it may be that it is the *attraction* of the opposite electricities that causes A and C to separate, and that there is *no repulsion* at all between the similarly electrified bodies A and C; and this, of course, is true whether A and C be spheres inside a conducting room with flat walls, ceiling, and floor, or whether they be conductors of any shape inside another of any other shape, as shown in Fig. 134.

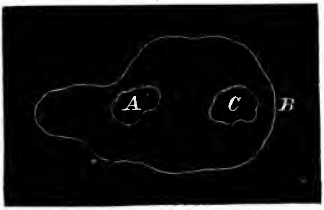

Fig. 134.

Example 102.—What is the capacity of the earth regarded as a sphere insulated in space?

Answer.—The mean radius of the earth is $6{\cdot}3703 \times 10^8$ centimetres, hence its capacity is 0·0007078 farads, or roughly 708 microfarads, which is the capacity of about 2,000 miles of ordinary submarine telegraph cable.

183. Condenser Method of Comparing the E. M. Fs. of Current Generators.—We have already seen (§ 132, page 234) that with cells which polarise, as it is called, the ordinary galvanometer methods of comparing E.M.Fs. cannot be employed to obtain accurate results, and that a null method like that of Poggendorff's is much to be preferred. When, however, a condenser and a suitable reflecting galvanometer for measuring capacity are at hand, the following method may be employed instead of Poggendorff's. Charge the condensers successively with the two current generators, and in each case measure the charge or discharge with the galvanometer, then, since the deflections are proportional to the charges or discharges (§ 160, page 299), and since these charges are proportional to the E. M. Fs. employed, it follows that the E. M. Fs. are proportional to the deflections.

If the plates of the cell have only a *very small* surface

in contact with the liquid, the polarisation arising from the flow of electricity into the condenser to charge it may be sensible if the condenser have a large capacity. Hence, in such a case, it is important to use a condenser of as small a capacity as can be employed to give a satisfactory deflection with the most delicate galvanometer available. Such a precaution is especially necessary when experiments on the E. M. Fs. of cells made of simple pieces of wire dipping into various liquids are performed.

184. Condenser Method of Measuring the Resistance of a Current Generator.—We have seen, § 115, page 204, that if a current generator having a fixed E. M. F. equal to E volts, and a resistance of b ohms, be shunted with a resistance of r ohms, the P. D. at the terminals will be

$$\frac{r}{r+b} \times \text{E volts.}$$

If, then, we employ first the generator unshunted to charge the condenser, and obtain, on charging or on discharging through a suitable galvanometer, a first swing d_1 of the spot of light; second, if the generator be shunted with a resistance r ohms, and we obtain, on charging or on discharging, a first swing d_2, we know that

$$\frac{d_1}{d_2} = \text{E} \div \frac{r}{r+b} \cdot \text{E},$$

$$\therefore\ b = \frac{r\,(d_1 - d_2)}{d_2}.$$

With cells that polarise it is very important that the battery should be shunted with the resistance r *only at the moment of charging the condenser*, and that the act of disconnecting the battery from the condenser should also disconnect the shunt. This may be conveniently effected, without the employment of any special key, by joining up the arrangement as shown in Fig. 135, the key in the

figure being exactly the same in principle as that shown in Fig. 121, page 320, but not possessing such high insulation, as this is unnecessary with the present experiment. One pole Q of the battery B is permanently connected with one end of the resistance r, with one coating C_1 of the condenser, and with the upper screw S_2 of the key; the other pole P of the battery is insulated as long as the contact at S_1 is broken. On depressing the lever the contact at S_2 is broken and that at S_1 made; this has the effect of connecting the pole P of the battery to the

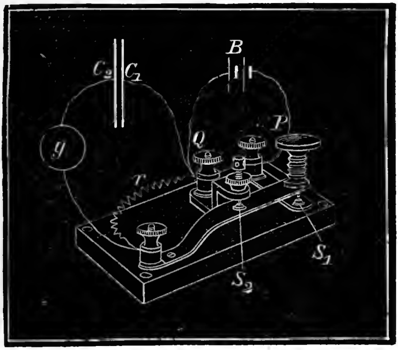

Fig. 135.

other end of the resistance r and to the other coating C_2 of the condenser through the galvanometer g, hence the condenser is charged through the galvanometer with the cell shunted. On liberating the key, *which should be done directly the first swing is completed*, the contact at S_1 is broken and that at S_2 made; P is therefore disconnected from the shunt and the galvanometer, and the condenser is discharged through the galvanometer.

To observe the charge with the battery unshunted, the infinity plug in r must be withdrawn, or one of the ends of the resistance r must be disconnected from the rest of the circuit.

185. Measuring a Resistance by the Rate of Loss of Charge.—When a resistance of not merely thousands of megohms, but of millions of megohms has to be measured, the galvanometer method described in § 151, page 278, is not sensitive enough, unless an enormously large battery be employed, and a mode of testing depending not on measuring the *rate of leakage* but on measuring the *amount that has leaked in a given time* has to be resorted to, as follows:—If a charged condenser have its two coatings connected by a resistance, it will be discharged with more or less rapidity depending on the magnitude of the resistance, and the capacity of the condenser. If F farads be the capacity, r ohms the resistance, and if the P. D. between the coatings be V volts at a certain time, and V′ volts t seconds afterwards, then we can prove that

$$r = \frac{0{\cdot}4343\, t}{\text{F} \log_{10} \dfrac{\text{V}}{\text{V}'}},$$

hence the resistance r may be ascertained if we know F, V, V′, and t.

To prove this formula we shall assume that the whole interval t seconds, during which the discharge is observed, is subdivided into a great number n of very small equal intervals of time τ, so small that during the whole of any one of these small intervals, the P. D. between the coatings may be supposed to remain constant, so that instead of the P. D. falling gradually from V volts to V′ volts, we suppose it to fall by n small jumps, one jump being made at the end of each interval. The same sort of approximation to the truth is made when a curve is supposed to be formed of a very great number of very short straight lines, each two adjoining straight lines differing very slightly from one another in direction, since, instead of the gradual change of direction which occurs in going along a real curve, we have a discontinuous change in moving along the succession of short straight lines.

At the commencement, the number of coulombs in one coating of the condenser is

$$\text{F V},$$

and during the first interval the quantity in coulombs that flows out of the one coating into the other is

$$\frac{\text{V}}{r}\tau,$$

so that the quantity that will remain in this coating is

$$\text{F V} - \frac{\text{V}}{r}\tau \text{ or } \text{F V}\left(1 - \frac{\tau}{\text{F}r}\right);$$

hence, the P. D. between the coatings at the end of the first interval equals

$$V\left(1 - \frac{\tau}{F\,r}\right).$$

During the second interval of τ seconds the number of coulombs that will flow from one coating into the other equals

$$\frac{V}{r}\left(1 - \frac{\tau}{F\,r}\right),$$

so that the quantity that will remain in each coating will be

$$F\,V\left(1 - \frac{\tau}{F\,r}\right) - \frac{V}{r}\left(1 - \frac{\tau}{F\,r}\right)\tau,$$

$$\text{or}\quad \left(F\,V - \frac{V}{r}\,\tau\right)\left(1 - \frac{\tau}{F\,r}\right),$$

$$\text{or}\quad F\,V\left(1 - \frac{\tau}{F\,r}\right)^2 \text{coulombs.}$$

Similarly, the number of coulombs remaining on each coating at the end of the third interval equals

$$F\,V\left(1 - \frac{\tau}{F\,r}\right)^3;$$

and at the end of the n, the interval that is at the time t,

$$F\,V\left(1 - \frac{\tau}{F\,r}\right)^n;$$

but this is equal to $F\,V'$,

$$\therefore\quad F\,V\left(1 - \frac{\tau}{F\,r}\right)^n = F\,V',$$

or dividing both sides by F, and substituting $\frac{t}{n}$ for τ, it follows that

$$V\left(1 - \frac{t}{F\,r}\,\frac{1}{n}\right)^n = V',$$

and this is more and more true the larger n be made. But it can be shown mathematically that when n is infinitely great

$$\left(1 - \frac{t}{Fr}\frac{1}{n}\right)^n = \epsilon^{-\frac{t}{Fr}}$$

when ϵ stands for $2{\cdot}7\dot{1}82\dot{8}$. So that

$$V\epsilon^{-\frac{t}{Fr}} = V',$$

$$\therefore \quad r = \frac{t}{F \log._\epsilon \dfrac{V}{V'}}.$$

Consequently, converting the logarithm to the base ϵ to a logarithm to the base 10, by the method given in § 158, page 296, we have

$$r = \frac{0{\cdot}4343\, t}{F \log._{10} \dfrac{V}{V'}}.$$

If an electrometer, with well-insulated quarter cylinders, be available, then the loss of potential can be easily observed by attaching the two coatings of the condenser to the opposite pairs of quarter cylinders, giving the condenser a charge, and observing the times at which the spot of light passes two definite positions on the scale, for V and V′ may be measured in any units, since we have merely to deal with the ratio of V to V′. In this way the insulation of even a short length of well-insulated cable can be measured. For, as the cable is shorter, and r is larger, F is proportionately smaller, so that the time the P. D. takes to fall from one given value to another is independent of the length of the cable.

186. Rate of Loss of Charge from Leakage through the Mass depends on the Nature of the Dielectric only, and not on the Shape or Size of the Condenser.—Not merely is the time the P. D. takes to fall from one given value to another independent of the area of the coatings of the condenser, but it is independent of the thickness of the dielectric. Take the case of a condenser with flat parallel plates. Then, if A be the area of one of the coatings in square inches, d the distance

between them in inches, and s the "*specific resistance*," or resistance per cubic inch of the dielectric.

$$r = \frac{d \times s}{A},$$

and from § 169, page 311, if i be the specific inductive capacity of the dielectric,

$$F = i \times \frac{A}{4 \cdot 452 \times 10^{12} \times d};$$

∴ if t be the time in seconds during which the P. D. falls from V to V′,

$$\frac{d \times s}{A} = \frac{0 \cdot 4343\, t \times 4 \cdot 452 \times 10^{12}\, d}{i\, A \log._{10} \dfrac{V}{V'}},$$

$$\therefore \quad \frac{\log._{10} \dfrac{V}{V'}}{t} = \frac{1 \cdot 934 \times 10^{12}}{s\, i},$$

the right-hand expression depending only on the specific resistance, and specific inductive capacity of the dielectric, and not on its shape or size.

So in the same way with a cylindrical condenser the capacity in farads, as we have seen from § 168, page 308, and § 169, page 311, is

$$i \times \frac{2 \cdot 413}{10^{13}} \times \frac{l}{\log._{10} D - \log._{10} d},$$

where i is the specific inductive capacity of the dielectric, l the length of the condenser in centimetres, and D and d the diameters of the coatings. It may also be shown that if s' be the resistance, in ohms, per cubic centimetre of the dielectric, r, the resistance of length l of the cylindrical condenser is

$$\frac{s'}{0 \cdot 8686\, \pi\, l} (\log._{10} D - \log._{10} d).$$

Consequently,

$$\frac{\log._{10} \dfrac{V}{V'}}{t} = \frac{4 \cdot 912 \times 10^{12}}{s'\, i}.$$

It has to be remembered that whereas for the condenser with flat parallel plates, s was the resistance per *cubic inch* of the dielectric, here s' is the resistance per *cubic centimetre.* Hence, since the resistance is proportional to the thickness, and inversely as the sectional area,

$$= \frac{2{\cdot}54}{2{\cdot}54^2}\, s',$$

$$\text{or} \quad s' = 2{\cdot}54\, s\,;$$

that is, *the resistance per cubic centimetre of any substance is* 2·54 *times the resistance per cubic inch.* The specific inductive capacity, i, is independent of the unit of length or area. Hence, substituting the value for s', we obtain

$$\frac{\log._{10} \dfrac{V}{V'}}{t} = \frac{1{\cdot}934 \times 10^{12}}{s\, i},$$

which is the same expression as that obtained with flat parallel plates.

187. Galvanometric Method of Measuring Resistance by Loss of Charge.—In the formula given in § 185, page 344, we may substitute for V and V′ the number of coulombs K and K′, on one of the coatings of the condenser when the P. D. between the coatings is V and V′ volts, so that

$$r = \frac{0{\cdot}4343\, t}{F \log._{10} \dfrac{K}{K'}}.$$

If the capacity of the condenser be sufficiently large, K and K′ can be measured by charging the condenser through a galvanometer at a certain moment, and discharging it again at the end of t seconds, using the arrangement shown in Fig. 123, page 322. To enable the lever L of the key, seen more plainly in Fig. 121, page 320, to be left without completing the contact at S_1 or at S_2 during the time the condenser is left insulated, the screw which makes the upper contact S_2 should be screwed out so far that it would require a slight upward pressure to be given to the lever to cause it to make this upper contact. If the resistance to leakage be very large, K and K′ will be nearly equal to one another unless t be taken inconveniently long. This difficulty may be overcome by using a large battery, and charging the condenser with the galvanometer shunted at the beginning of the

time t, and then charging it again with the galvanometer unshunted, and therefore in a much more sensitive condition at the end of the time t. In this way K and K − K′ will be measured, and by properly choosing the shunt, the second test may be made as delicate as the first. Since, however, as mentioned in § 174, page 319, a difficulty is introduced when comparing two quantities of electricity if the galvanometer be shunted in one case and not in the other, this method is not a perfectly accurate one unless the following correction be introduced.

188. Multiplying Power of a Shunt used in Measuring a Discharge.—When a quantity of electricity is passed through a shunted galvanometer, the quantities that pass respectively through the galvanometer and shunt are inversely as their resistances exactly as in the case of a steady current; but when, after the discharge has been completed, the needle begins to move, its motion induces a current in the galvanometer and shunt in such a direction as to tend to stop its motion. This induced current, therefore, *damps* the motion of the needle, and we have, therefore, to use the formula for damped vibrations given in § 157, page 296. It can, however, be proved mathematically that *with a given galvanometer, and with a given adjustment of the controlling magnet, &c., the damping in this case has simply the effect of increasing the resistance of the galvanometer by a definite amount, independently of the resistance of the shunt.* So that if g be the actual galvanometer resistance, and s that of the particular shunt employed, the multiplying power for a discharge is

$$\frac{s+g+g'}{s},$$

where g' has a definite value, independent of that of s, for a given galvanometer with a given adjustment of the controlling magnet, &c. Instead, therefore, of employing the formula for damped vibrations, to do which we must measure the decrement when its vibrations are damped, we may simply determine the constant g' in the following way:—

Charge a condenser with a small P. D., say of V_1 volts, through the galvanometer unshunted, obtaining a first swing d_1, say. Next, having discharged the condenser, shunt the galvanometer with any convenient shunt of resistance s, increase the P. D. to a suitably larger value V_2 volts, and charge the condenser through the shunted galvanometer, obtaining a first swing d_2. Then, since the quantities which pass into the condenser are proportional to V_1 and V_2,

$$\frac{V_2}{V_1} = \frac{s+g+g'}{s} \times \frac{d_2}{d_1},$$

or the multiplying power of the shunt,

$$\frac{s+g+g'}{s} = \frac{V_2}{V_1} \cdot \frac{d_1}{d_2},$$

$$\text{and} \quad g+g' = s\left(\frac{d_1}{d_2} \cdot \frac{V_2}{V_1} - 1\right).$$

As V_1 and V_2 only occur in a ratio, we do not require to know their absolute values in volts, and the simplest method of obtaining two P. Ds. having a known ratio is that given in § 150, page 278.

Example 103. — On charging a slightly leaky condenser through a galvanometer of 1,000 ohms' resistance, shunted with the $\frac{1}{100}$th shunt, a deflection of 230 scale divisions is obtained. The condenser is then insulated, and at the end of half a minute it is again charged but with the galvanometer unshunted, and a deflection of 112 scale divisions is obtained. What is the resistance of the condenser?

To ascertain the value of the first deflection in farads, as well as to find the increased multiplying power of the shunt for a discharge, let us charge a well-insulated condenser of known capacity, say $\frac{1}{3}$rd of a microfarad, with the same P. D. as was used in the previous experiment; let this give a deflection of 175 scale divisions with the galvanometer unshunted. Next discharge the condenser, shunt the galvanometer with, say, the same shunt as was used before, increase the P. D. employed, and again charge the condenser, obtaining, say, a deflection of 295 scale divisions. Let these two P. Ds. be those between the points S and T, Fig. 101, page 278, and L and M, and let the ratio of the resistances of q and p be in the ratio of 10 to 1,736.

The multiplying power of the shunt for a discharge equals

$$\frac{s+g+g'}{s} = \frac{1736}{10} \times \frac{175}{295},$$

$$= 103,$$

therefore the capacity of our slightly leaky condenser is

$$103 \, \frac{230}{175} \times \frac{1}{3 \times 10^6} \text{ farads,}$$

$$\text{or} \quad 45{\cdot}12 \text{ microfarads.}$$

Next, K being the number of coulombs in one coating of our

slightly leaky condenser at the moment of charging, and K the quantity at the end of half a minute,

$$\frac{K-K'}{K} = 112 \div 230 \times 103,$$

$$\therefore \log._{10} \frac{K}{K'} = 0 \cdot 0021.$$

Hence,

$$r = \frac{0 \cdot 4343 \times 30}{\frac{45 \cdot 12}{10^6} \times 0 \cdot 0021} \text{ ohms.}$$

Answer.—137·5 megohms.

Large Potential Differences.

189. Production of Large Potential Differences.—When any two *dissimilar* substances are brought into contact, there is a certain P. D. set up between them in consequence of what is known as the "*contact potential difference.*" The two substances, therefore, become charged, like the two coatings of a condenser, with equal and opposite amounts of electricity, depending on the *contact P. D.*, the proximity of the two bodies and their size. If either, or both, of these bodies be an insulator, or be held by an insulating handle, some, or all, of the charge will remain when the bodies are separated. If the bodies be separated in such a way that practically *all* the points of contact are broken at the *same time*, then *all* the charge will remain on each of the bodies if they be properly insulated. As the distance between the bodies increases the capacity of the condenser rapidly diminishes, hence the P. D. between the bodies rapidly increases. In this way a P. D. of many hundreds, or thousands, of volts can easily be produced by bringing a piece of dry, clean glass into close contact with a piece of silk, or a piece of dry, clean ebonite into close contact with a piece of cat's-skin, and then separating them; and

just as work has to be done in separating the two plates of a charged condenser (*see* Example 100, page 326), work has to be done in separating the glass from the silk, or the ebonite from the cat's-skin, and the power that the glass or ebonite has to give a spark when the knuckle is brought near it, arises from the condenser possessing a store of potential energy. (*See* § 176, page 322.) The ebonite forms one of the coatings of this condenser, and the surface of the room the other, because, as the cat's-skin is not a good insulator, the charge of positive electricity induced on it when it is in contact with the ebonite, spreads itself over the walls, ceiling, and floor of the room on the separation. As explained in § 61, page 115, the object of *rubbing* the glass with the silk is to bring all parts of the surface of the *insulating* glass into successive contact with the silk.

The well-known cylindrical and plate-glass frictional electrical machines are merely contrivances for bringing different portions of the surface of a cylinder, or a sheet of glass, successively into close contact with a silk rubber, and separating them again. The electrical energy produced by such an apparatus depends simply on the work required to perform the *separation* of the positively electrified portions of glass from the negatively electrified rubber, whereas the actual power expended in turning such a machine is mainly wasted in overcoming friction and producing heat. Hence, *such frictional machines are extremely inefficient converters of mechanical energy into electrical energy*, and they are, therefore, rapidly becoming obsolete, and being replaced by the much more efficient *influence machines*. (*See* § 194, page 361.)

190. Condensing Electroscope. — The increase of P. D. between the two coatings of a charged condenser, produced by separating the plates, may be employed to cause an ordinary gold-leaf electroscope to indicate the P. D. existing at the terminals of two or three cells in series. For, let the plate M, Fig. 42, page 120, be connected electrically with the tinfoil coating of the gold-

leaf electroscope, and placed close to the plate P; then let them be connected with the terminals of, say, three Daniell's cells in series, which will cause them to be charged with a P. D. of about 3·3 volts. Now, disconnect P from the cells, and remove M altogether, then the P. D. in volts between the gold-leaves and the tinfoil coating of the electroscope will become 3·3 multiplied by the ratio of the capacity of P when M was close to it, to its capacity when M has been removed far away, that is, when P forms a condenser with the walls and ceiling of the room, and with the tinfoil coating of the electroscope; since, with a given charge on the coatings of a condenser the P. D. between the coatings is inversely as the capacity (*see* § 167, page 308). This ratio will be the greater the nearer M was brought to P during the charging, and may easily be made 100 or more (so that the P. D. between the gold-leaves and the tinfoil coating is now between 300 and 400 volts) by having the surfaces of the plates carefully coated with a layer of shell-lac, and by simply resting M on P. Strictly speaking, the ratio of capacities to be considered is that of P plus that of the gold-leaves when M is close to P, to that of P plus that of the gold-leaves when M is far away; and although the capacity of the gold-leaves is insignificant in comparison with that of P when M is very near P, it is not so when M has been removed. The above will be practically the same whether M be disconnected or not from either the tinfoil coating or the cells, before it is removed.

In order that the distances separating all parts of M and P may be very small, their surfaces must be made quite plane, and it is difficult to do this unless the plates be fairly thick. But if they are thick they will be too heavy to rest on the stem of the electroscope, hence it is better to support P as the plate A (Fig. 29, page 88), is supported, by means of an insulating stand having a fairly strong glass rod, and to connect it with W of the electroscope by a thin piece of wire.

191. Calibrating a Gold-Leaf Electroscope.—If the ratio, r, say, that the sum of the capacities of P and of the gold-leaves when M is placed in a fixed position near P bears to the sum when M is far away, be accurately known, then a gold-leaf electroscope, which will not indicate directly a P. D. of less than 100 or 200 volts, may be calibrated for any divergence of the leaves by the employment of some ten or twelve cells. For if P and M, when near together, be charged with one cell, and then M be removed, and the divergence of the gold-leaves d_1 noted, then P and M be charged with two cells, M be removed, and the divergence d_2 noted, &c., these divergences d_1, d_2, &c., will correspond with a P. D. between the gold-leaves and the tinfoil coating of r E, 2 r E, &c., volts, where E is the E. M. F. of one cell, and which is 1·104 volts if the cells be Daniell's cells made with equidense solutions of copper and zinc sulphate, and if *pure* zinc and copper plates be employed (*see* § 119, page 211).

It would be practically impossible to determine this ratio, r, by calculation, owing to the difficulty of calculating the capacity of P, and the gold-leaves when M was removed. To determine it experimentally would be nearly as difficult as calibrating the gold-leaf electroscope directly by experiment. We must, therefore, employ some condenser, the capacity of which can be made to have two very distinct values, both of which are large compared with the capacity of the gold-leaves, having a known ratio to one another of about 100; or we may employ the arrangement suggested by Sir William Thomson, in 1885, for increasing a P. D. in a known ratio, and which is shown symbolically in Fig. 136. A, B, C, &c., are well-insulated condensers of not necessarily equal capacities, joined up in series, the outer coating of the first a being connected with the outside of the electroscope, and the inner coating z of the last with the gold-leaves. A well-insulated battery, S S, of a convenient number of cells, having an E. M. F. equal to E volts, has

its terminals connected, first with *a* and *b*, then, instead, with *b* and *c*, then with *c* and *d*, &c. On the battery terminals being connected with *a* and *b*, the coatings of the first condenser will have a P. D. of E volts produced between them, and similarly on the battery terminals being connected with *b* and *c* a P. D. of E volts will be produced between *b* and *c*, therefore the P. D. between *a* and *c* will be 2 E volts. Again, on connecting the battery terminals with *c* and *d*, the P. D. between *a* and *d* will become 3 E volts, &c. Hence, if there be 100 condensers in series, and if the battery be moved along so that its terminals make successive contacts with the pairs of coatings of each of the condensers, the P. D. between *a* and *z*, that is between the outer coating of the electroscope and the gold-leaves, will become 100 E, and by making E first, say 2 volts, next 3 volts, and so on, the electroscope can be calibrated with P. Ds. of 200, 300, &c., volts.

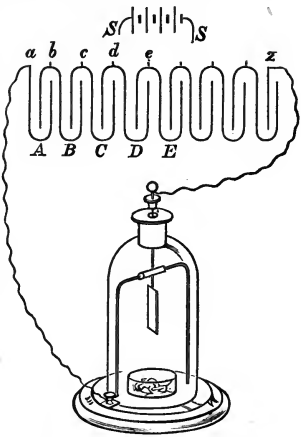

Fig. 136.

In the last paragraph it is stated that the coatings of the condensers are well insulated from one another, but if the battery terminals s s be *rapidly* moved backwards and forwards so as to make *rapid successive* contacts with the coatings of the various condensers, it will only be necessary for the insulation of the condensers to be fairly good, as there will be no time for leakage to take place between the successive contacts of the coatings of each condenser with the battery terminals.

The following gives the result of the approximate

calibration of a gold-leaf electroscope, the gold-leaves being about $1\frac{1}{4}$ inch long:—

Angle between the gold-leaves.	P. D. between the leaves and the tinfoil coating in volts.
26°	500
42°·6	750
60°·2	1,000
92°·7	1,500

192. Electrophorus.—The oldest form of *influence machine* is the "*electrophorus*," which consists of a plate of some insulating substance I (Fig. 137), usually ebonite in the modern electrophorus, fastened into a metal backing B, and a movable metal plate P, into which screws a metal ferrule attached to an insulating rod or handle R. The electrophorus can be made to give a succession of either positive or negative charges of high potential by the *variation of capacity* of the condenser formed of the ebonite and the plate P, produced by altering their distance from one another

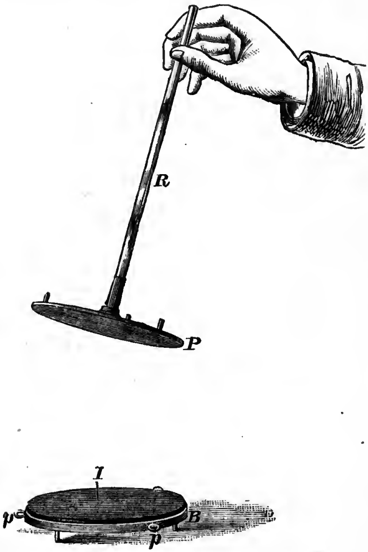

Fig. 137.

The ebonite, on being rubbed with a piece of cat's-skin, becomes negatively charged, and forms a condenser of fixed capacity with the uninsulated backing B, the upper surface of which is therefore charged positively. Further, this condenser-action causes the negative charge produced

on the upper surface of the ebonite to be attracted a small distance downwards into the insulating substance of the ebonite, and so prevents the charge being easily removed by the metal plate P when it is laid for a short time on the ebonite. If this plate be held by the insulating handle R, and placed on the ebonite, the potential of the ebonite will be slightly diminished numerically—that is, become less negative—(*see* § 67, page 120), and the plate P will be raised to a fairly high negative potential, the density on its lower surface being positive, and on its upper negative (*see* § 69, *8*, page 124); P, in fact, forms a condenser with the ceiling and walls of the room. If now, by means of the insulating handle, *held at the extreme end to diminish the surface leakage as much as possible,* P be removed again without being touched, its negative potential will grow less and less as its distance from the ebonite grows greater and greater, and the density on its upper and lower surfaces will also be diminished, until at last when P is beyond the range of the inductive action of the ebonite it will be simply an uncharged body at a potential nought.

But if, on the other hand, while P is resting on the ebonite, it be connected with the backing, B, or with the earth, by means of a wire, or more simply by touching it with one's finger, its potential will be reduced to nought, and the potential of the ebonite will be numerically diminished. Hence, some of the positive charge previously induced in the backing will flow away, all the negative charge on the upper surface of P will also disappear, and some more positive electricity will be attracted to the lower side of P, the density on its upper surface will become, therefore, nought, and on its lower surface more positive than before. P and B together now form the earth coating, and the ebonite the insulated coating, of a condenser. On removing P by means of the insulating handle R, its potential rapidly rises *positively*, and that of the ebonite increases negatively. When P has been removed some little distance from the

ebonite, its potential becomes high enough to enable it to give a *positive* spark* to a conductor brought near it. And as the ebonite is not sensibly discharged by the action of placing P on its surface and removing it, the operation of *inductively* giving P a large positive charge can be repeated again and again; and we may thus charge an insulated conductor with even a large capacity to a high positive potential.

To save the trouble of having to electrically connect P with B each time P is laid on the ebonite, it is desirable (if an electrophorus is made simply for practical use and not also for the purposes of instruction, as is the case with the one shown in Fig. 137) to drill a hole through the backing B (Fig. 138) and the ebonite I, and insert a small brass screw S into it of such a length that, when screwed in, its point is a *little below* the upper surface of the ebonite, for with this arrangement a spark passes across the small air space when P is laid on the ebonite in consequence of the high negative potential induced in P; but no spark passes on raising P, since its positive potential only becomes large when P is raised so far from the ebonite that a spark cannot pass to the screw. The presence, therefore, of this screw, with its slightly countersunk point, has precisely the same effect as connecting P with B when P is resting

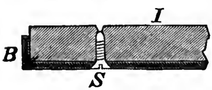

Fig. 138.

* When the P. D. between two conductors reaches a certain value, depending on their shapes, their distance apart, and the insulating material separating them, a crack or hole is found in the insulator, and a spark, produced by the burning of minute particles of the surfaces of the conductors, passes along the crack or hole. The P. D. required to produce a spark through air is given in § 196, page 370, but for paraffined paper, guttapercha, glass, &c., it is much greater. While the air is momentarily cracked, during the passage of a spark, its resistance is comparatively small, but after the spark has passed, the crack closes up, and the resistance regains its original value; if, however, the spark has passed through paper, a small hole may be seen, differing, however, from a hole made by a pin, in that the former is burred on both sides, as if the electric force making it had acted from the centre of the paper outwards towards each side.

on the ebonite, and removing this connection before P is raised.

If it be desired to charge an insulated conductor of large capacity to a high *negative* potential, we might use an electrophorus with I (Fig. 137) made of glass, which becomes charged *positively* on being rubbed with silk; but as glass is a much more hygroscopic body than ebonite, and therefore much more difficult to keep electrified when exposed to the air, it is better to use an ebonite electrophorus in the following manner.

193. Ebonite Electrophorus arranged to give Negative Charges.—Unscrew the handle from the plate P and screw it into the backing (Fig. 139). Excite the ebonite by rubbing it with cat's-skin, and suppose that the backing has been brought to a potential nought by connecting it for a moment with the ground when it was held at some distance from P, which is lying on the table. The ebonite is now the insulated coating of a condenser, the uninsulated one being B and the walls of the room. Next holding the backing and ebonite by the insulating handle R, place the ebonite on P (Fig. 140). The potential of the ebonite will then become less negative, the potential of B will be raised to a high positive value, the density on

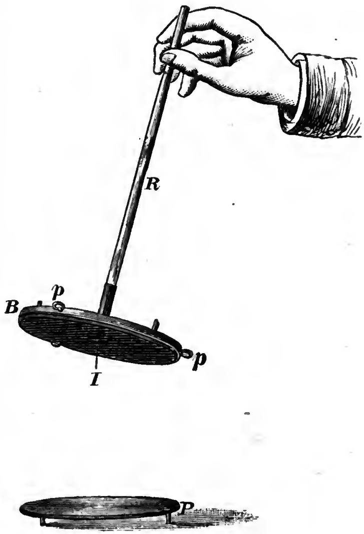

Fig. 139.

its upper side will become positive, the density on its lower side less positive than before, and the density of the upper surface of P positive. Connect B with P, the potential of B will be reduced to nought, the potential of the ebonite will be made still less negative, the density on the upper surface of P made less positive, the density on the upper surface of B nought, and on its lower surface more positive than before. Raise the backing and the ebonite by the handle, the potential of the ebonite will become more negative, and that of B will become *negative* and will reach a high negative value when the backing and ebonite are removed some little distance from P, so that a spark of negative electricity can be taken from B by a conductor brought near it.

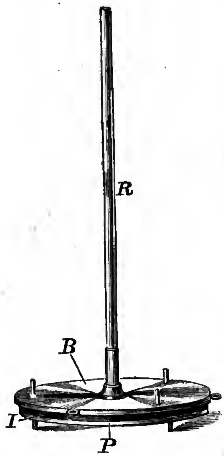

Fig. 140.

In the preceding we have considered the various electrical changes that take place on *all* the parts of an electrophorus when in use, but probably the simplest way of looking at the action of the electrophorus, whether it be used to give positive or negative charges to some conductor, is to remember that when P is in contact with the ebonite plate, and P and B are electrically connected together and with the earth, there are charges of positive electricity on the surfaces of P and B facing the ebonite, and these charges may in each case be regarded as being due to the *excess of the inductive action* of the *negative* charge on the ebonite over that of the positive charge on the other metal plate, the effect of the *negative* charge in each case *preponderating*. Consequently if both P and B could be separated from the ebonite by means of insulating handles, both would be found to have a *positive*

potential, and to be in a condition to give a *positive* charge to some other conductor. And if the ebonite and backing be removed without separation, P will, as before, have a positive potential; but the action on B will now be *quite different* from before, for, instead of the inductive action of the positive electricity on P, together with the preponderating inductive action of the negative electricity on the ebonite, being removed simultaneously, only the former is removed. Hence the inductive effect on B of this negative electricity on the ebonite will produce an effect *greater than before*, B will therefore have a *negative* potential, and be in a condition to give a *negative* charge to some other conductor.

In the electrophorus shown in the figures the ebonite is held to the backing by three pins *p p*, instead of being cemented to it as is usual in an electrophorus, and can be removed by withdrawing these pins. Hence we can examine the electrification of the ebonite or of the backing in any stage of the experiments described above. To charge a body of large capacity with a simple electrophorus is a slow process, and hence a "*rotatory electrophorus*" has been devised by Bertsch for enabling the operations described in § 192 to be rapidly performed, but even this apparatus is inferior to the machines described in the following sections.

194. Accumulating Influence Machines.—With the electrophorus we can, as we have seen, increase the potential of an insulated body until it is equal to that of P, when P, with its induced charge in it, has been removed far away from the ebonite, but we have no means of increasing the charge in the ebonite itself; and so, in order to use an electrophorus, it is necessary to commence by charging the ebonite by rubbing it with a piece of cat's-skin. With an "*accumulating influence machine*," on the other hand, we are able to increase the charge on the inductor, and hence to start such a machine with practically little or no charge on the inductor. The action of all such machines depends on the following prin-

ciple :—If A and B (Fig. 141), be two insulated metallic pots possessing a small P. D. between them, the potential of A being the higher, and if C and D be two uncharged conductors, C being placed near the outside of A, and D

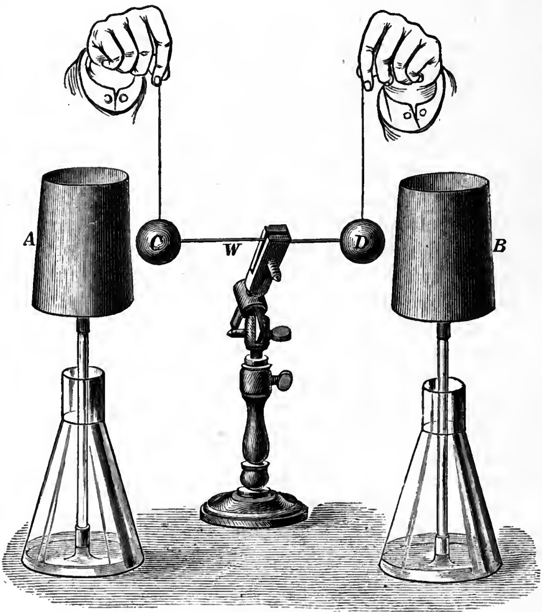

Fig 141.

near the outside of B, the potential of C will be a little higher than that of D; hence if C and D be connected by a piece of wire W, or other conductor, a small quantity of positive electricity will flow from C to D, so that there will be a small charge of positive electricity on D, and of negative on C. If, now, the wire be disconnected from C and D, and by means of insulating threads C be put in-

side B and be made to touch B near the bottom, while D is put inside A, and is made to touch A near the bottom (Fig. 142), the negative charge on C will be given up entirely to B, and the positive charge on D entirely to A (*see* § 64, page 118); hence the P. D. between A and B

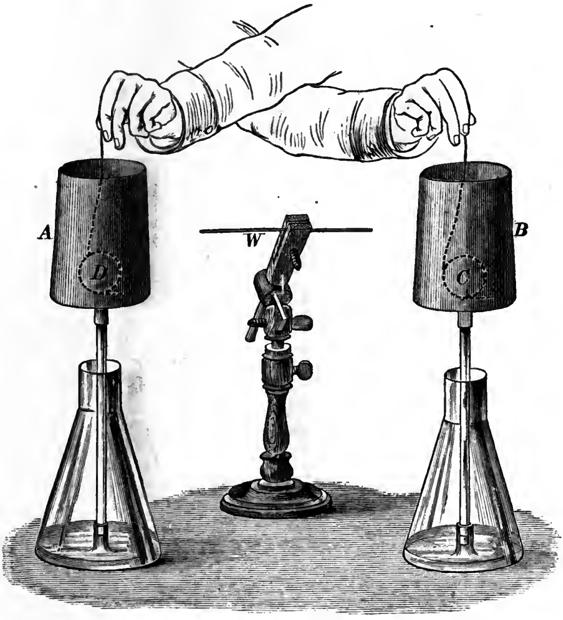

Fig. 142.

will be increased. C and D are now withdrawn, totally discharged from B and A, and on being put again into the position shown in Fig. 141, the operation is repeated. If this be performed a sufficient number of times, the P. D. between A and B may be made as large as we like; and as the charges induced in C and D depend on the P. D. already existing between A and B, it follows

that the increase of P. D. goes on more and more rapidly according to the "*compound interest law.*"

195. Thomson's Replenisher.—An accumulating influence machine for rapidly performing the operations

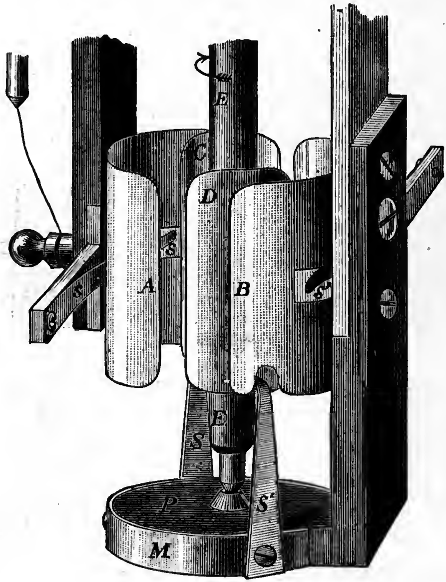

Fig. 143.

described in the last section was devised by Sir William Thomson about 1867, and has been much employed. The balls C and D, in Fig. 141, are replaced by two gilt brass "carriers" C, D, seen in perspective in Figs. 143, 145, and in plan in Fig. 144. These are carried eccentrically at the ends of an ebonite rod R, fixed to an ebonite spindle E, and by turning this spindle by means of the milled head M at the top (Fig. 145), the carriers are rapidly carried round. The metal pots A and B, of Fig. 141,

become the gilt brass "inductors" A B (Figs. 143, 144, 145), and the wire W is replaced by two springs S S′, connected by a strip of brass M fixed round the edge of the piece of ebonite P. This ebonite carries the springs and also the end of the spindle, and is itself supported as seen in Fig. 145. When the carriers C D simultaneously touch

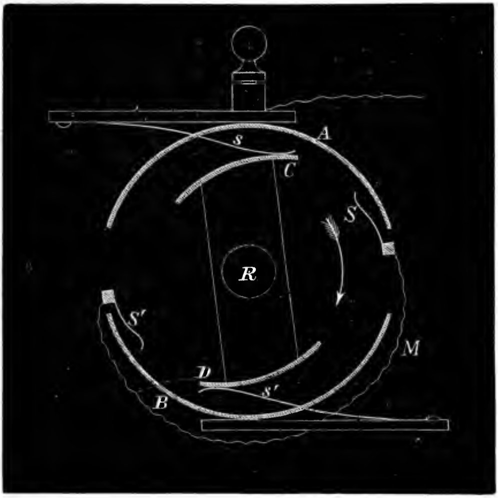

Fig. 144.

the springs S S′, they are practically in the same electric condition as are C and D (Fig. 141), and are acted on inductively by the charges in the inductors A B; while, on the other hand, when they have been turned round further in the direction of the arrow (Fig. 144) until they touch the springs *s′ s*, which are connected respectively with the two inductors, the carriers are electrically in the same condition as are C and D (Fig. 142)—that is, they are under cover of the inductors, and so part with their charges to these inductors.

It is found that there is always a sufficiently large

P. D. between the inductors A B (Fig. 143), no matter how well they may have been previously discharged, to start the action of the "*Thomson's replenisher*," and to enable the apparatus (if it be well constructed, and also clean and dry) to rapidly produce sparks on the compound interest principle.

To prevent the carriers C D causing the inductors A B to lose electricity by being left in contact with them, or by being electrically attracted round so as to come into contact with them, when the replenisher is not in use, the milled head M (Fig. 145) is fixed in the position seen in this figure by a pin attached to the farther side of the square head H, fitting into a hole in the head M. On turning the head H, this pin is withdrawn from the milled head M, which is then free to turn, and the spring K pressing against the square head H is for the purpose of holding the head in one or other of two definite positions—in one of which the pin locks the milled head M, and in the other leaves it quite free.

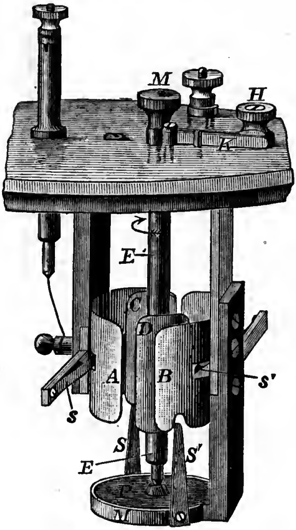

Fig. 145.

The earliest machine in which this compound interest principle of electrophoric action was used, was the "*revolving doubler*" invented by Nicholson more than one hundred years ago. This apparatus, however, seems to

have remained practically unknown, and unused. In 1860 C. F. Varley invented a somewhat similar apparatus, and still later a well-known machine was devised by Holtz, which, however, required an initial P. D. to be set up between the inductors by a piece of rubbed ebonite in order to start the action. So far the Holtz machine resembles the electrophorus, but while in a simple electrophorus, or even in Bertsch's rotatory electrophorus, there is no contrivance for even maintaining the P. D. between the inductors, the Holtz machine is so designed that the P. D. is increased by the action of the machine. This machine differs, however, from Thomson's replenisher: first, in that the carriers are practically infinite in number; secondly, in the connecting wire w (Figs. 141, 142), and s s′ (Figs. 143, 144), having a break in it so that it is divided into two parts, and the P. D. that is set up between these two parts when any pair of carriers are simultaneously in electrical contact with them, being the P. D. that is practically made use of.

The next improvement was made by Voss, who produced an accumulating influence machine which combined the advantages of the Thomson's replenisher and of the Holtz's machine, in that it required no initial P. D. to be given to the inductors to start the action, and produced considerable quantities of positive and negative electricity for an influence machine. It is, however, unnecessary to describe either this or the Holtz machine in detail, because the latest accumulating influence machine constructed by Mr. Wimshurst is not only extremely simple in construction, but is probably the most perfect machine of this type that has yet been devised.

196. Wimshurst Influence Machine.—This machine consists of two circular discs of ordinary window glass (Fig. 146), each attached to the end of a hollow boss of wood, or ebonite, upon which is turned a small pulley. These bosses are mounted on a fixed horizontal steel spindle, so that the glass discs are about one-eighth of an inch apart, and are rotated in opposite directions by the

cords which pass over the pulleys at the base of the instrument, one of the driving cords being crossed for this purpose. The glass discs are carefully coated with shell-lac varnish, and on the *outside* of each of them there are cemented an equal number of radial, sector-shaped plates of

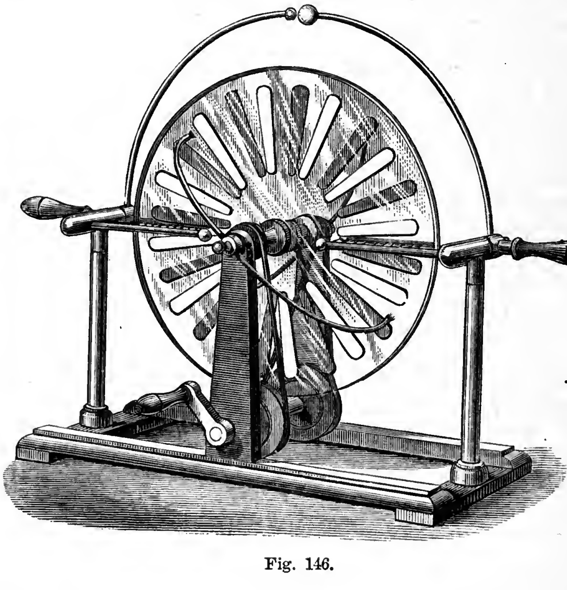

Fig. 146.

thin metal at equal distances apart, which act the part not only of the carriers C D (Figs. 143, 144, pages 364, 365), but also of the inductors A B, *the carriers on one disc acting as the inductors for the carriers on the other.* If only ten sectors be stuck on each of the glass discs, it is found that the machine will only excite itself under very favourable circumstances, whereas if there be sixteen or eighteen, it will excite itself under all atmospheric conditions. Two curved brass rods, terminating at their

ends in fine wire brushes, are placed, as seen in the figure, one at the front of the machine, and one at the back, making an angle of about 90° with one another, and about 45° with the horizontal "*collecting combs.*" These rods act like the springs s s′ (Figs. 143, 144) in connecting a pair of carriers when they are under the inductive action of the inductors, which in this machine are the adjacent carriers on the other plate. The combs are four in number, two being placed at the front of the machine, as seen in the figure, and two at the back, the points of the combs being directed towards the discs. The two combs at the left hand are connected together, and form one terminal of the machine, while the two at the right hand form the other. These combs are supported in position by the brass cylinders to which they are attached, and which stand on glass legs. These cylinders carry the two "*discharging rods*" which terminate in two balls, and in order to charge any two bodies (the inside and outside of a Leyden jar, for example) to a high P. D., they must be connected with pieces of wire to the brass cylinders, and the balls at the ends of the discharging rods separated.

It does not appear that the collecting apparatus takes any important part in the inductive action of the Wimshurst machine, for if it be removed and the glass discs made to spin round in opposite directions, their whole surface is seen to glow with a luminous discharge, and a sharp crackling sound is heard. The collection of the positive and negative charges might be effected by attaching springs to the horizontal rods so as to touch the carriers as they pass instead of using the combs which collect by a "*brush** *discharge,*" but the combs introduce, of course, far less frictional resistance to the motion of the plate, and act very well, because when a carrier comes

* If the P. D. between two conductors be raised, it is found that before it reaches the value that will cause a spark to pass between the conductors, a hissing sound is heard, and a "*brush*" or "*glow discharge*" takes place, rendering the space between the conductors luminous in the dark.

between a pair of combs, it is practically *inside* a conductor; and we have seen that when a body is inside a conductor, no charge that the conductor may have can prevent the body discharging itself into the conductor, and as, in addition, the density is very great at a point (*see* § 63, page 118), the charge easily passes across the small air space separating the points of the teeth of the comb from the surface of a carrier when it is passing the comb. Hence, in all modern frictional or influence machines, such combs have been used as the collectors.

By attaching the inner coatings of Leyden jars to the sets of collecting brushes, the outer coatings of the jars being connected together, the capacity of the collectors is much increased, hence the brightness of a spark and the noise that it makes in passing from one of the balls to the other is also much increased. As, however, we cannot augment the rate of work done by the machine in this way, and as the work given out by each spark equals

$$\frac{F V^2}{2 \cdot 712} \text{ foot lbs.,}$$

(*see* § 176, page 323), where F is the capacity in farads of one of the Leyden jars that is discharged, and V the P. D. between their inner coatings, it follows that for a given influence machine and for a given rate of turning, the rapidity of producing sparks will be diminished by connecting Leyden jars with the collecting combs.

The P. D. produced between the terminals of an influence machine can send a spark from one of the balls to the other when they are separated by a distance of several inches. When the surfaces of two metallic balls are separated by more than about one-tenth of an inch, the experiments made by Drs. De la Rue and Hugo Müller, show that the P. D. required to produce a spark is nearly proportional to the distance between their surfaces, and increases at the rate of, roughly, 10,000 volts per one-tenth of an inch, so that it

would require a P. D. of about 100,000 volts to start a spark between two metal balls separated by a distance of one inch. If the bodies between which the spark passes be a point and a plate, the "*striking distance*"* is greater for the same P. D., being at the rate of one inch for every 23,400 volts P. D. between the point and the plate. From this it will be seen that an influence machine can produce a P. D. between its terminals of some hundreds of thousands of volts; consequently, the quantity of electricity that passes in the sparks must be very small, since the work, in foot pounds, done per minute by the machine, equals

$$44{\cdot}25 \text{ A V},$$

(*see* § 114, page 201), where A is the mean value in amperes of the current passing, and V the mean P. D. in volts between the terminals, and this product cannot exceed about 5,000, the greatest work, in foot pounds per minute, that a man can do in turning the machine. Hence, although brilliant sparks and powerful shocks can be produced with such a machine, we cannot expect that it will produce any visible decomposition in a voltameter used to join its terminals, or that it will cause a deflection of the needle of even a sensitive galvanometer. A galvanic cell of small resistance can produce a current of many amperes through a small external resistance, and yet can only produce a maximum P. D. of a volt or two, whereas an influence machine is, to a certain extent, like a very large number of cells in series, each cell having a very high resistance, for such a battery can produce a very high P. D. between its terminals

* The *striking distance* is the distance that separates two conductors when a spark is *started* between them. To maintain a continuous "*electric arc*" between two conductors requires a much smaller P. D. than to start a spark between them; for example, to maintain an arc one inch long between two carbon rods only requires a P. D. of about 118 volts if the carbons be hard, and a less P. D. if they be soft. (*See* "The Resistance of the Electric Arc." *Phil. Mag.*, May, 1883.) Hence in all "*arc lamps*" there must be some mechanism for first bringing the carbons into contact, to start the arc, and then separating them.

if they be insulated, but only a very weak, steady current, even if its terminals be joined together with a short thick piece of wire, and the battery short-circuited. The low resistance cell is in fact analogous with a *large shallow* reservoir of water which is constantly kept filled with a big supply tap, while an influence machine with the balls at some distance apart is analogous with a *very tall, very narrow* tube, into which water slowly but steadily *trickles.* If a tap at the side of the former be opened and left open, there will be a *large, steady* stream of water, but the *distance through which the stream will spurt from the side of the reservoir will be small,* whereas if a tap at the side of the tall, narrow tube, near the bottom, be opened, *the water will spurt out through a distance of many feet,* but the stream will rapidly fall off as the tube empties, and the spurt can only be repeated by keeping the tap at the bottom of the tube closed, while the tube is refilling.

The distance at which the balls of an influence machine are separated, determines the *maximum* P. D. that can be set up between the discharging rods, or between any two conductors connected with them; hence, by placing the balls at a given distance apart, and then turning the machine until a spark is just going to pass between them, we know approximately the P. D. set up between two conductors connected with them.

197. Dry Pile.—When it is desired to maintain a high P. D. between two conductors that are well insulated from one another, as, for example, the outside of an electrometer, and the needle inside (*see* § 75, page 130), a battery consisting of a large number of cells in series, each cell having a high resistance, may be employed, since, as the resistance external to the battery is infinite, the P. D. at its terminals will be simply the E. M. F. of the battery, no matter how high may be the resistance of each cell. Fig. 147 shows a section of such a battery, consisting of a large number of small, simple voltaic elements, joined up in series. The liquid part

of each cell may be made smaller and smaller without affecting the P. D. at the terminals of the battery, provided that it is not required to send any current, and it may be reduced to simply the moisture which exists in ordinary paper when exposed to the air. In that case the zinc and copper plates may be pieces of metallic foil stuck on to the two sides of each piece of paper, or the cell may be formed simply of a piece of paper with a little powder rubbed on each side. In Zamboni's con-

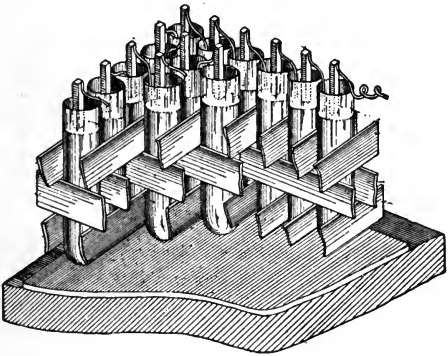

Fig. 147.

struction of a *dry pile*, sheets of paper are prepared by pasting *finely laminated zinc* or *tin* on one side, and rubbing *manganese peroxide*, or what is sometimes called *black oxide of manganese*, on the other. Discs are punched out of this paper, and several hundred of them are piled up into a column, with their similar sides all facing the same way, inside a glass tube T T (Fig. 148), which has been carefully coated inside and out with shell-lac varnish. The discs are kept in contact with one another, and electric connection is made with the two outside ones by their being pressed between the brass plate P and the brass cap B, cemented to the bottom of the tube. The plate P is pressed down by the wire W, which is held

in position by a small pinching screw s (Fig. 149), which fixes it in a collar c soldered to the inside of the other brass cap A, which latter is cemented to the tube at the top.

The dry pile may be conveniently hung by one of its terminal wires from the outside of the Edelmann electro-

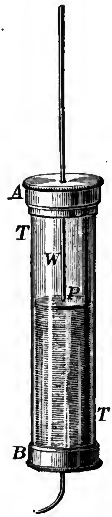

Fig. 148.

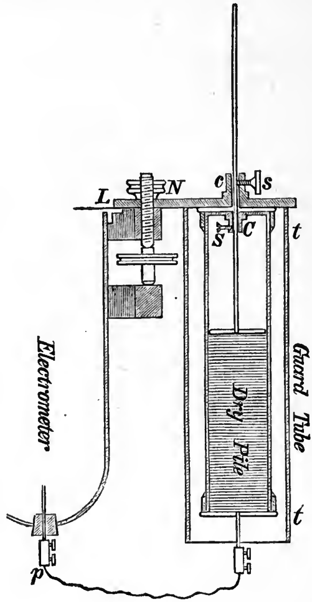

Fig. 149.

meter seen in Fig. 48, page 132, and its lower wire connected with the wire p of the electrometer. Although the pile will bring any two insulated bodies attached to its ends to a fixed P. D., its resistance is too high to enable it to instantly supply the electricity necessary to do this if the capacity of one of the bodies be suddenly changed, therefore, to avoid the capacity of the brass end of the

pile, which is electrically connected with p, being suddenly increased by some conductor in connection with the earth being brought near it, which would have the effect of momentarily lowering the potential of this end, and therefore of the electrometer needle attached to it, it is desirable to enclose the pile in a brass tube $t\,t$ of somewhat larger diameter than the glass one, and to support the pile inside this metal "*guard tube.*" This may be done by fixing the end of one of the terminal wires by a pinching screw s to a collar c, soldered to the outside of the end of the guard tube as seen in section in Fig. 149. The brass cap B at the bottom of the pile forms a condenser of fixed capacity with the brass tube, and must not, of course, even momentarily, touch this tube. The whole apparatus may then be conveniently supported from the outside of the electrometer, by placing a lug L projecting from the metal top of the guard tube, under the clamping nut N of one of the levelling screws of the electrometer (Fig. 149).

A dry pile is much more simple and compact than a battery, consisting of some hundreds of cells, but experience shows that when considerable accuracy is desired, it is better to use some form of battery (such as that illustrated in Fig. 147, for example) than a dry pile to keep the electrometer needle charged.

Ayrton Practical Elect.

CHAPTER VIII.

COMMERCIAL AMMETERS AND VOLTMETERS.

COMMERCIAL instruments for the *accurate direct* measurements of amperes and volts are quite as important as boxes of resistance coils accurately graduated in ohms; but while the construction of resistance coils has engaged the attention of manufacturers for the last twenty years, it is only since about 1880 that the construction of commercial ammeters and voltmeters has been considered. This, combined with the fact that it is far more easy to construct a coil of wire that will have a perfectly constant resistance at a fixed temperature, and even a fairly constant resistance within a considerable range of temperature, than a measuring instrument that will be constant in its indications, makes it desirable to devote a chapter to commercial ammeters and voltmeters.

198. Defect of Permanent Magnet Meters.—The ammeters and voltmeters described in §§ 36, 72, pages 73 128, have the disadvantage that, if they be placed *too near* a large powerful magnet, such as a dynamo machine

or an electromotor, not only is the strength of the controlling field, and consequently the sensibility of the instrument, *temporarily* varied, but the permanent magnet of the ammeter, or voltmeter, may have its magnetism *permanently* altered, in which case the sensibility of the instrument will also be permanently altered without the user being in many cases aware that any such change has taken place.

To avoid the possibility of this very serious error arising, the permanent magnet must be dispensed with, and the controlling force produced in some other way. Three forms of controlling force not produced by permanent magnets have been made use of, namely :—

1. The pull of a spring;
2. The attraction of gravity ;
3. The attraction of an electro-magnet temporarily magnetised by the whole or a portion of the current to be measured.

Spring Control Meters.

199. Siemens' Electro-Dynamometer.—Probably the oldest form of commercial current measurer, employing a spring to produce the controlling force, is "*Siemens' electro-dynamometer*," shown in perspective in Fig. 150, and symbolically in Figs. 151 and 152. It consists of a fixed coil A B C D (Fig. 151), and a movable coil E F G, which latter is frequently made of a single stiff wire. The current passes round the fixed coil and through the movable coil or wire in series, electric connections with the two ends of the latter being maintained by their dipping into mercury cups $m\,m'$ (Fig. 151).

The movable coil is suspended by a thread and by a delicate spiral spring N (Fig. 151), which latter can be twisted by turning the milled head T (Figs. 151 and 152) through an angle, which is measured by the pointer M attached to the head T, turning over a scale graduated in degrees, or, instead, in 400 equal divisions, and seen in Fig. 152. The instrument having been

levelled by means of the plumb-line, seen to the right of Fig. 150, the head T is turned until the plane of the movable coil E F G is at right angles to that of the fixed coil A B C D, which is indicated by the pointer P attached to the movable coil (Figs. 151 and 152) coming opposite

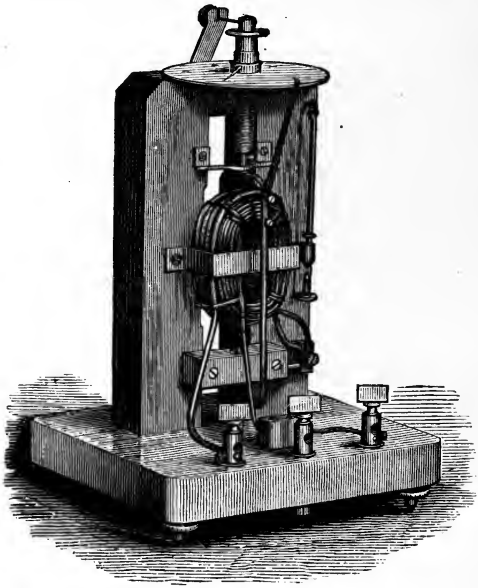

Fig. 150.

the 0° on the dial. Should the pointer M not now also point to the 0°, a small pinching screw which clamps the pointer M to the head T is loosened, and M is turned to the 0° without turning the milled head T, or twisting the spring N. If a current be sent through the instrument entering at the left-hand binding screw (Fig. 151), and following the path A B C D m E F G m', and leaving

therefore by the right-hand binding screw, the movable coil turns, tending to place its plane parallel with that of the fixed coil, until the pointer P comes up against the right-hand stop S (Fig. 152). On turning the head T, and the pointer M attached to it, through an angle, say, of 50°, P can be again brought to 0°. The couple exerted between the coils is balanced by the couple exerted by the twisted spring, and the moment of the

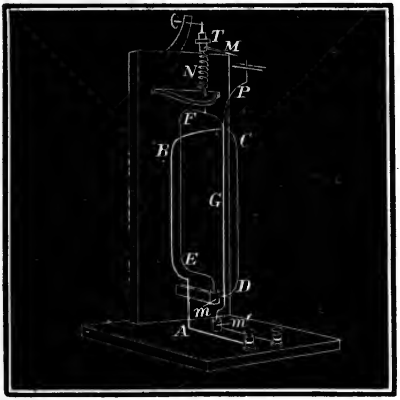

Fig. 151.

latter is proportional to the angle through which M has been turned.

To compare the current now passing through the dynamometer with some other current, exactly the same adjustment is made when the other current is passing, and since the movable wire, or coil, is always brought back to the *same position relatively to the fixed one*, the couple exerted between the coils is proportional simply to the product of the current passing through one coil into the current passing through the other—that is, to the *square* of the current passing through them *in series*. Hence, *the angle through which* M *has to be turned from*

the zero position to bring the pointer P *to* 0°, *is proportional to the square of the current.*

In the actual instrument, as seen in Fig. 150, there are two fixed deflecting coils having a different number of convolutions, and either of which can be employed by using the middle and the right-hand binding screw, or the middle and the left-hand one. The two coils have usually the one about five times as many convolutions as the other, so that the sensibility of the instrument when

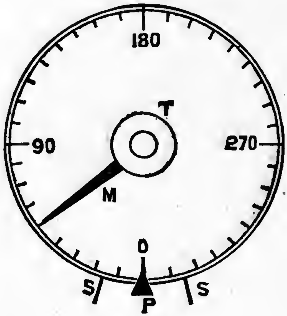

Fig 152.

using the one is about five times as great as when using the other.

The advantages of this instrument, in addition to the one already mentioned that it contains no permanent magnet, are :—First, since the fixed and moving parts between which the electric attraction is exerted always occupy *exactly the same position relatively to one another* when an observation is being made—that is, since the dynamometer is a "*zero instrument*"—one experiment is all that it is necessary to make to enable the graduation of the *whole* scale to be effected with great accuracy, since the law of the instrument is known *exactly*, arising from the fact that as long as two wires occupy exactly the same relative positions the force exerted by each on

the other is directly proportional to the product of the currents passing through them respectively; second, this dynamometer can be used with considerable accuracy to measure an *alternating* current—that is, one the direction of which undergoes rapid reversals, since the direction of the current in both the moving and stationary coils will be reversed simultaneously, and the force between them will therefore remain the same as before the reversal.

The disadvantages of the Siemens' dynamometer are:—First, the instrument being one in which the moving coil has always to be brought to zero, cannot show at once, without adjustment, the strength of a current, and as a little time is necessary to enable this adjustment to be made, the instrument cannot be used for measuring sudden variations in the strength of a current; second, owing to the moment of inertia of the suspended coil being rather large, the instrument is not dead-beat; third, the readings are much affected by neighbouring magnets, or wires conveying currents; indeed, the wires leading the current into and out of the dynamometer must be carefully twisted together, so that their mean distance from the moving coil may be the same, and the action of the current in the one leading wire balanced by the action of the equal and opposite current flowing in the other; further, as the suspended coil when traversed by a current is acted on by the earth's magnetism, the instrument must always be placed so that *the plane of the suspended coil*, when P is at 0°, *is at right angles to the plane of the earth's magnetic meridian*, since this is the position in which the coil desires to place itself as far as the action of the earth's magnetism is concerned when a current is passing through it; fourth, as the instrument must be placed in this particular position before use, also as it must be levelled and mercury poured into the cups m and m' (Fig. 151) if it has been spilt when the instrument is carried about, it is not very portable; fifth, the movable coil being quite uncovered,

is blown about by draughts of air, and the spring is liable to be accidentally damaged by things being knocked against it; sixth, the scale, being graduated in degrees, or arbitrary divisions, is not direct-reading; and lastly, the instrument gives no indication of the *direction of the current*, which, in electroplating, electrotyping, the charging of accumulators, &c., is as important as the strength of the current.

Shortly, therefore, we may say that the Siemens' dynamometer is an extremely valuable *standard instrument* when it can be kept and used in a *fixed position* in a laboratory far away from all moving magnets, or wires in which strong currents are passing, &c., and its constant experimentally determined in that fixed position; but for a portable instrument to be carried about in a workshop or room containing dynamos in motion, and used wherever required, there are other instruments more convenient.

200. Cunynghame's Ammeter and Voltmeter.—These *zero* instruments are a modification of the Siemens' dynamometer, an electro-magnet E E (Fig. 154) being substituted for the stationary deflecting coil, and a *pivoted soft iron needle* N (Figs. 153 and 154) for the movable one, the magnetic axis of the needle, as seen in Fig. 153, which shows a sectional plan of the instrument, making an angle of about 30° with the line joining the poles F F of the electro-magnet, when a pointer attached to the moving needle is at 0°. The *soft iron core* C C of the electro-magnet, seen in sectional elevation in Fig. 154, is made *massive*, in order that a considerable magnetic force may be produced by it for a comparatively small magnetic action of the current, because experiment shows that when the core of an electro-magnet is only *slightly magnetised, the strength of the magnet is directly proportional to the current*, the strength of the magnet being measured by the force with which it attracts or repels one end of a hard steel permanent magnet, put in a given position relatively to the electro-

magnet; whereas if the magnetic action of the coil be great, the soft iron core becomes "*saturated*," and its

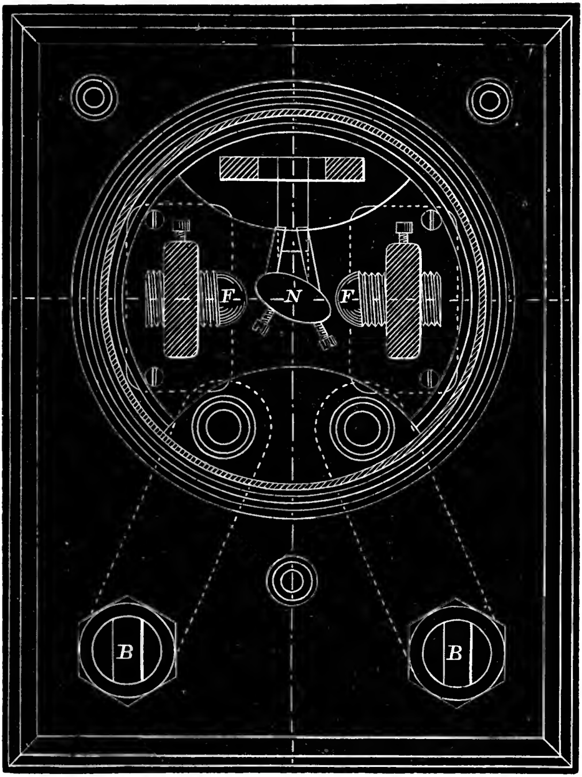

Fig. 153.

strength hardly increases with an increase in the current. The soft iron needle is magnetised inductively by the electro-magnet, and for a given relative position of the

two the amount of magnetism induced in the iron needle will be directly proportional to the strength of the electro-magnet, provided the needle is so *massive* that it is far from being "*saturated*" (*see* page 388). Under these circumstances *the couple exerted by the electro-magnet on the needle will be proportional to the square of the current.*

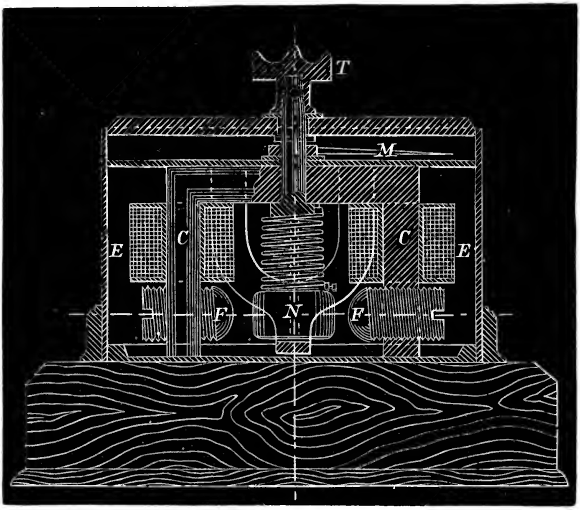

Fig. 154.

This couple is balanced by the twist given to the spiral spring, as in the Siemens' dynamometer, and therefore is also proportional to the angle through which the pointer M, attached to the milled head T, has been turned. As long, therefore, as we are dealing with currents not strong enough to saturate the iron core and the iron needle, the angle through which the pointer attached to the milled head has to be turned to bring the pointer attached to the moving needle to 0° is proportional to the square of the current.

The scale is, therefore, graduated not in degrees, but in numbers proportional to the square roots of the number of degrees, and the adjustable pole-pieces F F enable the instruments to be made direct-reading (*see* § 37, page 76). The wires leading the current to and from the instrument are fastened to the binding screws B B (Fig. 153).

The advantages of this type of instrument are:—First, the controlling force not being produced by a permanent magnet, the sensibility cannot be *permanently* changed by placing the instrument near a powerful magnet; second, its indications are but little affected by an outside magnet, as the mass of soft iron in the core and pole-pieces of the electro-magnet shields the needle to a great extent from external magnetic disturbance (*see* § 52, page 102); third, it is direct-reading; fourth, it is dead beat; fifth, it has no mercury cups, does not require levelling, can be used in any position, is not likely to be damaged, as the pointers and spring are all boxed in; and hence the Cunynghame instruments are very portable.

The disadvantages are:—First, being a zero instrument, an adjustment has to be made before the value of a current can be read, and therefore the magnitude of sudden changes in a current cannot be measured; second, it can only be used to measure currents in one direction; third, in spite of the mass of iron the current is not quite proportional to the square root of the angle, and therefore the reading is a little too small for large currents (*see* § 208, page 401); fourth, in consequence of "*residual magnetism*,"* the value of a current corresponding with a particular reading depends somewhat on whether the currents previously passing through the instrument were larger or smaller than the one being measured (*see* § 208, page 401); fifth, in consequence also of residual magnetism, a reverse current sent for a short time through the

* "*Residual magnetism*" is the name given to the magnetism that remains in a substance after the magnetising force has ceased. With very soft iron the amount of residual magnetism is small, whereas with hard steel it is very large.

instrument diminishes the subsequent indications for small direct currents (*see* § 209, page 403).

Shortly, therefore, we may say that while the instrument has not an *exact* law, and cannot, therefore, like a Siemens' dynamometer, be used as a standard instrument, it is far more convenient for general use in the workshop and in an electric lighting establishment.

201. Instruments with Magnifying Gearing.—We have seen (§ 20, page 46) that if all the deflections of a galvanometer are *small*, the deflections will be *directly proportional* to the current whatever be the shape of the coil and needle; hence, attempts have been made by M. Deprez to use a form of portable current galvanometer, in which the needle could only deflect through a small angle, and to magnify this deflection by attaching the pointer to a small grooved pulley geared by a fine endless thread to a much larger grooved wheel attached to the needle. A similar result has been attained by the author by using instead of the small and large grooved wheels a small toothed wheel, or pinion, attached to the pointer, and a larger toothed wheel to the axle or staff of the needle. Such contrivances, however, for magnifying the motion by means of *pivoted gearing* cannot be recommended, as they introduce friction as well as add to the moment of inertia of the moving parts, and so diminish the dead beat character of the apparatus. These difficulties, however, have been overcome in the following apparatus :—

202. Magnifying Spring Ammeter and Voltmeter.—In these instruments, devised by the author, a special form of spring is employed, shaped like a narrow shaving curled up into a cylinder of very small diameter (Fig. 155). Such a spring, quite unlike an ordinary spiral spring, has the peculiarity that for a *small increase in length along the axis there is large rotation of one end of the spring relatively to the other, the angle of rotation being directly proportional to the axial extension.* Hence, if one end of the spring be fixed and the other be slightly

pulled axially, a pointer attached to this end will turn through a large angle, and so will measure in a very magnified way the axial extension of the spring, without the employment of a rack and pinion, or of levers, or of any other magnifying arrangement, and without, therefore, the cost or the friction attending the use of such magnifying arrangements.

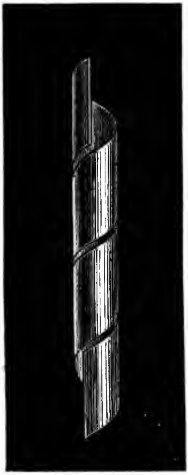

Fig. 155.

The instrument is shown in Fig. 156, where T T is a thin tube of charcoal iron, attached at its lower end to a brass cap C, terminated in a brass pin P, guided at the bottom in the way shown. To C is attached the lower end of the spring S (made of hard phosphor-bronze), the upper end of which is attached rigidly to a brass pin *p*, passing through a hole in the glass top of the apparatus G G, and fastened by means of a screw and nut to the brass milled head H outside the glass top. This pin *p*, to which the upper end of the spring is attached, also serves as a guide to the top of the iron tube. In the space W W a "*solenoid*" * wire or strip is wound, its ends being attached to the terminals shown. Hence, when a current is passed through this solenoid, the iron tube is sucked down into the solenoid, and its lower end C, to which the spring is attached, receives a large rotatory motion, which is communicated directly to the pointer attached to the top of the iron tube. Parallax, in taking readings of the pointer, is avoided by the horizontal scale having a piece of looking-glass let in it in the well-known way. (*See* § 12, page 28.)

By making the iron tube T T very thin, so that it is

* A coil of wire wound as cotton is on a reel, is called a "*solenoid*" when the length of the coil is not small compared with its diameter.

"*magnetically saturated*" for a comparatively weak current—that is, so that a current passing round the coils much weaker than the instrument is intended to measure

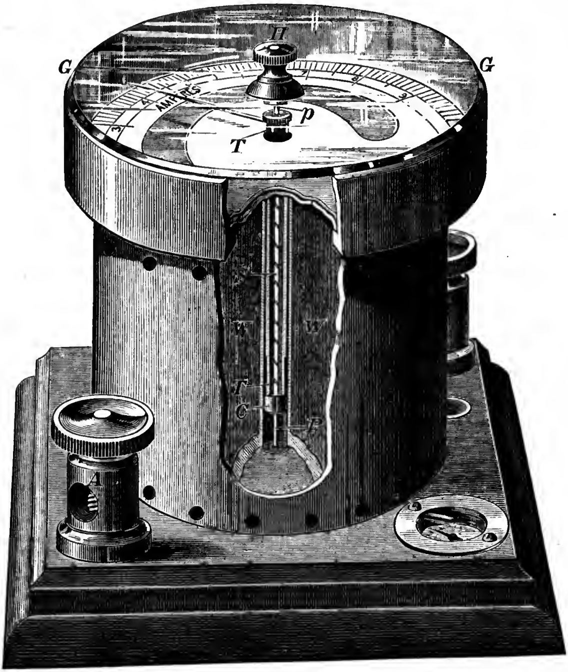

Fig. 156.

is able to impart to the iron as much magnetism as it is possible for any current to give to it—also by fixing the iron tube so that it projects into the solenoid a definite distance, which has been carefully determined, partly by calculation and partly by experiment, and lastly by constructing the spring so as to produce a large rotation

with the minimum pull, and with not too much axial motion of the free end of the spring, deflections up to 270° can be obtained *directly proportional to the current*, excepting for the first 15°, where the scale is not graduated.

This instrument being direct-reading has to be provided with an adjustment for sensibility, and this is obtained partly by the amount of wire or strip that is wound on the bobbin, and partly by means of a small movable bobbin, wound with a coil of fine wire of the same length as that employed in winding the main coil, joined up in parallel with the main coil. This movable coil slides up and down on the main bobbin, and by trial a position is found for it such that the readings on the dial are correct, and in that position this auxiliary coil is permanently fixed by the maker of the instrument.

The pointer will deflect in the same direction, no matter which way the current passes through the instrument, and owing to the softness of the iron used in making the tube T T, and the smallness of its mass, there is but very little *residual magnetism* left in it; hence the pointer indicates the correct strength of the current, no matter which way it passes through the instrument. To ascertain the direction of the current, a small compass needle is let into the base of the instrument, as seen in Fig. 156, which is deflected when the current passes through the instrument in such a way, that when the blue-coloured end of the compass needle points inwards, the current enters at that one of the binding screws that has an A marked on it, the nearer binding screw in this figure.

As, however, experience shows that the compass needle may have its magnetism reversed by a sudden very strong current sent through the ammeter (in spite of the needle being surrounded by iron to partially shield it from the action of the current), and as, in addition, its position cannot be very easily seen by an observer unless close to the

instrument, the direction of the currents in the latest magnifying spring instruments is indicated by a much larger magnet, suspended on a horizontal axis in front of the instrument, which points to the binding screw at which the current enters.

The advantages of this instrument are :—First, owing to the controlling force not being produced by a permanent magnet, the sensibility of the instrument cannot be *permanently* affected by placing it near a powerful magnet; secondly, the sensibility will not be even *temporarily* affected, no matter how strong this outside magnet may be, provided that it is so far away that the magnetic field is *uniform* throughout the small space in which the little iron tube T T moves (*see* § 15, page 36). For example, although an ordinary compass needle is turned round by a uniform magnetic field, there is no force tending to pull the compass needle bodily along, as may easily be proved by floating a compass needle on a piece of cork in a basin of water, when it will be found that while the needle will place itself at once so that its axis points north and south, it will not move towards the side of the basin as it would if it were pulled as a whole in some direction. Or the experiment may be tried thus :—suspend a bar of unmagnetised hard steel by one of its ends from the pan of a delicate balance, so that the bar hangs vertically downwards, and weigh it, then magnetise the bar, and weigh it again, when it will be found that its weight is neither increased nor diminished in the slightest by the magnetic action of the earth. This fact is expressed by saying that *a uniform magnetic field can produce a motion of rotation, but not a motion of translation of a magnet.* Now, the magnet that is moved in the magnifying spring instrument is the soft iron tube T T, which has a north-seeking pole induced on its lower end, say, and a south-seeking pole on its upper end, or *vice versâ*, by the current passing round the coil of wire or strip, and this tube is simply pulled downwards by the attraction of the current passing round this coil. Hence,

this pulling action is neither increased nor diminished by the magnetic action of the earth, nor by the action of any magnet, no matter how strong it may be, if the field it produces is uniform over the space in which the iron tube moves; second, by using the magnification introduced by the special form of spring, the distance moved through by the attracted iron tube is not large, so that the instrument has much of the advantage of a *zero instrument* (*see* § 199, page 380), that is, the force depends simply on the current, and is practically unaffected by the motion of the attracted soft iron tube. This, combined with the small mass of iron, causes *the increase of force to be directly proportional to the increase of current.* The scale is therefore *long*, and the distances corresponding with a given fraction of an ampere or of a volt are *equal* throughout the *whole length* of the scale, which not only facilitates the manufacture of the scale, but greatly increases the power of estimating by eye the decimal parts of a division. Hence, a current, or a P. D., can be read to a very small fraction of its total value.

The main disadvantage of the instrument is that currents or P. Ds. less than about one-fifth of the maximum current or P. D. that the instrument is intended to be used for cannot be measured, since for currents under this value the iron tube is not magnetically saturated.

Gravity Control Meters.

203. Gravity Control Meters. — Instruments in which the controlling force is produced by a weight attached to the needle have been devised by Sir William Thomson, Messrs. Schuckert, Edelmann, Statter, and others.

The advantages of such instruments are: first, as the controlling force is *absolutely constant*, the sensibility of

the instrument cannot vary from time to time on account of a variation in the force; second, the price is low, arising from the simplicity of construction.

The disadvantages are: first, the readings usually are easily varied by extraneous magnetic disturbance; second, there is generally a certain want of quickness of action, so that any small temporary change in the strength of the current or P. D. that is being measured is not instantly recorded. For this purpose the needle and pointer must not only be very light, but the controlling force must be great (*see* § 38, page 78). Now, if gravity be used, the only way to obtain a large controlling force is to use a large mass to be attracted, but if a large mass be attached to the needle and pointer, the moment of inertia will be seriously increased, and slow motion will be the result; whereas, by using a powerful controlling magnet or a comparatively strong spring, we obtain a dead-beatness so great that the number of times the joint in the driving-belt passes over the dynamo pulley can be easily counted, every adjustment in the carbons on an arc lamp be seen on the ammeter and voltmeter, and even the effect on an arc lamp produced by whistling may be instantly observed on the distant ammeter.

The gravity *control meters* of Sir William Thomson not yet being in common use, the author has had no experience with them, and, therefore, cannot speak of their advantages or disadvantages.

Electro-magnetic Control Meters.

204. **Crompton and Kapp's Meters.**—The third device, which consists in using for the controlling force that produced by an electro-magnet, round which flows the whole or a portion of the current to be measured, appears at first sight to be the best; but it is attended with very serious practical difficulties. The possibility of using a current to deflect a needle, and the very same current to

resist its being deflected, without obtaining the same deflection for all currents (a result which would occur if the deflecting and controlling forces varied proportionally to one another as the current was increased), arises from the fact that whereas the magnetic force exerted on a magnetic pole at a particular point by a current flowing round a coil of wire is directly proportional to the current, the force exerted on the same magnetic pole by the iron core of an electro-magnet round which the current is flowing increases nearly proportionately to the current when the current is small, but becomes nearly constant for all values of the current above a certain value, in consequence of the magnetic saturation of the iron core. Hence, by using the force due to a coil without an iron core for the deflecting force, and the force due to the iron core of the electro-magnet for the controlling force, Messrs. Crompton and Kapp have made extremely ingenious current and P. D. meters, which require the employment of neither permanent magnets, springs, nor weights.

The coil of the electro-magnet has a magnetic action as well as its iron core, and as the former increases in direct proportion to the current, its action must be neutralised if we wish the controlling force to be constant. This can be done either by the use of a third coil of a suitable size and number of convolutions, placed in such a position that when the current flowing round the electro-magnet also flows round this coil, its action exactly neutralises that of the electro-magnet coil, or the neutralisation may be more simply effected by placing the deflecting coil in such a position that it is equivalent to two coils, one the deflecting coil, and the other a coil whose effect neutralises that of the coil round the electro-magnet.

205. Paterson and Cooper's Electro-magnetic Control Meters.—These are the same in principle as those invented by Messrs. Crompton and Kapp, with the addition of *movable* pole-pieces similar to those shown in

Fig. 25, page 74, for adjusting the sensibility of the instrument.

The advantage of electro-magnetic control meters is that, as neither permanent magnets nor springs are employed in their construction, their sensibility cannot be affected by variations in their strength, and hence their behaviour from year to year remains exactly the same.

The disadvantage arises from the fact that as the entire controlling force, corresponding with that produced by the powerful permanent magnet in the apparatus shown in Fig. 23, page 70, for example, has to be produced by an iron core of the electro-magnet, the mass of iron must not be too small, otherwise any external piece of iron or magnet will affect the indications of the instrument. But it is found by experiment that unless the iron be not only very soft, but *also be very small in mass*, there is considerable *residual magnetism*, which causes the magnetic force exerted by the iron to depend not merely on the strength of the current passing round it at any particular time, but also on the strength of the previous currents, and this is the case even when the iron is still too small to prevent very serious variations in the reading of the instruments being produced by the presence of a neighbouring magnet (*see* § 210, page 407). The readings, therefore, in the lower part of the scale, instead of corresponding with definite values of the current, or of the P. D., correspond with currents or P. Ds. differing in some of these *electro-magnetic control* instruments by as much as thirty per cent., depending on whether it is an increasing current or a decreasing current that is being measured. (*See* § 208, page 402.)

206. Testing Ammeters.—The faults to be looked for in an ammeter, and for which it must be carefully tested, are :—

1. An error arising from the ampere-standards employed by different makers differing from one another.

2. An error arising from a current producing a

different deflection, depending on whether the previous currents passing through the instrument were much smaller or much larger than the current being measured.

3. An error arising from the instrument indicating a different number of amperes for the same current when it is reversed in direction.

4. An error arising from the sensibility of the instrument being *temporarily* varied by external magnetic disturbance.

5. An error arising from a *permanent* alteration of sensibility, due, for example, to the demagnetisation of a steel magnet.

207. Test for Accuracy of the Graduation.—It has been explained in § 6, page 11, that the standard ampere is that which deposits 0·00111815 grammes of silver per second. Makers of commercial instruments, however, do not calibrate each ammeter by comparing it with a silver voltameter, but only compare it with some standard current meter which has at some previous time been compared with a silver voltameter, but which may have changed its sensibility in the interval. To check the accuracy of any ammeter, therefore, it is desirable to compare it directly with a silver voltameter, and in Fig. 157 the apparatus is shown arranged for calibrating a magnifying spring ammeter A, in this way. D is a platinum dish, containing a 25 per cent. solution of silver nitrate, into which is placed a *thick* silver disc P, wrapped in filtering paper, to prevent particles of oxide of silver which may become detached from the silver plate dropping on to the platinum, and making the weight appear to be too great. It is better to use a platinum dish than a silver one, because the silver deposited at the bottom of the platinum dish can be removed, and re-formed into silver nitrate by pouring a little nitric acid into the dish. This could not be done with a silver dish, as the nitric acid would probably burn holes in it; hence the silver dish would gradually grow thicker and heavier. The platinum dish should be made as thin and as *light* as possible, so that it may be

accurately weighed; with a diameter of 4 inches, and a depth of rather more than $1\frac{1}{2}$ inches, it need not weigh more than 78 grammes.

This silver disc is held in position by a strip S, attached to it, held in a clamp C, the two sides of which are pressed together by turning the nut N. The disc and the strip S are in one piece, cut out of a *thick*,

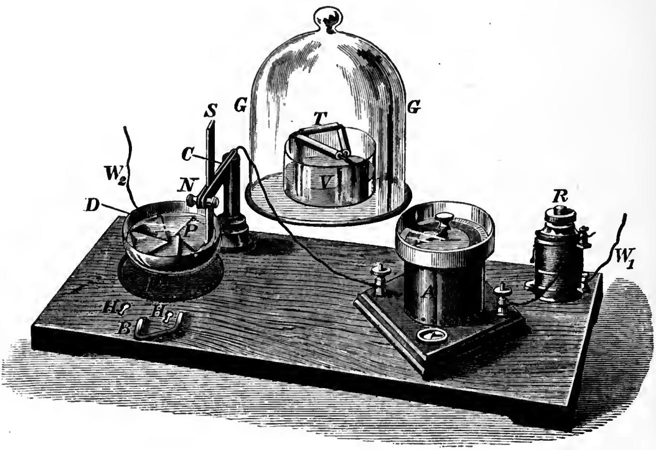

Fig. 157.

flat sheet of silver, the strip being bent up at right angles to the disc after it is cut out.

Electric connection is made with the platinum dish D, by its resting on three metal pins p, connected with the wire W_2, and connection is made with the silver disc by the wire soldered to C, the other end of which is connected with one terminal of the ammeter. The other terminal of the ammeter is connected through an adjustable carbon resistance R with the wire W_1, and the circuit is closed by putting the metallic bridge-piece B into the small mercury cups H H. The current produced by a current

generator, the terminals of which are attached to the wires w_1 and w_2, can be conveniently varied within wide limits by screwing or unscrewing the nut at the top of R, shown at *n* (Fig. 158), separated from the rest of the apparatus. Screwing this nut *n*, presses down more or

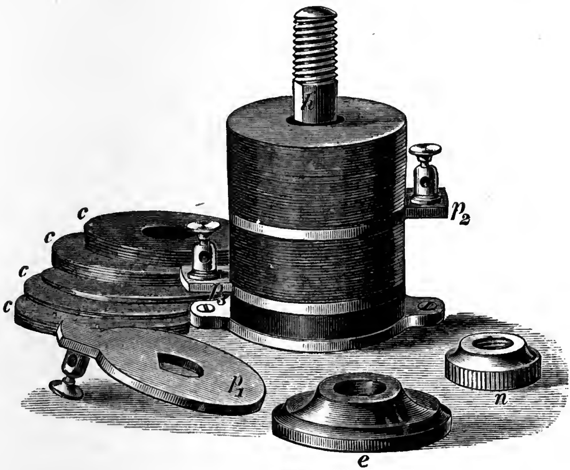

Fig. 158.

less a wooden washer *e*, which, in its turn, compresses more or less a pile of discs of carbonised cloth, some of which, *c*, *c*, *c*, *c*, are seen, in Fig. 158, separated from the carbon resistance. This cloth is specially prepared by Mr. Varley, by heating ordinary cloth to an *extremely high temperature* in a vacuum, which carbonises the cloth without destroying its flexibility and elasticity. The carbon discs are piled up in a heap by slipping them over a thin wooden tube which surrounds the brass rod *h*, terminated at the top in a screw thread for the nut *n* to screw on, and contact is made with the discs by one or other of three plates of brass, p_1, p_2, p_3, one of which, p_1, is seen separated in Fig. 158. These plates of brass

are of about the same size as the carbon discs, and the hole in the centre of each is shaped like the section of the rod h—that is, not quite round, so that p_1 and p_2 can slide up and down this rod without being able to turn round it.

Starting with a pressure sufficiently great to keep the discs fairly well in contact, so that they cannot shake about and thus produce a varying resistance, and gradually increasing this pressure, but not to such an extent as to damage the discs, the resistance of the *whole* column can be varied from about $\frac{1}{2}$ to $9\frac{1}{2}$ ohms when the discs are about $1\frac{1}{2}$ inch in diameter, and when the height of the column of them is about 3 inches. A resistance still less can be obtained by attaching the wires to the plates p_2 and p_3 (Fig. 158), instead of to the top and bottom plates as in Fig. 157.

When adjusting the carbon resistance R so as to obtain the desired current, it is desirable that no decomposition should take place in the silver voltameter, for in that case the drying and weighing of the platinum dish D would have to be carried out after the carbon resistance was adjusted, and it would probably be found that a fresh adjustment was required when it was desired to start the decomposition. To avoid this difficulty, the circuit through the silver voltameter should not be closed during the adjustment, but W_2 and the left-hand terminal of the ammeter should be joined instead by a piece of German silver wire, having the same resistance as the voltameter. A third mercury cup, not shown in the figure, but which we may call H′, may be easily arranged so that when the bridge-piece B is put into the holes H and H′, the circuit through the German silver wire is closed, whereas when one of its ends is shifted from H′ to H, the other being left in the other hole H, the circuit through the voltameter is closed.

At the commencement of the experiment the platinum dish D (Fig. 157) should be carefully washed with distilled water, to remove any dust or dirt, then dried over a

spirit lamp, and placed on the triangle T over the vessel V of strong sulphuric acid, and the glass cover G left over while the platinum dish is cooling. When it is cool it should be carefully weighed. The dish is now put in position on the pins *p*, the silver disc placed so that its edges are equally distant from the sides and bottom of the dish, and the solution of silver nitrate poured in. Next, a current is sent through the carbon resistance R, the ammeter A, and the German silver wire above referred to, and the carbon resistance adjusted until the current, as observed on the ammeter, has the right value. The maximum value that may be given to the current so as to obtain a good adherent deposit with a particular platinum dish is (as stated in the foot-note, § 6, page 11) one ampere per six square inches of surface. At a time noted on a watch the current is sent through the voltameter instead of through the German silver wire, and its strength is kept constant by slightly turning from time to time the nut *n* (Fig. 158) at the top of the carbon resistance so as to keep the ammeter deflection constant, and at a noted time, at the end of from ten to thirty minutes, depending on the current used, the circuit is interrupted. The silver nitrate solution having been put back into the bottle, the platinum dish, with the layer of deposited silver in it, is carefully rinsed out with distilled water; next it is filled with distilled water, and left standing for ten or fifteen minutes to remove traces of the silver nitrate solution, then having been rinsed out again with distilled water, it is rinsed out with alcohol to remove the water, and with ether (which evaporates with great rapidity) to remove the alcohol, and finally it is dried over a spirit lamp, and left to cool under the desiccator G, when it is again carefully weighed. Then, if W be the increase in weight in grammes produced in t seconds by a current of mean strength, A amperes,

$$\mathbf{A} = \frac{W}{0{\cdot}00111815\, t}.$$

It is desirable to repeat this test for two or three very different currents that the ammeter is adapted to measure, as the calibration may be right in even two very different parts of the scale, and not at some intermediate part, arising from the law of the instrument not being exactly what the maker has supposed; for example, he may have determined accurately the currents corresponding with two points of the scale, and have interpolated the intermediate graduations on the assumption that the increase of deflection was directly proportional to increase of current, which may not be quite true with the particular instrument.

208. Test for Residual Magnetism.—In order to ascertain whether a current produces the same deflection on an ammeter, independently of whether the currents previously passing through the instrument were much smaller or much larger than the particular current in question, the instrument should be joined up in series with a Siemens' dynamometer, or other current meter containing absolutely no iron or steel, and, therefore, having no error due to residual magnetism, together with an adjustable carbon resistance, care being taken to put the dynamometer so far away from the other instrument that any magnetism produced in the latter will not affect the dynamometer. Then, starting with the carbon resistance unscrewed, so that its resistance is great, the circuit should be closed, and successive simultaneous readings of the two instruments taken; first, as the carbon resistance is gradually screwed down, and the current increased up to the maximum current the instrument is intended to measure; then, as the carbon resistance is gradually unscrewed, and the current diminished again.

The following are the results of such tests made with a *strongly magnetised* permanent magnet ammeter, like that shown in Fig. 26 page 76; with a spring control meter, like that shown in Fig. 154, page 384; with a magnifying spring ammeter, like that shown in Fig. 156, page 388; and with an electro-magnetic control meter.

Amperes as measured by a *Permanent Magnet* Ammeter; reading from 0 to 25 amperes.		Amperes as measured by a Siemens' Dynamometer.
6·1	Ascending.	6·58
12·2		12·31
18·3		18·32
About 24·4		Not read
18·3	Descending.	18·3
12·2		12·3
6·1		6·4

Amperes as measured by a *Spring Control* Meter, with *massive iron needle*, and deflecting electro-magnets with *massive cores*; reading from 0 to 100 amperes.		Amperes as measured by a Siemens' Dynamometer.
20	Ascending.	19·6
25		25·3
35		36·2
45		47·1
55		58·1
58·5		61·4
55	Descending.	57·4
45		46·0
35		34·4
25		23·2
20		17·2

That it required a smaller current at the end of the experiment to produce the same deflection as was produced at the beginning, showed that the iron core of the *deflecting* electro-magnet retained some of the magnetism put into it when the strong current was flowing round it.

Amperes as measured by the *Magnifying Spring* Ammeter; reading from 4·5 to 25 amperes.		Amperes as measured by a Siemens' Dynamometer.
5	Ascending.	4·95
10		9·9
15		15
20		20·4
23		24·45
20	Descending.	20·85
15		15
10		9·87
5		4·85

Amperes as measured by the *Electro-Magnetic Control* Meter; reading from 0 to 100 amperes.		Amperes as measured by a Siemens' Dynamometer.
10		8·82
25	Ascending.	27·6
30		32
40		41·9
50		52·3
60		63·5
60	Descending.	64·4
50		53·6
40		44·3
30		34·7
20		24·9
10		11·5

That it required a much higher current at the end of the experiment to produce the same deflection as was produced at the beginning, showed that the iron core of the *controlling* electro-magnet retained some of the magnetism put into it when the strong current was flowing round it.

209. Test for Error on Reversing the Current.—Certain instruments, such as the spring instrument of Mr. Cunynghame, and the electro-magnetic control instruments of Messrs. Crompton and Kapp, are intended to be used only when the current flows through them in one direction, and therefore they ought not to be intentionally used with the current flowing through them in the wrong direction. As, however, in the charging and discharging of accumulators, &c., the current is liable to be reversed, it is desirable to try experimentally the kind of error that would be produced if the current were reversed, and then reversed back again so as to again flow through the instrument in the proper direction. To make the experiment, the instrument to be tested should be joined in a series with some standard instrument, like a Siemens' dynamometer, and the direction of the current through the former instrument *only* should be reversed, to avoid the possibility of any error being introduced into the readings by the reversal of the current through the latter. The two instruments

must, of course, be placed so far apart that the reversal of the magnetic action of the one, when the current passing through it is reversed, does not affect the other directly.

With instruments having much iron, it is found that not merely are the readings which are obtained with the same current when flowing in different directions very different, but that even when the current has been twice reversed, so as to flow again in its original direction, the value of a small current, as determined from the indication of the instrument, is very different from its true value, and this is especially the case when a strong current was used in the first reversal, and only a weak one in the second.

Amperes as measured by a *Spring Control* Meter, with *massive soft iron needle* and deflecting electro-magnet with *massive cores;* reading from 0 to 100 amperes.	Amperes as measured by a Siemens' Dynamometer.
At first 21.	20·8
A *reverse* current of 100 amperes was now sent through the instrument for 30 seconds, then the original current in the original direction, the deflection now was 18·8.	20·8
A *reverse* current of 85 amperes was next sent for 30 seconds, next the original current in the original direction, the deflection was still 18·8.	20·8
A *direct* current of 100 amperes was sent for 30 seconds, and then the original current the deflection now became 19·75,	20·8
and slowly increased to 20.	20·8

210. Test for Error Produced by External Magnetic Disturbance.—To test this a steady current should be

sent through the instruments, and the readings taken first with no outside magnet near, then, when a fairly strong bar magnet is moved round in a plane passing through the centre of the instrument, the magnet being held so as to always point towards the centre of the instrument, and with its end at always the same distance

Fig. 159.

from the centre. A foot is found to be a convenient distance to take, and the plane in which the bar magnet is moved should be that in which the magnet must produce the greatest disturbance; for example, with an instrument having a needle turning round on a vertical axis, the plane in which the magnet is moved should be horizontal, as shown in Fig. 159, whereas with a magnifying spring instrument in which the soft iron tube

TT (Fig. 156) is pulled downwards, the plane should be a vertical one, as seen in Fig. 160.

The experiment should be made with a weak current passing through the instrument, and also with a strong one, as frequently the magnetic disturbance differs in

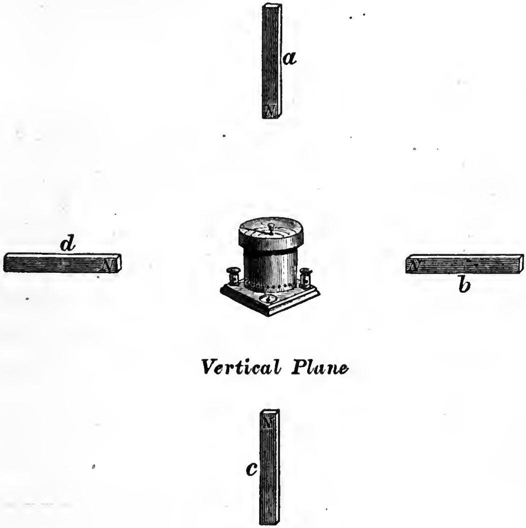

Fig. 160.

amount for different currents, and in both cases the constancy of the current during the experiment should be assured by its passing also through some other instrument, such as a Siemens' dynamometer, placed so far away that the motion of the magnet does not affect it.

The following show the results of this test made with several instruments, always using the same permanent magnet to produce the disturbance at the same distance from the centre of the instrument.

Magnet moved round a Permanent Magnet Ammeter in a Horizontal Plane.

Amperes as measured by the *Permanent Magnet* Ammeter; reading from 0 to 25 amperes.	Amperes as measured by a Siemens' Dynamometer.		
22·2	22·0	No magnet near.	
22·1	22·0	Magnet in position *a*	Fig. 159.
22·4	22·0	" " *b*	Fig. 159.
22·3	22·0	" " *c*	Fig. 159.
22·0	22·0	" " *d*	Fig. 159.
22·2	22·0	No magnet near.	

Magnet moved round a Magnifying Spring Ammeter in a Vertical Plane.

Amperes as measured by the *Magnifying Spring* Ammeter; reading from 4·5 to 25 amperes.	Amperes as measured by a Siemens' Dynamometer.		
6	5·59	No magnet near.	
6	5·59	Magnet in position *a*	Fig. 160.
6	5·59	" " *b*	Fig. 160.
6	5·59	" " *c*	Fig. 160.
6	5·59	" " *d*	Fig. 160.
6	5·59	No magnet near.	
21	21·5	No magnet near.	
21	21·5	Magnet in position *a*	Fig. 160.
21	21·5	" " *b*	Fig. 160.
21	21·5	" " *c*	Fig. 160.
21	21·5	" " *d*	Fig. 160.
21	21·5	No magnet near.	

Magnet moved round an Electro-magnetic Control Meter in a Horizontal Plane.

Amperes as measured by the *Electro-magnetic Control* Meter; reading from 0 to 100 amperes.	Amperes as measured by a Siemens' Dynamometer.	
10	9·2	No magnet near.
10·1	9·2	Magnet in position *a*.
14·6	9·2	" " *b*.
10·9	9·2	" " *c*.
7·9	9·2	" " *d*.
9·5	9·2	No magnet near.
82	90	No magnet near.
81·8	90	Magnet in position *a*.
84·6	90	" " *b*.
84·6	90	" " *c*.
81·3	90	" " *d*.
82·2	90	No magnet near.

211. Test for Permanent Alteration of Sensibility.—This test is one that must necessarily extend over a long period, as permanent magnets are found to slowly demagnetise, springs to become permanently strained, or, as it is called, get a "*permanent set,*" &c. Frequent comparisons should, therefore, be made between the readings of an ammeter, and the amount of silver deposited in a given time by the currents giving these readings.

Errors in Voltmeters.

212. Testing Voltmeters.—In addition to the five errors given in § 206, page 394, and which affect voltmeters equally with ammeters, there is a most important sixth error arising from the sensibility of a voltmeter varying with its resistance, and, therefore, with its temperature. This change of resistance is due partly to the variation of the temperature of the room, and partly to the coils of the instrument becoming heated by the

passage of the current through them. Voltmeters in this respect differ entirely from ammeters; an increase of resistance of an ammeter may diminish the current in the circuit, but the ammeter will accurately measure the current so diminished; consequently, *the sensibility of an ammeter is unchanged by a change in the resistance alone.* For example, if two exactly similar ammeters be wound, the one with copper, and the other with German silver wire of the same gauge, and with the same number of convolutions, the sensibility of the one will be exactly the same as that of the other, in spite of the resistance of the latter instrument being thirteen times that of the former; whereas an increase in the resistance of a voltmeter causes a less current to pass through it for the same P. D. at its terminals, and hence *the sensibility of a voltmeter varies with change in its resistance.*

213. Test for Accuracy of the Graduation. — From the definition of a volt (§ 81, page 141), it follows that if we know the current in amperes passing through a resistance, the value of which is known in ohms, we know the P. D., in volts, at its terminals, since this is equal to the product of the number of amperes into the number of ohms. This leads to a very simple and accurate method for calibrating voltmeters, and which is shown symbolically in Fig. 161. V is the voltmeter to be calibrated, r_1 a resistance formed of a long coil of fairly thick copper, or better of platinoid wire wound double so as not to produce any external magnetic action, and coiled up loosely so as to cool fairly quickly. A is an ammeter which has been accurately graduated, and W a Wheatstone's bridge, or differential galvanometer, with battery complete for measuring the parallel resistance between the points C and B, and which is made up of r_1 and of V. Between the terminals T_1 and T_2, there is some suitable current generator, not shown in the figure, which will send a current through the arrangement on inserting the plug P_1; r_2 is an adjustable, but not necessarily a known, resistance for varying this current,

and P_2 is a plug key for completing or interrupting the circuit through the measuring apparatus W.

The experiment is performed thus :—P_2 being opened and P_1 closed, r_2 is adjusted so that a convenient deflection is obtained on V. This deflection is read by one observer, and simultaneously the deflection on A by another observer, when, on a signal being given at which the time is noted, P_1 is opened, P_2 is closed, and *time measurements* of the parallel resistance between C and B taken. These resistances being plotted as ordinates on

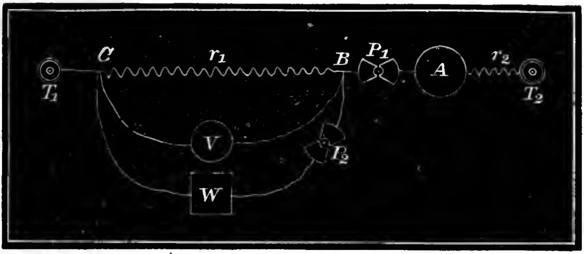

Fig. 161.

a sheet of squared paper with the times, from the moment of opening P_1, as abscissæ, a curve can be drawn, and on producing it backwards it is easy to ascertain what was the exact resistance in ohms and fraction of an ohm of the circuit between C and B *at the moment* the simultaneous readings on V and A were taken, then the product of this resistance into the number of amperes gives the exact number of volts corresponding with the deflection on V. r_2 is now varied so as to produce a different deflection on the voltmeter V, and the number of volts corresponding with it ascertained as before, and so on for as many readings as it is necessary to take to determine the absolute calibration of the voltmeter.

If the coil r_1 be made of very thin German silver wire, and the current sent through it be only a small

Ayrton, Practical Elect.

one, the resistance may not alter by the passage of the current; but if it be desired to produce a P. D. of 100 or more volts between the points C and B, and to use an ordinary ammeter A, graduated up to, say, 20 amperes, the resistance r_1 would have to be something like 10 ohms, and able to take a current of 10 amperes without heating at all. Such a wire would have to be very long and thick, and, therefore, expensive, whereas the device of taking *time measurements* of the resistance enables the coil to be made of even copper wire.

The preceding method is based on our knowing the exact value of a current and of a resistance, but we may calibrate a voltmeter by comparing its readings with the E. M. F. of a cell, if this E. M. F. be accurately known in volts. The cells best suited for this purpose are a "*Latimer Clark's cell*," or some form of gravity Daniell, in which the copper sulphate and zinc sulphate solutions mix very slowly.

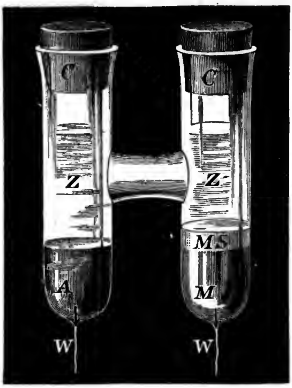

Fig. 162.

214. Latimer Clark's Cell.—These cells are made in a variety of forms,* but probably what is called the H form, shown in Fig. 162, is the best. One of the legs is partially filled with an "amalgam of zinc" A, formed by putting some pure zinc into pure mercury, which has been previously distilled in a *vacuum*, the other with pure mercury M, which has been similarly distilled, covered with a layer of "*mercurous sulphate*" M S.

* Phil. Trans. Roy. Soc., vol. xvii., p. 411. Part II., 1884.

The whole is then filled up above the level of the cross tube with pure *saturated* zinc sulphate z, and a few crystals of zinc sulphate are added. Evaporation is prevented by the insertion of paraffined corks c, and electrical contact is made with the amalgam, and with the pure mercury, by platinum wires w w, sealed into the glass. Marine glue may be employed instead of paraffin wax to make the corks c air-tight, or, best of all, the upper ends of the tubes may be hermetically sealed (*see* note, page 20). If the zinc sulphate be *saturated*, but not "*super-saturated*,"* the experiments of Lord Rayleigh † show that when this cell is not allowed to send currents, its E. M. F., after it has been set up for some weeks, is *extremely constant* for the same temperature, and has a very exact value for any particular temperature; its value in legal volts being equal to

$$1{\cdot}438\,\{1-0{\cdot}00077\,(t-15^\circ)\},$$

where t is the temperature of the cell in degrees Centigrade.

As in the Daniell's cell (*see* § 119, page 211), a diminution in the density of the zinc sulphate solution increases the E. M. F. of the Latimer Clark's cell.

215. Standard Daniell's Cell.—In spite of the great value of the Latimer Clark's cell, it has two defects, the one that it polarises rapidly, and its E. M. F. temporarily falls off if a current be allowed to pass through the cell, the other that the variation of its E. M. F. with temperature is considerable, and therefore for accurate work the temperature of the cell must be accurately known. These

* When a *saturated* solution of a salt is cooled, some crystals are formed so as to leave the liquid simply saturated at the lower temperature; but if the liquid be closed up so that the air does not get to it, and if it be cooled without shaking, crystallisation may not take place, and the liquid is then said to be "*super-saturated*," for on dropping a crystal of the salt into it, crystallisation immediately occurs. The presence, therefore, of crystals in a liquid is a proof that it is saturated and not super-saturated.

† Proc. Roy. Soc., vol. xl., p. 79.

objections are overcome by the employment of a form of gravity Daniell, in which the solutions can only mix very slowly. If the plates, or rods, be formed of *clean, pure* zinc, and of freshly "*electrotyped*" copper—that is, copper on the surface of which a layer of copper has been deposited by putting the plate, or rod, into a bath of copper sulphate, and sending a current through the bath, so that it leaves by the plate or rod—and if the solutions used in the Daniell's cell be formed of *pure* crystals of copper sulphate and zinc sulphate, then the E. M. F. will be 1·104 volts when the solutions are equally dense, and 1·074 volts if the copper sulphate solution has a specific gravity of 1·100 at 15° C., and the zinc sulphate solution 1·400 at the same temperature. A form of gravity Daniell's cell, specially designed by Dr. Fleming,* to be used as a standard, is shown in Fig. 163, and consists of a U-tube $\frac{3}{4}$ inch in diameter, and 8 inches long, provided with glass taps, &c., as shown. To use the cell, the tap A is opened, and the whole U-tube filled with the denser zinc sulphate solution; the zinc rod which is kept in the test tube L, when the cell is not in use, is now inserted in the left-hand tube, and its indiarubber stopper P fitted

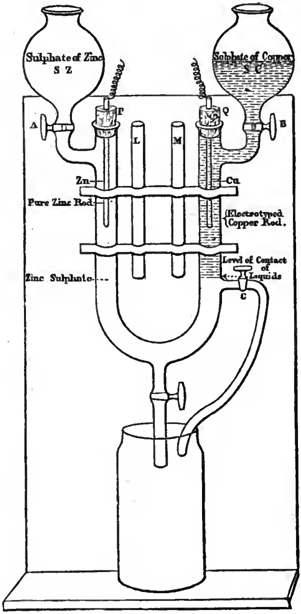

Fig. 163.

* Phil. Mag., S. 5, vol. xx., p. 126.

tightly into this tube. Now, on opening the tap C, the level of the liquid will begin to fall in the right-hand limb, but no liquid will flow out of the left-hand one. As the level commences to sink in the right-hand limb, copper sulphate solution can be allowed to flow in gently to replace it by opening the tap B; and this operation can be so conducted that the surface of demarcation of the two liquids remains quite sharp, and gradually sinks to the level of the tap C. When this is the case, all the taps are closed and the copper rod is removed from the test tube M, in which it is kept, and, after having been freshly electrotyped, is fitted into the right-hand tube Q.

It is impossible to stop the liquids mixing together at the surface of contact, but whenever the surface of contact ceases to be sharply defined, the mixed liquid at the level of the tap C can be drawn off, and fresh solutions supplied from the reservoirs above.

Experiment shows that the effect of oxidation of the zinc is to lower the E. M. F., while oxidation of the copper raises it.

In order that the E. M. F. of a Latimer Clark's cell should be quite constant, it is absolutely necessary that the cell should not be allowed to send any appreciable current, and even with the Daniell's cell better results will be obtained if the cell be not sending a current when the test is made, since in that case the P. D. at its terminals will be equal to the E. M. F., independently of the internal resistance of the cell, which will be rather high if it be so constructed that the solutions can only mix slowly. Hence, Poggendorff's method (*see* § 132, page 234), or the condenser method (*see* § 183, page 341), must be employed, care being taken to determine accurately the multiplying power for a discharge of the shunt employed (*see* § 188, page 349).

The complete arrangement for calibrating a voltmeter by Poggendorff's method is shown in Fig. 164, the figure being somewhat distorted so that the details of the key can be easily seen. In actual practice the

board and the wires on it are much longer than they appear to be in the figure. J K is a long German silver, or platinum-silver, or platinoid wire, very carefully drawn so as to have, as nearly as possible, the same diameter everywhere, and as it is very difficult, if not impossible, to draw a long wire having exactly the same diameter at all points in its length, the resistance of each five or six inches of the wire should be carefully

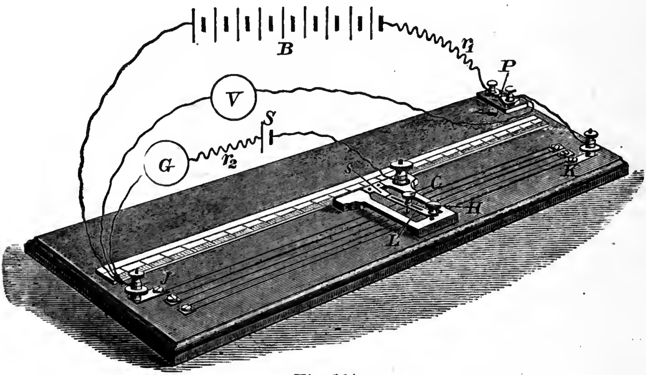

Fig. 164.

measured and recorded. B is a large battery of any kind of cells that will send through r_1, and the wire J K, a current that will remain constant for at any rate a few seconds. G is a sensitive high resistance galvanometer, S the standard cell, and r_2 is a high resistance inserted in this circuit to keep the current that would flow through the cell on closing the key quite a weak one, even if the point of contact of the key with the wire J K be far away from the position that gives no current through the galvanometer. The test is made by inserting the plug P, the handle H of the key being up, and adjusting r_1 until the P. D. between the points J and K, that is, between

the terminals of the voltmeter V, produces about the desired deflection; the key is then closed for a moment, when, if there be any deflection on G, the key is slid, in the proper direction, along the wire J K, and contact again made, and so on until a point M is found such that no current passes through G; the reading on V is taken at that moment, and we know that it corresponds with a P. D. equal to

$$\frac{\text{resistance of J K}}{\text{resistance of J M}} \times \text{E. M. F. of the standard cell.}$$

In order to enable the contact-maker C to touch any one of the five wires composing J K, C can be slid along the slot in the lever; and, to prevent the platinised knife-edge attached to the lower part of C being pressed too hard against the stretched wire, and damaging it, the flat spring *s* is made rather weak. Hence, on depressing H, the knife-edge attached to C first comes into contact with the wire, and, on still further depressing H until it comes against the stop placed underneath it, the lever turns about the knife-edge.

216. Test for Heating Error.—The various errors found in ammeters occur, as already explained, also in voltmeters, and may be tested for in the same manner by using a voltmeter with no iron employed in its construction, as the instrument of comparison, instead of an ammeter. As, however, the heating error (*see* § 212, page 408) is one peculiar to voltmeters, and may exist in the voltmeter which we use as our standard when testing for the other errors, it is desirable to consider how it may be reduced to a minimum, since the existence of this heating error in the standard voltmeter might easily mask all the other errors in the voltmeters that are being tested. The first point to determine is the way in which the sensibility of a galvanometer, with coils of a given shape and size, and with a given needle and controlling force, varies with the resistance of the

wire employed in winding it; next, how the rate of production of heat, when a given deflection is produced, also varies with the resistance of the wire employed in winding the galvanometer, because it may be that by winding it with some special form of wire, we may obtain considerable sensibility with but little heating of the coils.

217. Variation of the Sensibility of a Galvanometer with its Resistance.—When all the convolutions of wire occupy the same position relatively to the needle,

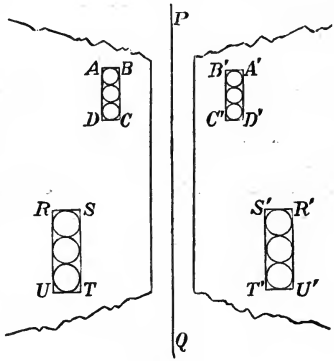

Fig. 165.

we nave seen (§ 22, page 51) that the sensibility of a galvanometer is directly proportional to the number of convolutions—that is, to the length of wire employed in winding the bobbin. This conclusion is also true, no matter what be the shape of the coil, or what the distances of the various convolutions from the needle, provided. that the coil is *fixed in size and shape;* for let A B C D, A′ B′ C′ D′ (Fig. 165), be a *small* bit of a section of a galvanometer coil taken through the axis P Q of the coil; A B C D being so small that all the three wires that pass through it are at practically the same distance from the needle, and therefore produce the

same magnetic effect when the same current passes through each of them. Now, if the bobbin were wound with wire of half the diameter, there would be four wires for each of the three wires that pass through A B C D, or twelve altogether, as in Fig. 166, hence the magnetic effect due to the wires that pass through the small bit A B C D, A′ B′ C′ D′ would, for the same current, have been increased four times. And so for the wires passing through any other bit R S T U, R′ S′ T′ U′ of the section. Hence, although

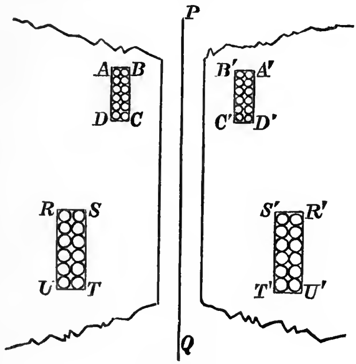

Fig. 166.

the magnetic effect of one convolution passing through A B C D, A′ B′ C′ D′ may, for the same current, be very different from the effect of a convolution passing through R S T U, R′ S′ T′ U′, we may say that the *whole* magnetic effect for the same current is directly proportional to the number of convolutions, or to the length of the wire; and it will be observed that this result remains true even if the diameter of the wire at different parts of the coil be quite different, provided that the law of winding be maintained when the gauge is changed—that is to say, the diameter of the wire used in winding the three convolutions passing through A B C D may be quite different

from that employed in winding the three convolutions passing through R S T U, and yet the whole magnetic effect for the same current will be directly proportional to the length of the wire, provided that when we halve, double, or treble the diameter of one set of wires, we do the same for every other. We may conclude, therefore, that when we have a galvanometer with coils of a given shape and size, wound according to a given law, and fitted with a given needle, or set of needles, and controlled by a given force, *the sensibility of the galvanometer is directly proportional to the number of convolutions, or to the length of wire used in winding it.*

But the resistance of the wire used in winding a given coil is, for the *same material*, copper, German silver, &c., proportional to the square of the number of convolutions—that is, to the square of the length—because when we replace each convolution by four, we make the length of the wire used in winding the bit A B C D, A′ B′ C′ D′ four times as great, and the sectional area of each wire one-quarter, therefore the resistance of the wire passing through A B C D, A′ B′ C′ D′ becomes sixteen times as great, and so for the wire used in winding any other small bit R S T U, R′ S′ T′ U′, hence *the sensibility of a galvanometer is directly proportional to the square root of its resistance*, and *the magnetic effect is directly proportional to the product of the current into the square root of the resistance.*

Therefore, with coils of a given shape and size, wound according to a given law, with wire of a given material, and fitted with a given needle, or set of needles, controlled by a given force, *the current required to produce a given deflection is inversely proportional to the square root of the galvanometer resistance.*

And since the current passing through a galvanometer is equal to the P. D. maintained at its terminals, divided by its resistance, it follows that *the P. D. required to be maintained at the terminals of a given voltmeter, wound*

with wire of a given material, to produce a given deflection is directly proportional to the square root of the resistance of the voltmeter.

And these two last conclusions may be shown to be true whether the needle be a hard steel magnet or a piece of soft iron magnetised by the current passing round the coils of the instrument.

218. Rate of Production of Heat in Galvanometer Coils.—We have seen in the last section that the current required to produce a given deflection is inversely proportional to the square root of the galvanometer resistance, and this is the same thing as saying that to produce a given deflection the product of the current into the square root of the resistance must be constant. But the rate of production of heat in the galvanometer is, by § 113, page 198, proportional to the product of the square of the current into the resistance of the galvanometer. Hence, with coils of a given shape and size, wound according to a given law, with wire of a given material, and fitted with a given needle, or set of needles, controlled by a given force, *the rate of production of heat, when a given deflection is being produced, is a constant and is independent of the gauge of wire used in winding the coils.*

Hence, we see that if the following things be fixed in a voltmeter :—

1. The shape and size of the coils ;
2. The material of which the wire is made ;
3. The law of winding, *i.e.*, the variation of the thickness of the wire with the diameter, or position, of a convolution ;
4. The needles and the controlling force ;

we cannot diminish the error arising from the heating of the coils when a current passes round them by winding the instrument with finer or with thicker wire.

We have next to consider whether it may be diminished by varying 2, 3, or 4. As to 4, it is quite clear that the smaller the controlling force, and the more astatic the system of needles (*see* § 152, page 282), the smaller will

be the current required to produce a given deflection, and therefore the less the heating error. As to the material, if we are merely concerned with variations of resistance of the voltmeter arising from changes of temperature of the room, then it is better to use German silver, platinum-silver, or platinoid wire (*see* § 94, page 160), or we may add a small piece of carbon in series with the voltmeter coils of such a length and size that its diminution of resistance for an increase of the temperature exactly balances the increase of resistance of the coils; but if it is the increase of resistance due to the heating of the coil by the passage of the current that we wish to have as small as possible, then it is easy to show that it is better to wind the whole of the coils with copper wire than with German silver. For, since the resistance of German silver for the same length and thickness is about thirteen times as great as that of copper, it follows, if two exactly similar voltmeters be wound, the one with German silver wire, and the other with copper wire of the same length and thickness, that the rate of production of heat when there is the same deflection in the two instruments (which will be produced by the same current) will be about thirteen times as great in the one that is wound with German silver wire, as in the one that is wound with copper wire, whereas for the same rise of temperature the increase of resistance of copper is only about 8·8 times that of German silver (*see* § 94, page 160). We cannot, of course, say that the rise of temperature is proportional to the rate of production of heat (*see* § 111, page 194), but it is probable that the rise of temperature of the German silver coils will be more than 8·8 times that of the copper ones, and, therefore, as far as the heating due to the passage of the current is concerned, copper is to be preferred to German silver wire.

The law of winding that will give a minimum heating error will depend on the dimensions of the instrument, and for a magnifying spring voltmeter of the

dimensions shown in Fig. 156, and where the radius of the central part not wound with wire is one-eighth of the radius of the cylinder formed by the outside of the wire, it may be shown that, if a be the sectional area of the copper wire at any distance d from the axis of the instrument, and a_0 the sectional area of the first layer of the copper wire nearest the central portion, to have a minimum heating error the following condition should be satisfied :—

$$a = a_0 d^{0 \cdot 42},$$

so that the sectional area of the outside wire should be 2·395 times the area of the inside wire.

The particular sectional area given to the innermost wire must depend on the strength of the spring and the P. D. that it is desired shall produce a particular deflection.

A method that is frequently employed for diminishing the heating error, is to wind the voltmeter so that a comparatively small P. D. maintained at its terminals will produce a large deflection, and then to add a separate resistance coil joined in series with the voltmeter, when the practical terminals of the instrument become the free end of this resistance coil and the free end of the voltmeter. If V_1 be the P. D. in volts required to produce a deflection of, say, 100° when applied directly to the terminals of the voltmeter of resistance r_1, and V_2 be the P. D. required to produce the same deflection when a coil of resistance r_2 is put in series with the voltmeter,

$$\frac{V_2}{V_1} = \frac{r_1 + r_2}{r_1}.$$

Hence, by giving a proper value to r_2, we can make V_2 have any value we like, but what is even more important, the temperature error arising from changes of temperature of the room as well as from the heating of the coils by the passage of the current round them, will depend

not merely on the variation of r_1, but on the variation of $r_1 + r_2$, and this we can keep as small as we like by making r_2 large compared with r_1, and by constructing the extra resistance of *thick* German silver wire, so that the proportional increase in the total resistance $r_1 + r_2$ shall be small, even if the increase in r_1 alone be considerable. It might be asked, why not make the voltmeter itself large, and wind it with such thick wire that the heating would be small. The answer is that if we did so we should remove the outer layers of wire so far away from the attracted needle, that the effect of a current passing round them would be very small, and hence we should seriously diminish the sensibility of the instrument. The separate resistance coil has to produce no magnetic action, hence the objection to using very thick German silver wire in it, and making it very large, is merely increase in cost and diminution in portability.

There is, however, one objection to making r_2 large compared with r_1, and that is, that the energy expended in the voltmeter itself, and which is equal to $44{\cdot}25\ A^2r_1$ foot pounds per minute (*see* § 114, page 201), is only a small fraction of the energy expended in the extra resistance, and which is equal to $44{\cdot}25\ A^2r_2$ foot pounds per minute. The former waste we cannot help, as it is a constant depending on the construction of the spring and the shape of the voltmeter (*see* § 218, page 419), but the latter is a *large* waste introduced *solely* to diminish the heating error. *Hence, a voltmeter, with a powerful controlling force, wound with thick wire of low resistance, and furnished with a separate coil of high resistance, can only be used in electric light, or power, installations where a small waste of energy is unimportant.*

219. Standard Voltmeter.—But if the controlling force be weak, then the total waste of energy will be so small as to be negligible, and hence we are led to the best form to give to a standard voltmeter: suspend the needle as delicately as possible, and use a controlling force as weak as is compatible with the instrument

retaining a fixed constant, wind the instrument with not very fine copper wire, and place in series with it a *large* resistance made of as thick platinoid wire as is obtainable. When, as explained in § 11, page 23, a galvanometer has a single suspended magnetic needle, the alteration of its strength will not affect the sensibility of the instrument; but if there be an astatic combination, an increase or diminution of strength of either of the needles will affect the sensibility of the instrument, hence it is better to use a single needle galvanometer when we desire great constancy in the sensibility, as in a standard voltmeter.

220. Cardew's Voltmeter.—This voltmeter, designed by Captain Cardew, R.E., differs from all the instruments previously described in that the *heating* and not the magnetic action of a current is employed, and the elevation of temperature of the conductor is measured by its expansion. The conductor consists, in the newest form of the instrument, the back of which is seen in Fig. 167, of about thirteen feet of platinum-silver wire 0·0025 inch in diameter. This wire, which is fixed at one end to a screw A, passes, at the top of the instrument, over a pulley P_1, made of bone so as to be an insulator, then down under a small bone pulley p_1, then up again over a bone pulley P_2, and lastly is fastened to a screw B. The pieces of brass into which the screws A and B are fastened, are connected with the terminals T_1 and T_2, and on a P. D. being set up between these terminals a current flows through the stretched wire, the strength of which depends on the P. D. maintained between the terminals of the voltmeter, and on the resistance of the wire. The wire becomes hot and expands, and as it is *very thin*, it *very quickly* acquires the temperature corresponding with the particular current passing through it. The support carrying the little pulley p_1, is pulled down by a thread wrapped round the grooved wheel W and fastened to the spring S_1; hence when the wire lengthens, and the little pulley p_1 descends,

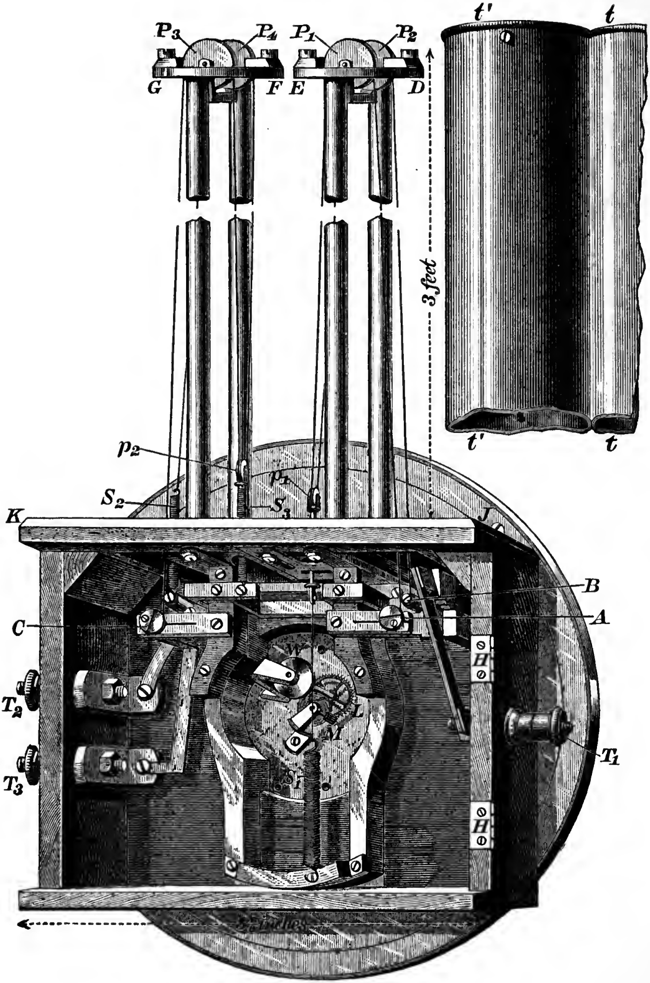

Fig. 167.

the wheel W is turned. The staff (or little shaft) carrying the wheel W also carries a toothed wheel L, geared into a small pinion M, hence when, by a slight lengthening of the wire, W is turned through a small angle, the pinion turns through a large one. On the farther end of the staff carrying the pinion there is fixed, in the front of the instrument, a pointer moving over a dial graduated in volts, the back of which is seen in the figure; consequently the pointer is caused to move right round the scale by a comparatively small descent of the pulley p_1.

It will be observed that the pull of the spring S_1 is balanced by twice the tension in the stretched wire, and that the descent of the pulley p_1 is due to the expansion of only half the total length of wire employed, that is, the expansion of only about six feet six inches of wire. The advantage, however, of using a long wire, fixed in the way shown, instead of a wire half as long, and of twice the sectional area, which would enable the same spring S_1 to be used and cause the same motion of the pointer for the same elevation of temperature, is that the fine wire heats and cools much more rapidly than the thicker one, and so makes the voltmeter much more dead-beat. If the P. D. to be measured is between 30 and 120 volts, the stretched wire alone may be used, but for larger P. Ds., an extra resistance (*see* § 218, page 421) is added, and the terminals of the voltmeter are now T_1 and T_3. If the extra resistance be equal to that of the voltmeter itself, not merely when the wires are cold, but also when they are heated by the passage of the current, the readings on the scale will correspond with exactly twice the number of volts; or a double scale somewhat similar to that seen in Fig. 26, page 76, can be employed, the numbers on the one being twice the corresponding ones on the other. To insure the resistance of the added wire being *always* exactly equal to that of the voltmeter itself, Captain Cardew uses for the extra circuits a stretched wire of the same length and section, and placed under similar conditions as regards cooling as the

wire of the voltmeter itself, both sets of wires being surrounded with metal tubes, as will be described farther on, and the tubes, like the metal rods supporting the pulleys P_1, P_2, P_3, P_4, being lamp-blacked on the surface. This extra wire, which has one end attached to the screw C, passes over a bone pulley P_3 at the top of the instrument, then down and under the little bone pulley p_2, then up and over the bone pulley P_4, and lastly is attached to the spring S_2. The support carrying the pulley p_2 is also attached to a spring S_3, hence the stretching of the second wire which occurs when the current passes through it is taken up by the contraction of both the springs S_2 and S_3, and the wire is kept tight. To prevent draughts of air cooling the stretched wires they are enclosed in metal tubes $t\ t$, $t'\ t'$, shown in the figure separated from the rest of the apparatus. The internal diameter of these tubes is only a little greater than that of the circular metal plates, D E, F G, carrying the bearings on which the pulleys P_1, P_2, P_3 and P_4 turn, so that when the tubes are slipped over the plates and screwed on to J K, the top of the box, they prevent these plates having any lateral motion.

To prevent the rods which support the pulleys P_1, P_2, P_3, P_4 expanding and contracting more or less than the stretched wires when the temperature of the room changes, which would cause the pointer to move, these rods may be composed partly of brass and partly of iron, so that their mean co-efficient of expansion is the same as that of platinum-silver.

The mechanism contained in the wooden box in the lower part of the instrument is protected from damage by the box being closed with a wooden back (not shown in the figure) which turns on the hinges H H.

The two *great* advantages of this instrument are :—First, it has no heating error, since the elevation of the temperature produced by the passage of the current is the property of the current made use of; second, it can be used for measuring alternating P. Ds. (*see* § 113,

page 198). As already stated (§ 100, page 174), when a current is started round an electro-magnet, it takes a certain time to reach its maximum value, so that with an alternating current, which is continually being started in opposite directions, the effect of the self-induction of the coil is to practically increase its resistance by an amount which varies with the rapidity of the alternations; hence, apart from the fact that the rapid reversals of magnetism, which are produced by an alternating current, prevent an ordinary galvanometer being used for measuring such a current, even a high resistance dynamometer, which can be used for measuring an alternating current (*see* § 199, page 381), cannot be used for measuring an alternating P. D., for its self-induction would cause it to practically have a variable resistance, and we have seen (§ 212, page 408) that any variation in the resistance of a voltmeter varies its sensibility. But as the self-induction of a straight wire bent back on itself is very small, the error in Captain Cardew's voltmeter, arising from self-induction, is negligible, and so this instrument is much used for measuring an alternating P. D. It is also dead-beat, direct-reading, not disturbed by magnets, and fairly portable, although large.

The disadvantages of the instrument, as usually made, are :—First, it absorbs a good deal of energy; second, it cannot be used for measuring a small P. D., for we cannot make it of thicker wire as we should do in the case of an ordinary voltmeter intended to measure small P. Ds., as this would render it sluggish, since a thick wire traversed by a current heats and cools slowly on starting and stopping the current; third, there is considerable vagueness in the readings near the zero point, and sometimes inaccuracy in the upper parts of the scale.

221. Commutator Ammeter and Voltmeter.—With any of the magnetic instruments already described, the following commutating device, due to the author, may be employed, and which enables the same instrument to be used with two degrees of sensibility, the one exactly a

certain *known* number of times the other. This arrangement is very convenient when an ammeter has at one time to be used to accurately measure, say, the current passing through an arc-lamp, which may be 20 or more amperes, and at another time to measure with *equal* accuracy that passing through an incandescent lamp, which will most probably be less than one ampere, or when the same voltmeter is to be employed to measure

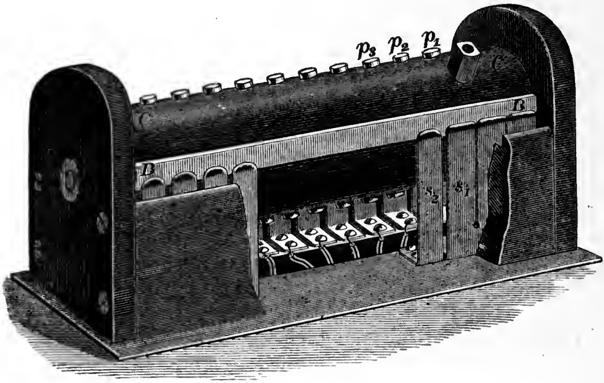

Fig. 168.

the P. D. at the terminals of a dynamo machine, and which may be 100 or more volts, and the P. D. at the terminals of five or six cells. Further, this power of varying the sensibility in a *known* ratio is of especial convenience in enabling an ammeter which is to be employed for measuring strong currents, or a voltmeter that is used for measuring large P. Ds., to be *accurately* calibrated by using, in the one case, known currents, and in the other known P. Ds. only one-tenth as large as the instrument can be employed to measure. The device consists in winding the instrument with a *strand* of *separate*

wires instead of merely one wire, and employing a "*commutator*," by means of which the current can be made to go either through all the wires *in parallel*, as if through a single thick wire, or, instead, through the wires one after the other *in series*, as if the instrument were wound with one long fine wire. Such a commutator is seen under a cover at the back of the ammeter shown in Fig. 26, page 76, and the commutator with the cover

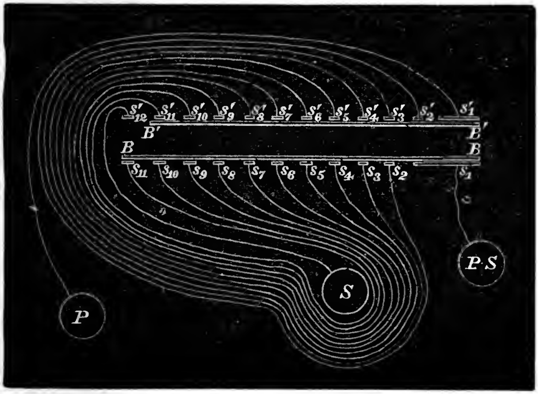

Fig. 169.

removed is shown in Fig. 168, part of the side and some of the springs being removed to show the remainder more clearly. One end of each of the wires is permanently attached to the upright springs s_2, s_3, s_4, &c. (Fig. 169), on one side of the ebonite barrel of the commutator C C, and the other end of each of the wires to one of the upright springs s'_2, s'_3, s'_4, &c., on the other side of the barrel. In one of the positions of the commutator, all the springs on one side are electrically connected together by a platinised strip of brass B B, inserted in the barrel of the commutator parallel to its axis of rotation, and all the springs on the farther side

are also connected by a similar piece of metal B′ B′, inserted in the other side of the ebonite barrel, the tips of the springs being also platinised to insure good contact. The terminals marked P, P S, seen at the back of Fig. 26, are permanently connected, by pieces of thick wire in the base of the instrument, to the first of each of the springs s_1, s'_1, on the two sides of the barrel, hence the connections are now as shown symbolically

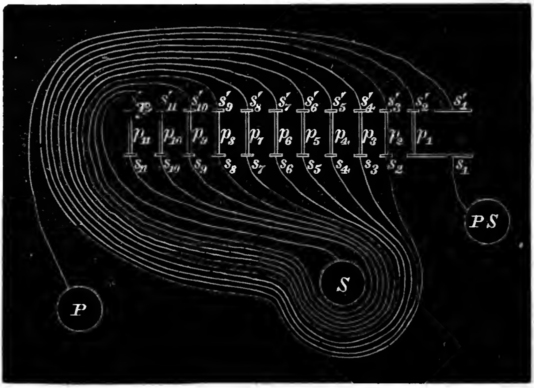

Fig. 170.

in Fig. 169. If, however, the barrel of the commutator be turned through a right angle, the metal bars B B, B′ B′ are removed from the position in which they touch the springs, and, instead, pins p_1, p_2, p_3, &c., inserted through the barrel at right angles to its axis, now make the following connections as seen in Fig. 170; the broad spring s_1 is connected with s'_2, the spring s_2 with s'_3, &c., so that the coils are connected in series, and a current entering the instrument at the terminal P S leaves it by that marked S, which is connected with s'_{12}, having passed through all the wires in succession.

The terminal S in the symbolical figures 169, 170 is

drawn inside the wires in the position that the needle would occupy in the actual instrument. This is merely to prevent the wire which connects S with the spring s'_{12} having to cross the other wires, and so producing confusion in the figures.

If all the coils were far away from the needle, and all occupied practically the same position relatively to it, *the sensibility of the instrument when all the wires were in series would bear to the sensibility when they were all in parallel, a ratio simply equal to the number of separate wires employed*, quite independently of the way the current divided itself among the wires when they were in parallel. But if to obtain greater sensibility they be wound on the bobbin *close* to the needle, some of them will be, on the whole, nearer to it, and therefore have a greater magnetic effect for the same current than the rest, and it will be necessary, in order that the simple ratio of the sensibilities given above shall exist, that the current shall divide itself equally among the wires when in parallel. For, since the same current passes, of course, through each of the wires when they are in series, it follows that, if matters be so arranged that equal currents also pass through them all when in parallel, any particular coil will produce the *same proportion* of the total magnetic effect whether the commutator be turned to series or to parallel. Now, to insure that when the commutator is turned to parallel, equal currents shall pass through all the coils, it is necessary that they should be of exactly the *same resistance*, and this, therefore, is the condition fufilled in constructing commutator instruments.

In the ammeter seen in Fig. 26, page 76, ten coils of equal resistance have been wound on the bobbin, and hence the ratio of the instrument in the two positions of the commutator are as 1 to 10; the scale, therefore, has two sets of graduations, the angular deflection on the one to be used when the commutator is turned to parallel, corresponding with ten times the number of amperes indicated by the other.

The binding screws P and P S are made so that a thick wire can be attached to them, while S has so small a hole in it that only a fine wire can be put in it; hence the wires used to convey large currents, which come from a dynamo machine, for example, can only be attached to P and P S, and not to S, hence there is no fear of either of them being attached to the wrong binding screw; and, further, as will be seen from Figs. 169, 170, the strong current can only pass through the instrument when the commutator is turned to parallel. Hence, even if it be accidentally turned to series while the instrument is connected with a dynamo, for example, or a large battery of cells, the current will be interrupted instead of being allowed to pass through all the coils in series, which would probably burn them up, or would, at any rate, in consequence of the sensibility of the instrument being increased, say ten times, knock the pointer violently against the stops, which are inserted to limit its motion, and damage it.

222. Calibrating a Commutator Ammeter.—First plan: Turn the commutator to series so that only a *small* current is required to produce a fairly large deflection, place the ammeter in series with a silver voltameter, and calibrate by the method described in § 207, page 395. Second plan: Turn the commutator to series, connect the terminals of a cell of which the E. M. F. is known *accurately*, with the binding screws S and P S (Fig. 26, page 76). Let it be E volts, and let a reading a_1 on the ammeter be obtained. Take out the plug, seen to the left-hand side of Fig. 26, which has the effect of introducing a resistance of one ohm into the circuit when the commutator, as at present, is turned to series. Let the reading on the ammeter be now a_2. The ammeter we will suppose to be so constructed that the angular deflection is proportional to the current (*see* § 35, page 71), and to have been originally direct-reading; but from the permanent magnet having become, say, weakened since the instrument was adjusted, the readings are now

too large, so that K times the reading gives the current in amperes where K is a constant, less than unity, the value of which we have to determine. If r be the resistance of the cell, together with that of the instrument (both of which may be unknown), when all the coils are in series, the current in the first case is

$$\frac{E}{r} \text{ amperes,}$$

and in the second case

$$\frac{E}{r+1} \text{ amperes,}$$

$$\therefore \quad \frac{E}{r} = K\,a_1,$$

$$\text{and} \quad \frac{E}{r+1} = K\,a_2.$$

Eliminating the unknown resistance r, we have

$$K = E\,\frac{a_1 - a_2}{a_1\,a_2}.$$

The soft iron cores F (Fig. 27, page 77) should now be adjusted until, on making the preceding experiment, K is found to equal unity, when the instrument will be properly adjusted for both the "series" and "parallel" scales (Fig. 26, page 76).

Of these two plans of calibration the first is more accurate than the second.

223. Calibrating a Commutator Voltmeter. — A voltmeter is more sensitive when all the coils are in parallel than when they are in series; hence, turn the commutator to parallel, and attach to the proper binding screws the terminals of a cell of known E. M. F.; then, if b be the resistance of the cell, and r that of the volt-

meter when all the coils are in parallel (both of which resistances may be unknown), the P. D. maintained at the terminals of the voltmeter will be

$$\frac{r}{r+b}\,\text{E volts}$$ (*see* § 128, page 224).

Remove the plug, which, in the case of a commutator voltmeter, inserts a resistance equal to that of the instrument when all the coils are in parallel. The P. D. maintained at the terminals of the voltmeter is now

$$\frac{2\,r}{2\,r+b}\,\text{E volts},$$

but the P. D. at the terminals of the coils of the voltmeter, which is sending the current through them, is only

$$\frac{r}{2\,r+b}\,\text{E volts}.$$

Hence, if a_1 and a_2 be the deflections produced in the two cases on the direct-reading voltmeter, and if we suppose that they require multiplying by an unknown constant K to convert them into volts,

$$\frac{r}{r+b}\,\text{E} = \text{K}\,a_1,$$

$$\frac{r}{2\,r+b}\,\text{E} = \text{K}a_2,$$

$\therefore$ eliminating the unknown resistances r and b, we have

$$\text{K} = \text{E}\,\frac{a_1 - a_2}{a_1 a_2},$$

and, as in the case of the ammeter, the soft iron cores F (Fig. 27, page 77) must be adjusted until K equals unity.

224. Best Resistance to give to a Galvanometer. —The considerations given in § 217, page 418, enable us to solve this question, for it was shown there that the magnetic effect produced by the coils of a galvanometer of given shape and size is proportional to G $\sqrt{g}$, where G is the current flowing through the galvanometer, and g the resistance of the coils. Our object, now, is to see what should be the value of g, or, in other words, what gauge of wire should be used in winding the galvanometer, in order that G $\sqrt{g}$ may be a maximum. To solve this problem we must consider what are the conditions as to the rest of the circuit.

1st. Let the circuit be a simple one, consisting of a battery of *fixed* E. M. F. equal to E volts, and resistance of b ohms in series with a *fixed* resistance of r ohms, and the galvanometer, then

$$\mathrm{G}\sqrt{g} = \frac{\mathrm{E}}{b + r + g}\sqrt{g}.$$

The expression on the right-hand side is of the same form as the expression in § 136, page 244, the s of that expression being replaced by $\sqrt{g}$ in the above. Consequently the value of $\sqrt{g}$ that will make the above expression a maximum, can be found by giving fixed numerical values to E and $b + r$, and then drawing a curve similar to that shown in Fig. 93, page 245, having for its abscissæ values of $\sqrt{g}$, and for its ordinates the corresponding values of

$$\frac{\mathrm{E}}{b + r + g}\sqrt{g}.$$

From this curve we should see that g equal to $b + r$ makes the ordinate a maximum, *and hence to obtain the maximum magnetic effect with a galvanometer in a simple circuit, the gauge of wire wound on the coils of the galvanometer should be such as will make the resistance*

of the galvanometer equal to that of the rest of the circuit. When measuring, therefore, a resistance of many megohms by the method described in § 149, page 277, the wire used in winding the galvanometer coils should be as fine as can be conveniently wound on.

2nd. Let the galvanometer be a differential one, the resistance of each of its coils being g ohms; let the resistance of the portion A (Fig. 59, page 149) be fixed and equal to r ohms, and that of the portion B equal to $r + p$ ohms, p having a very *small* fixed value relatively to r, since it is only when balance is nearly established, and the deflection on the galvanometer is very small, that it is of interest to determine what is the best value to give to the galvanometer coils. Let E volts be the E. M. F. of the battery inserted between the points P and Q, and b ohms its resistance.

Then from § 137, page 253, the current through the circuit A is

$$\frac{(r + g + p)\,\mathrm{E}}{b\,(2\,r + 2\,g + p) + (r + g)\,(r + g + p)} \text{ amperes,}$$

and that through the circuit B is

$$\frac{(r + g)\,\mathrm{E}}{b\,(2\,r + 2\,g + p) + (r + g)\,(r + g + p)} \text{ amperes.}$$

Hence the magnetic effect on the needle, which is equal to the difference of the magnetic effects of the two coils, is

$$\frac{p\,\mathrm{E}\,\sqrt{g}}{b\,(2\,r + 2\,g + p) + (r + g)\,(r + g + p)}.$$

Since p is very small compared with r, this is approximately equal to

$$\frac{p\,\mathrm{E}\,\sqrt{g}}{2\,b\,(r + g) + (r + g)^2},$$

which may be written in the form

$$\frac{p\,\mathrm{E}\,\sqrt{g}}{(r+g+b)^2 - b^2}.$$

To determine the value of g that makes this a maximum, we may give fixed numerical values to p, E, r, and b, and draw a curve. Generally, however, b, the battery resistance, is small compared with r and g, hence for all practical purposes we may regard the magnetic effect on the needle of a differential galvanometer as being approximately equal to the simple expression

$$\frac{p\,\mathrm{E}\,\sqrt{g}}{(r+g)^2}.$$

Drawing a curve with values of g for abscissæ, and the corresponding values of the expression for ordinates, we find that the maximum ordinate corresponds with the value of g equal to $\frac{r}{3}$. Hence, *to obtain the maximum magnetic effect with a differential galvanometer, the two coils should be wound with such a gauge of wire that the resistance of each of them equals one-third of the resistance to be tested.*

3rd. Let the galvanometer be that used on a Wheatstone's bridge. This problem is more difficult to solve, but the mode of determining the value of g that makes the magnetic effect a maximum for a fixed small inequality in the ratios of the resistances of the pairs of arms, is given in § 237, page 466, and the result has already been indicated in § 98, page 172.

Example 104.—An ammeter has been constructed so as to measure currents varying between 10 and 50 amperes, and it is desired, without altering anything but the gauge of wire used to wind it, to adapt it to measure, instead, currents varying between 3 and 15 amperes.

What should be the resistance of the instrument after re-winding compared with its present resistance?

Let a be the present resistance in ohms, and x the required resistance after re-winding, then from § 217, page 418,

$$3\sqrt{x} = 10\sqrt{a},$$

$$\therefore \quad \frac{x}{a} = \frac{100}{9}.$$

Answer.—The resistance after re-winding should be 11·11 times the present resistance.

Example 105.—An ammeter so constructed that the deflections are proportional to the currents, and having a resistance of 0·0015 ohm, gives a deflection of 40° when a current of 22 amperes passes through it. What should be the resistance of another ammeter in every way similar to the former one, except as to the gauge of wire employed in winding the coils, so that it may be direct-reading, degrees corresponding with amperes?

We wish that a deflection of 40° shall be produced by 40 amperes instead of by 22 amperes, therefore, if x be the required resistance in ohms,

$$40\sqrt{x} = 22\sqrt{0{\cdot}0015},$$

$$\therefore \quad x = 0{\cdot}0004536.$$

Answer.—453·6 microhms.

Example 106.—When a P. D. of 120 volts is maintained at the terminals of a certain voltmeter having a resistance of 1,235 ohms, the pointer is deflected to the end of the scale. The instrument has to be re-wound with wire of such a resistance that the same deflection shall be produced with a P. D. of 170 volts. What should be the resistance after re-winding?

The current that deflects the pointer to the end of the scale in the first case is $\frac{120}{1{,}235}$ amperes, and after re-winding

it will be $\frac{170}{x}$ amperes, if x be the new resistance of the instrument in ohms, therefore

$$\frac{120}{1{,}235}\sqrt{1{,}235} = \frac{170}{x}\sqrt{x},$$

$$\text{or } \frac{120}{\sqrt{1{,}235}} = \frac{170}{\sqrt{x}}.$$

$$\text{Hence } x = 2{,}477.$$

Answer.—2,477 ohms.

Example 107.—With a voltmeter wound according to a certain law as regards variation of thickness of wire with the radius of the convolution, and having wire 0·012 of an inch thick for the innermost convolution, the pointer deflects over the portion of the dial that is graduated when the P. D. varies from 30 to 150 volts. If the instrument be re-wound according to the same law, but with the innermost layer consisting of wire 0·015 of an inch thick, what will be the range of the instrument?

If r be the resistance of the voltmeter before re-winding, and r' that after re-winding,

$$\frac{r'}{r} = \left(\frac{0{\cdot}012}{0{\cdot}015}\right)^4,$$

and if V be the P. D. in volts required to deflect the pointer to the higher end of the scale after re-winding,

$$\frac{V}{\sqrt{r'}} = \frac{150}{\sqrt{r}},$$

$$\therefore \quad V = \left(\frac{0{\cdot}012}{0{\cdot}015}\right)^2 \times 150$$

$$= 96 \text{ volts.}$$

Similarly the P. D. corresponding with the lower end of the scale will be

$$\left(\frac{0{\cdot}012}{0{\cdot}015}\right)^2 \times 30, \text{ or } 19{\cdot}2 \text{ volts.}$$

Answer.—After re-winding, the range of the voltmeter will be from 19·2 to 96 volts.

Example 108.—When a P. D. that will deflect the pointer to the end of the scale is maintained at the terminals of a voltmeter, it is found that, due to the heating of the coil by the passage of the current, the reading diminishes by 5 per cent. at the end of a considerable time. If the resistance of this voltmeter be 100 ohms, and if, instead of using the voltmeter alone, it be put in series with an outside resistance of 1,000 ohms made of platinoid wire, by how much per cent. will the voltmeter reading fall off on account of the heat produced, when the platinoid wire is of such a thickness that its resistance is only increased by $\frac{1}{10}$th per cent. on a current being kept for a considerable time passing through it strong enough to deflect the voltmeter pointer to the end of the scale?

When the pointer is deflected to the end of the scale, the voltmeter resistance increases by 5 per cent., that is, from 100 to 105 ohms, while the resistance of the outside coil of platinoid wire increases by only $\frac{1}{10}$th per cent., that is, from 1,000 to 1,001 ohms. Therefore the total increase of resistance is from 1,100 to 1,106, or an increase of 0·55 per cent.

Answer.—With the outside resistance the maximum deflection will fall off by 0·55 per cent.

Example 109.—If the voltmeter referred to in example 106 be re-wound so that the pointer is deflected to the end of the scale when a P. D. of 3 volts is maintained between the voltmeter terminals, by how much will the E. M. F. of an accumulator having a resistance of 0·001 of an ohm appear to be lowered if it be measured with the voltmeter so re-wound?

Let x ohms be the resistance of the voltmeter after re-winding, then

$$\frac{3}{\sqrt{x}} = \frac{120}{\sqrt{1235}},$$

$$\therefore x = 0{\cdot}7716.$$

If E volts be the E. M. F. of the accumulator, and V the P. D. between its terminals when they are joined by the voltmeter, then

$$V = \frac{0{\cdot}7716}{0{\cdot}7716 + 0{\cdot}001}E.$$

Answer.—The E. M. F. is diminished by 0·12 per cent.

CHAPTER IX.

POWER AND ITS MEASUREMENT.

225. Power.—"*Power*" is the name given to the *rate of doing work*, and it must be carefully distinguished from the amount of work done, there being the same sort of difference between *power* and *work*, that there is between a *velocity* and a *distance.* When a constant current is flowing through a circuit, at the terminals of which a constant P. D. is maintained, the power given to that circuit and expended in its circuit is constant, and is measured by the *ratio* of the work done in *any* time,

divided by the time in which the work is done. If, however, either the current or the P. D. be fluctuating, the power is also varying in amount, and the rate of doing work at one moment is greater or less than that at a subsequent one. In such a case we mean by the power expended at any moment, not the actual work done in a minute or even in a second, but the following :—Measure the work done in a *very short* time, a portion of which precedes, and the remainder of which follows the instant at which we wish to measure the power, divide the work done in the *very short* time by that time, then the ratio more and more nearly represents the power being expended at the moment in question as we make the very short time shorter and shorter.

Whether, however, the power given electrically to a circuit and expended in it be constant or not, it is very easily measured, for the work done in t minutes in a circuit through which A amperes flow, and at the terminals of which a P. D. of V volts is maintained, is

$$44{\cdot}25 \text{ A V } t \text{ foot pounds}$$

(*see* § 114, page 201), therefore it follows that the power, measured in foot pounds per minute, equals

$$44{\cdot}25 \text{ A V};$$

or, measured in foot pounds per second, equals

$$0{\cdot}7375 \text{ A V};$$

or, measured in horse-power, equals

$$\frac{\text{A V}}{746} \text{ or } 0{\cdot}00134 \text{ AV},$$

A and V being the amperes and volts obtained by a *simultaneous* measurement of the current and P. D.

226. Watt.—A "*watt*" is the power developed in a circuit when one ampere flows through it, and when the P. D. at its terminals is one volt, hence the number of watts developed in any circuit equals the product of the current in amperes flowing through it into the P. D. at its terminals in volts. Therefore

1 watt is the power developed when 44·25 foot pounds are done per minute.

1 watt is the power developed when 0·7375 foot pounds are done per second.

1 watt equals $\frac{1}{746}$th of a horse-power.

Example 110.—If an incandescent lamp give an illumination of 16 candles when 0·7 ampere passes through it, and 110 volts are maintained at its terminals, how many watts are required per candle?

Answer.—4·8.

Example 111.—How many watts must be expended to send a current of 5 amperes through a resistance of 10 ohms?

If V be the P. D. maintained at the terminals of the resistance,

$$V = 5 \times 10,$$

$$\therefore \quad \text{the power} = 5 \times 50.$$

Answer.—250 watts.

This question may also be solved thus:—If a circuit consist simply of a conductor of resistance r ohms, and in which there is no E. M. F., so that the power is simply expended in heating the conductor, the work done in t minutes equals

$$44{\cdot}25\, A^2 r t$$

(*see* § 114, page 201). Hence, in such a case, the number of watts equals

$$A^2 r.$$

Hence, in the present example,

$$\text{the power} = 25 \times 10.$$

Answer.—250 watts.

Example 112.—If each incandescent lamp require 3 watts to make it glow properly, how many such lamps can be illuminated by the expenditure of one horse-power in the circuit?

Answer.—248 lamps if a very little too bright, or 249 if a very little too dull.

Example 113.—If an electric horse-power cost £15 per annum for 5 hours per night, what is the value of one watt-hour?

One watt being the $\frac{1}{746}$th part of a horse-power, and one hour the $\frac{1}{1825}$th part of the time during which the horse-power is supplied, it follows that the value of one watt hour is

$$£\,\frac{15}{746 \times 1825}, \text{ or about the } \tfrac{1}{100}\text{th of a farthing.}$$

227. Wattmeter.—The watts being expended in any circuit can, as we have seen, be ascertained by a *simultaneous* measurement of current and P D., and since we generally desire to know the current and the P. D. as well as the watts, the *simultaneous* measurement of the two former is usually employed to give us the latter. By the employment, however, of a "*wattmeter,*" it is possible to measure the watts directly. This instrument consists of two coils, one of *thick* wire placed in the main circuit, like C (Fig. 71, page 190), and therefore traversed by the whole current, the other of *fine* wire put like *c*, as a shunt to *o*, the part of the circuit in which we wish to ascertain the expenditure of power. Instead, however, of the currents passing through these two coils acting on a needle as they do in the ohmmeter, *they act on one another in a wattmeter* in the same way as do the currents flowing

through a Siemens' dynamometer (§ 199, page 377). In fact, if one of the coils in a Siemens' dynamometer be made of fine wire, and if instead of the two coils being connected in series with one another they be placed as are the coils C and *c* relatively to *o* (Fig. 71, page 190), the instrument becomes a *wattmeter*, for the couple measured by the rotation of the pointer M (Figs. 151, 152, pages 379, 380) will, in this case, measure the product of the whole current passing through *o* into the P. D. maintained at its terminals — that is, the number of watts expended in *o*.

Instruments of this kind have been made by M. Deprez, the author, Sir William Thomson, and the late Sir William Siemens.

The main error in wattmeters is the heating error that occurs in voltmeters (*see* § 212, page 407), and it may be overcome by using the same means as are employed for obviating this defect in voltmeters (*see* § 218, page 421).

228. Distribution of Power in a Circuit.—Of the power developed by a current generator, say P watts, when it is sending a current through an external circuit, one portion, say P_1 watts, is wasted in heating the generator itself, and the remainder, say P_2 watts, is utilised in the external circuit. And in *all* cases

P is equal to the product of the current, in amperes, into the E. M. F. of the generator, in volts.

P_1 is equal to the product of the square of the current, in amperes, into the resistance of the generator, in ohms.

P_2 is equal to the difference between P and P_1.

If the outside circuit consist simply of a conductor having resistance, but containing no voltameters nor electromotors in motion—in fact, nothing that can produce an E. M. F.—the power P developed by the generator will be divided between the generator and the outside

circuit directly as their resistances, so that, if R be the resistance of the generator, and r that of the outside circuit,

$$P_1 = \frac{R}{r + R} P,$$

and $$P_2 = \frac{r}{r + R} P.$$

But if there be an E. M. F. in the outside circuit, then neither of the two last equations will be true, *but in all cases the three relationships given above will hold*—that is, if the E. M. F. of the generator be E volts,

$$\begin{aligned} P &= A\,E, \\ P_1 &= A^2 R, \\ P_2 &= A\,E - A^2 R \\ &= A\,(E - A\,R). \end{aligned}$$

Example 114.—A battery consisting of 4 Daniell's cells in series, and 2 in parallel, is employed in sending a current through a simple conductor, having a resistance of 2 ohms. If the E. M. F. of each cell be 1·07 volts, and the resistance 0·8 ohm, how many watts are developed by the battery, how many are employed in heating the external resistance, and how many are wasted in heating the battery?

$$\text{The current produced} = \frac{4 \times 1{\cdot}07}{2 + \dfrac{4 \times 0{\cdot}8}{2}}$$

$$= 1{\cdot}189 \text{ amperes};$$

$$\left.\begin{array}{l}\text{the power developed by}\\ \text{the battery}\end{array}\right\} = 1{\cdot}189 \times 4{\cdot}28$$

$$= 5{\cdot}089 \text{ watts};$$

$$\left.\begin{array}{l}\text{the watts employed in}\\ \quad\text{heating the external}\\ \quad\text{resistance}\end{array}\right\} = \frac{2}{3\cdot6} \times 5\cdot089$$

$$= 2\cdot827 \text{ watts};$$

$$\left.\begin{array}{l}\text{the watts employed in}\\ \quad\text{heating the battery}\end{array}\right\} = 5\cdot089 - 2\cdot827$$

$$= 2\cdot262 \text{ watts.}$$

Example 115.—What must be the resistance of a current generator so that 95 per cent. of the power produced by it shall be given to the outside circuit, consisting of a simple conductor having a resistance of 35 ohms?

$$\text{We have } \frac{35}{35 + R} = \frac{95}{100},$$

if R be the resistance of the generator;

$$\therefore \quad R = 1\cdot842.$$

Answer.—1·842 ohms.

Example 116.—If a Cardew's voltmeter be used to measure the P. D. of the incandescent lamp referred to in example 110, how many watts are absorbed in the voltmeter, and what is the ratio of the watts absorbed in the voltmeter to the watts used in the lamp?

From Table I., page 154, we see that the resistance at 0° C. of 13 feet of platinum-silver wire 0·0025 of an inch in diameter is

$$\frac{9\cdot603 \times 13 \times 12 \times 4}{\pi \times 0\cdot0025^2} \text{ microhms,}$$

or if we assume that the resistance is increased by 5 per cent. by the elevation of temperature, the resistance will

be 319 ohms Therefore the number of watts absorbed in the voltmeter equals

$$\frac{110^2}{319} \text{ or } 37{\cdot}93 \text{ watts.}$$

The number of watts used in the lamp is 77, therefore about half as many watts are absorbed in the voltmeter as are used in this lamp.

229. Current that Develops the Maximum Useful Power.—The power used in heating a current generator is generally entirely wasted, and, in addition, if allowed to become excessive, will prevent the generator working properly, whereas all the power given to the outside circuit may be utilised with proper arrangements. The problem of ascertaining the current that will develop maximum useful power may be solved either on the assumption that the generator is fixed and the external circuit variable, or on the assumption that it is the external circuit that is fixed, and the generator is the thing to be varied.

1st. Let the generator have *fixed* values of E and R (which will be the case for a battery, a set of accumulators, or a magneto-electric machine running at a constant speed, but not usually for a dynamo machine), then the equation

$$P_2 = A\,(E - A\,R)$$

shows us that we must determine the value of A that makes this expression a maximum, in order to find the current that develops the maximum useful power. To do this, give numerical values to E and R, and plot a curve, having the values of A for abscissæ, and of P_2 for ordinates. Such a curve is shown in Fig. 171, the values of P_2 being calculated on the supposition that E and R are equal to 2 and 3 respectively, and it will be seen that the value of A that makes P_2 a maximum is

$$A = \frac{E}{2\,R},$$

that is, *a current generator having a fixed E. M. F. and resistance gives maximum power to the external circuit, when that circuit is such that the current that flows is half the current that would flow if the generator were short-circuited.*

We do not say that the conductor must have a resistance equal to that of the generator, since, although this

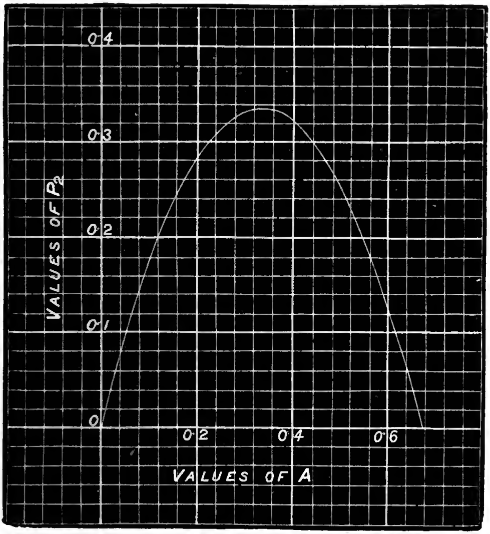

Fig. 171.

will undoubtedly reduce the current to one-half if the outside circuit be a simple conductor, there are other ways of reducing the current to one-half, such as the insertion of an *opposing E. M. F.* equal to half that of the generator.

When A has the value given by the last equation,

$$P = \frac{E^2}{2R},$$

$$P_1 = \frac{E^2}{4\,R},$$

$$\text{and} \quad P_2 = \frac{E^2}{4\,R};$$

therefore, *with a current generator having a fixed E. M. F. and resistance, maximum power will be given to the outside circuit when the power developed by the generator is expended half in the outside circuit and half in heating the generator itself.*

On the other hand, maximum power will be developed by the generator when maximum current flows through it—that is, when the generator is short-circuited.

2nd. Let the external circuit consist of a simple conductor, and let its resistance be *fixed* and equal to r ohms; also let the current generator be a battery consisting of a fixed number of cells N, each having an E. M. F. of e volts, and a resistance of b ohms. Then, since the power developed in the external circuit equals the square of the current into r, and since r is a constant, it follows that the arrangement of cells that will give maximum power to the external circuit, is that which will produce the maximum current. Now, this arrangement we have seen (§ 136, page 245), is that which makes the resistance of the battery equal to the external resistance. Hence, *the arrangement of a given number of cells that gives maximum power to a simple conductor having a fixed resistance, is that which makes the resistance of the battery equal to the resistance of the conductor.*

With this arrangement of cells, it is easy to see that the power developed by the battery will be *twice* that given to the external circuit, one-half being wasted in heating the battery. But this arrangement of cells will not, as a rule, make the power developed by the battery

a maximum. For, as shown in § 136, page 244, the current equals

$$\frac{s\,e}{r + \dfrac{s^2\,b}{\mathrm{N}}} \text{ amperes,}$$

and, therefore, the power developed by the battery equals

$$\frac{s^2\,e^2}{r + \dfrac{s^2\,b}{\mathrm{N}}} \text{ watts,}$$

which equals

$$\frac{e^2}{\dfrac{r}{s^2} + \dfrac{b}{\mathrm{N}}} \text{ watts,}$$

and this obviously has its least practical value when s equals unity, that is, when all the cells are in parallel, and has its largest practical value when s equals N, that is when all the cells are in series. Hence, if the resistance of the outside circuit be less than that of the battery, putting all the cells in series will not only give less power to the outside circuit than if the cells be so arranged that the battery resistance is equal to that of the outside circuit, but it will waste much more power, since the total power produced by the battery will be greater.

230. Efficiency.—The "*efficiency*" of a system consisting of a current generator supplying power to an outside circuit, is *the ratio of the power given to the outside circuit to that developed by the generator.*

From the equations

$$\mathrm{P} = \mathrm{A\,E},$$
$$\mathrm{P}_2 = \mathrm{A\,(E - A\,R)},$$

we see that in all cases the *efficiency* equals

$$\frac{\mathrm{E - A\,R}}{\mathrm{E}};$$

hence, the efficiency will be the greater the larger we make E, and the smaller we make A and R.

From § 229 we see that if we wish a current generator having a given E. M. F. and resistance to develop *as much power as possible* in the outside circuit, we arrange matters so that the current is half that which would be produced if the generator were short-circuited, or, what is the same thing, so that half the power is wasted in the generator itself; hence, when a Grove's battery is employed to produce a *bright* electric light, we regulate the lamp so that the P. D. at its terminals is *half* the E. M. F. of the battery. Whereas we have just seen that if we wish a current generator to give power *economically* to the outside circuit, we employ a generator having a very large E. M. F., and allow it to produce only a small current; hence, in the recent electric transmission of 50 horse-power by M. Deprez from Creil to Paris, a distance of about 37 miles, he employed an E. M. F. of between 6,000 and 7,000 volts, and a current of only 10 amperes.

231. Measuring the Efficiency of an Electric Light. —The "*efficiency of an electric light*" is *the ratio of the illuminating power of the light to the watts supplied to it.* To measure the illuminating power we use a "*photometer,*" the simplest, and at the same time one of the most accurate, being that designed by Rumford. A form of "*Rumford's photometer*" is seen in Fig. 172, E being the electric light, an incandescent lamp, for example, held in a convenient adjustable holder H, and C a "*standard candle,*" which is a special form of candle made so as to burn 120 grains of spermaceti wax per hour.* The lamp is placed at a convenient distance *e* from the screen S, which is covered with a sheet of white blotting

* For rough experiments on illuminating power, *No.* 8 *sperm candles,* costing 11d. per pound, may be used satisfactorily instead of standard candles costing 2s. 9d. per pound, since experiments show the No. 8 sperm candles do not differ much more from one another, or from a standard candle, in illuminating power, than standard candles are said to differ among themselves.

paper, and the candle is moved backwards and forwards along the graduated arm G G, until, by trial, a position for it is found, at a distance, c, say, from the screen, such that the two shadows cast by a vertical rod of blackened wood R, about the thickness of an ordinary pencil, and fixed at about two inches from the screen, appear to

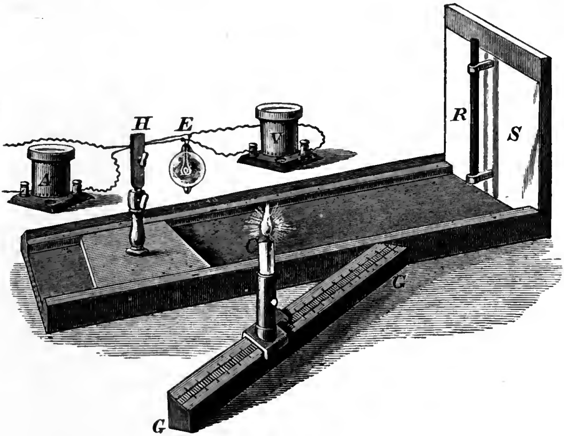

Fig. 172.

be equally dark. Under these circumstances, as the portion of the screen not in shadow is illuminated by *both sources* of light, whereas the two parts in shadow are each illuminated by *only one*, and as the screen S and the rod R are so placed that lines drawn through the rod and through each of the lights make equal angles with the screen, it follows that, when the shadows are equally dark, the quantities of light falling on a square inch of the screen, due to each of the lights, are equal to one another, hence

$$\frac{\text{the illuminating power of the electric light}}{\text{,, \quad ,, \quad ,, \quad ,, \quad standard candle}} = \frac{e^2}{c^2}.$$

The correct position of the candle which produces equality in the darkness of the shadows can be best detected not by gradually moving the candle continuously towards or away from the screen, but by trying to find a position such that, if the candle be put a little nearer, one of the shadows becomes distinctly too dark, whereas if it be put a little farther away, that one of the shadows becomes distinctly too light.

The current passing through the lamp is measured by an ammeter A, and the P. D. maintained at the lamp terminals by the voltmeter V, the product of the amperes and the volts giving P, the watts furnished to the lamp. Hence, the efficiency of the lamp equals

$$\frac{e^2}{c^2 P}.$$

232. Dispersion Photometer.—In the preceding section we have spoken of one type of electric lamp—the incandescent one. This consists of a hermetically sealed glass bulb (*see* note, § 10, page 20) containing usually a very fine filament of carbon, which becomes luminous when a suitable current passes through it, but does not burn away as there is a *very perfect vacuum* inside the glass bulb. But there is a much more powerful electric light—the arc light, in which the light is produced by a current passing between two pieces of carbon slightly separated from one another, the resistance of the heated air between the carbons taking the place of that of the carbon filament in the incandescent lamp. As an arc light has often an illuminating power of several thousand candles, it would have to be put many feet away from the screen of the photometer in order that the light cast by it on a given area should be equal to that produced by the standard candle. To avoid the inconvenience of having to put an arc light so far away from the screen, the "*dispersion photometer*" shown in Fig. 173 was devised by the author. Instead of the light from the

electric lamp being allowed to fall directly on the screen, it is allowed to pass through a double concave lens L,* which *disperses* the light, so that the screen is illuminated by only a *small fraction* of the light that would come to it from the powerful electric lamp if the lens L

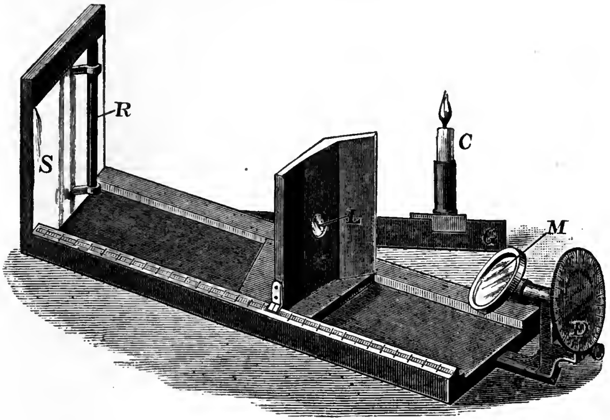

Fig. 173.

were removed. Let the electric light and the lens be at distances e and l respectively from the screen, and, when the standard candle is at a distance c from the screen, let the shadows be equally dark, then the light from the electric lamp which would have illuminated an area A (Fig. 174) is now dispersed so as to illuminate a much larger area A′, so that

$$\frac{\text{the illuminating power of the electric light}}{\text{,, \quad ,, \quad ,, \quad ,, \quad standard candle}} = \frac{A'}{A} \cdot \frac{e^2}{c^2}.$$

To find the ratio of A′ to A, let a be the area of

* Dr. J. Hopkinson uses a double convex lens, and forms a real image of the electric arc between the lens and the screen.

the double concave lens filled by the pencil of light which would have illuminated the area A, and let the

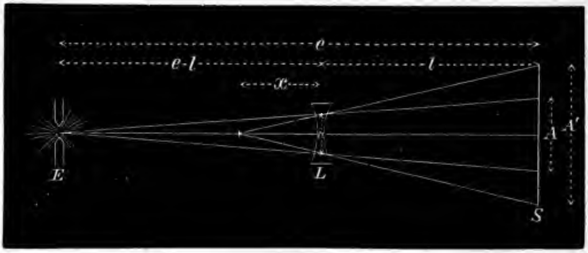

Fig. 174.

light after dispersion appear to come from a distance x behind the lens, then

$$\frac{\mathrm{A}}{a} = \frac{e^2}{(e-l)^2},$$

$$\frac{\mathrm{A}'}{a} = \frac{(l+x)^2}{x^2},$$

and $$x = \frac{f(e-l)}{f+e-l};$$

where f is the "*focal length*" of the lens—that is, the distance from the lens of the point from which light, after passing through the lens, would appear to come if the source of light were very far away, like the sun. Hence, eliminating a and x from the preceding three equations, we have

$$\frac{\text{the illuminating power of the electric light}}{\text{,, ,, ,, ,, standard candle}} = \left\{\frac{l(e-l)+fe}{cf}\right\}^2.$$

A great difficulty in comparing an electric light with a candle arises from the difference in colour of these two sources of light, an *arc light being much bluer than a candle.* To partially overcome this difficulty, two distinct comparisons of the electric light with the candle should be made when the screen is looked at successively through *green* and *red* glass. Pieces of what are known in the trade as *signal green* and *ruby red* answer very well for this purpose, but they should be selected so that a bright light is hardly visible when looked at through the two pieces placed one over the other, as then the green glass allows practically no red, and the red glass practically no green light to pass. The two comparisons made with green and red glass will give very different results for the illuminating power of a powerful arc light in terms of that of a candle, because the ratio of green to red rays in the former is so much larger than in the latter.

It is important to be able to measure the illuminating power of an arc lamp, not merely in a horizontal plane, but for rays making various angles with the horizontal. This can be conveniently done by placing the arc lamp so that its rays come in any desired direction to the mirror M (Fig. 173), and turning the mirror, with the graduated disc D attached to it, until the rays pass through the concave lens L, and fall properly on to the screen S. The angle that the beam of light under observation now makes with the horizontal plane at the electric light, can be read off directly by the position of the graduated disc. By causing the mirror M to turn about an axis which makes *an angle of* 45° *with its plane,* the light reflected from it always makes *the same angle with its surface* when it passes after reflection through the lens, hence the portion of the light absorbed by the mirror is *constant,* and may be determined once for all experimentally. So also the portion of the light absorbed by the lens will be constant, and may be determined experimentally, and both these fractions can easily be allowed for in any

measurements made of the illuminating power of an arc lamp.

233. Efficiency and Life of Incandescent Lamps.—The heat produced per second in a conductor is proportional to the square of the current passing through it (*see* § 113, page 198), and is therefore proportional to the product of the current into the P. D. maintained at the ends of the conductor—that is, to the number of watts given to it. The temperature of the conductor will depend on the heat produced in it per second, and on its facility for cooling (*see* § 111, page 195). But experiments show that the light emitted by a body increases very much more rapidly than the heat given to it per second; for example, the heat given to a kettle of boiling water per second may be considerable, but is not sufficient to cause the kettle to emit any light at all, whereas if the metal of the kettle be made a good deal hotter, it will begin to glow and commence emitting light, and when it becomes white-hot, the light emitted will be considerable. So it is found that the light emitted by an electric lamp increases *much more rapidly* than the watts given to it—that is, the *efficiency increases with the power* supplied to it. As far then as the cost of producing the power is concerned, it is more economical to cause the carbon of an electric lamp to have an intensely white-hot temperature than merely to allow it to glow at a dull red heat. But, on the other hand, the number of hours during which an incandescent lamp can be used before the carbon filament breaks, depends on the temperature of the filament. If the temperature be kept *always* low enough, the filament will last for several thousands of hours, the lamp emitting light all the time, whereas if the temperature be too high, the life will be reduced to a few hundred, or less number of hours. Hence it is an important question to decide how bright we should make the filament of an incandescent lamp when in use, or, in other words, what P. D. we should maintain at its terminals. This question is one that must be solved for each particular case, depending on

the efficiency and life of the lamp for different P. Ds., on the cost of a new lamp, and on the cost of power at the particular place where the lamp is used.*

Example 117.—If power cost £15 per horse-power per annum supplied for 5 hours per night, and if a new incandescent lamp cost 3s., further, if when used so as to require only $2\frac{1}{2}$ watts per candle it lasts for 500 hours, whereas when used with a lower P. D. at its terminals it requires $3\frac{1}{4}$ watts per candle, but lasts 1,500 hours, determine which is the more economical of the two modes of using the lamp?

First case :—

Cost for *power* per candle per hour $= \dfrac{15 \times 20 \times 2\frac{1}{2}}{746 \times 5 \times 365}$ shillings,

Cost for *lamp renewals* per candle per hour $= \dfrac{3}{500}$ shillings,

Total cost per candle per hour $= 0{\cdot}00655$ shillings.

Second case :—

Cost for *power* per candle per hour $= \dfrac{15 \times 20 \times 3\frac{1}{4}}{746 \times 5 \times 365}$ shillings,

Cost for *lamp renewals* per candle per hour $= \dfrac{3}{1500}$ shillings,

Total cost per candle per hour $= 0{\cdot}00272$ shillings,

therefore using the smaller P. D., and the larger number of watts per candle, is much the more economical arrangement.

Example 118.—An arc lamp through which 8 amperes are passing, and at the terminals of which 50 volts

* *See* "The Most Economical Potential Difference to employ with Incandescent Lamps." *Phil. Mag.*, April, 1885.

are maintained, produces 750 candles, while an incandescent lamp through which 0·6 of an ampere is passing, and at the terminals of which 70 volts are maintained, produces 17 candles. Compare the efficiency in the two cases.

For the arc lamp the efficiency is $\frac{750}{8 \times 50}$ or about 1·9 candle per watt.

For the incandescent lamp the efficiency is $\frac{17}{0{\cdot}6 \times 70}$ or about 0·45 candle per watt,

therefore the efficiency of the arc lamp is more than four times that of the incandescent.

Example 119.—A battery having a resistance of 4 ohms, and an E. M. F. of 30 volts, is sending a current through an outside circuit consisting of leading wires having a resistance of 1 ohm and 4 incandescent lamps arranged in parallel, and at the terminals of which 12 volts are maintained. If each lamp produces $3\frac{1}{2}$ candles, calculate the efficiency of the arrangement.

The current $= \frac{30 - 12}{4 + 1}$

$= 3{\cdot}6$ amperes.

The power produced by the battery $= 3{\cdot}6 \times 30$

$= 108$ watts.

The power wasted in the battery $= (3{\cdot}6)^2 \times 4$

$= 51{\cdot}84$ watts.

The power wasted in the leading wires $= (3{\cdot}6)^2 \times 1$

$= 12{\cdot}96$ watts.

The power given to the 4 lamps in parallel $= 3 \cdot 6 \times 12$

$= 43 \cdot 2$ watts.

Therefore, of the 108 watts produced by the battery, 64·8 watts, or 60 per cent., are spent uselessly in heating the battery and leading wires; 43·2 watts, or 40 per cent. of the total power, are given to the lamps; and, as $4 \times 3\frac{1}{2}$ or 14 candles' illumination is produced, the efficiency of the lamps is 0·324 candles per watt.

When a power of 1 watt is being developed *the work done per second* is sometimes called a "*joule.*" Hence 1 *joule* equals 0·7375 foot pounds. And

1 watt-second = 1 joule.
1 watt-minute = 60 joules.
1 horse-power hour = 1,980,000 foot pounds.
" " " = 2,685,600 joules.

Appendix to the Section on Shunts.

234. Kirchhoff's First Law—235. Kirchhoff's Second Law—236. Current through the Galvanometer of a Wheatstone's Bridge—237. Best Resistance for the Galvanometer with a Wheatstone's Bridge—238. Best Arrangement of the Battery and Galvanometer with a Wheatstone's Bridge—239. Measuring a Resistance containing an E. M. F.

In the case of even a somewhat complicated circuit like that shown in Fig. 175, there is no difficulty in calculating the current flowing in every part, if we use the principles developed in § 103, page 177, and in § 137, page 253, to solve the problem step by step. Let capital letters stand for the currents flowing in the several branches, and small letters for the resistances of these

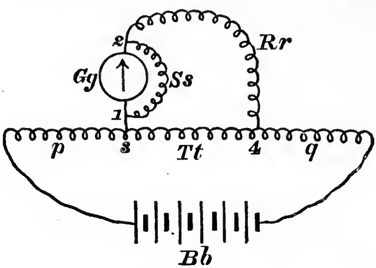

Fig. 175.

branches; let x be the resistance between the points 1 and 2, and y that between 3 and 4, and let E be the E. M. F. of the battery; then

$$x = \frac{g\,s}{g+s},$$

$$y = \frac{t\,(x+r)}{t+x+r},$$

$$B = \frac{E}{b + p + q + y},$$

$$T = \frac{x + r}{x + r + t} B,$$

$$R = \frac{t}{x + r + t} B,$$

$$G = \frac{s}{s + g} R,$$

$$S = \frac{g}{s + g} R.$$

Hence, the currents B, T, R, G, and S are expressed in terms of E, and the various resistances b, p, q, t, r, g, and s.

But if we try to do the same thing for the circuit shown in Fig. 176, and which at first sight appears

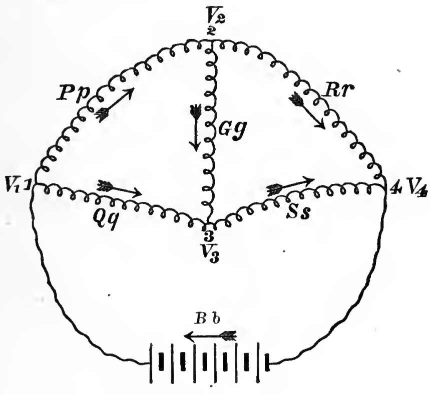

Fig. 176.

equally simple, it will be seen that the method previously employed is inapplicable. We may say that

$$B = \frac{E}{h + \text{resistance between 1 and 4}},$$

but how are we to express the resistance between the points 1 and 4 in terms of p, q, r, s, and g. To do this we require to use what are known as "*Kirchhoff's first and second laws.*"

234. Kirchhoff's First Law.—This is very simple, and merely expresses the fact that if there is one current B (Fig. 176) that flows towards a point 1, and two currents P and Q that flow away from this point

$$B = P + Q \quad . \quad . \quad . \quad . \quad (1)$$

Similarly,

$$P = G + R \quad . \quad . \quad . \quad . \quad (2)$$
$$S = G + Q \quad . \quad . \quad . \quad . \quad (3)$$
$$B = R + S$$

These equations are not, however, all independent, as any one could be obtained from the other three.

Kirchhoff's first law is sometimes stated thus :—*The algebraical sum of all the currents meeting at a point is nought*, the "*algebraical*" sum meaning that the currents that flow away from the point must be taken with a *negative* sign if those flowing towards it be taken with a *positive sign*, or *vice versâ.*

235. Kirchhoff's Second Law.—*In any closed circuit the algebraical sum of the products of the currents into the resistances equals the E. M. F. in the circuit.* In using this law the currents are to be taken with a positive or a negative sign according as they flow in the same or in opposite directions round a circuit; and the E. M. F. is to be taken with a positive or a negative sign according as it assists or opposes the currents that are arbitrarily taken as positive.

Let V_1, V_2, V_3, &c., be the P. Ds. at the points 1, 2, 3, &c., then from Ohm's law (*see* § 74, page 130) it follows that

$$Pp = V_1 - V_2,$$
$$Gg = V_2 - V_3,$$
$$Qq = V_1 - V_3,$$
$$\therefore \quad Pp + Gg - Qq = 0 \quad . \quad . \quad . \quad . \quad (4)$$

Similarly, $\quad \mathrm{R}\,r = \mathrm{V}_2 - \mathrm{V}_4,$

$$\mathrm{G}\,g = \mathrm{V}_2 - \mathrm{V}_3,$$

$$\mathrm{S}\,s = \mathrm{V}_3 - \mathrm{V}_4,$$

$$\therefore \quad \mathrm{G}\,g + \mathrm{S}\,s - \mathrm{R}\,r = 0 \quad . \quad . \quad . \quad . \quad (5)$$

As to the circuit containing the battery and the points 1, 3, 4,

$$\mathrm{Q}\,q = \mathrm{V}_1 - \mathrm{V}_3,$$

$$\mathrm{S}\,s = \mathrm{V}_3 - \mathrm{V}_4,$$

$$\mathrm{B}\,b = \mathrm{E} - (\mathrm{V}_1 - \mathrm{V}_4),$$ (*see* § 116, [page 205).

$$\therefore \quad \mathrm{Q}\,q + \mathrm{S}\,s + \mathrm{B}\,b = \mathrm{E} \quad . \quad . \quad . \quad . \quad (6)$$

Three independent equations (1), (2), (3), therefore, may be obtained by using Kirchhoff's first law, and three more (4), (5), (6), by using his second law, or six equations altogether. From these the six currents B, P, Q, R, S, and G can be found in terms of E, the E. M. F. of the battery, and the six resistances b, p, q, r, s, and g.

236. Current through the Galvanometer of a Wheatstone's Bridge.—The current of most interest to us is G, because this is the current that will pass through the galvanometer in a Wheatstone's bridge when balance is not obtained. The value of G* so obtained is

$$\frac{\mathrm{E}\,(q\,r - p\,s)}{b\,\{g(p+q+r+s) + (p+q)(r+s)\} + g\,(p+r)(q+s) + r\,(p+q)\,(q+s) - q\,(q\,r - p\,s)}$$

And this we see equals nought when

$$q\,r = p\,s,$$

* A very convenient method based on Kirchhoff's laws, but involving the use of determinants for solving such questions, was suggested by the late Professor Clerk Maxwell, and has been recently extended by Dr. Fleming in the Proc. Phys. Soc., vol. vii., part 3, page 215.

$$i.e. \quad \text{when } \frac{p}{q} = \frac{r}{s}$$

This result for the law of the Wheatstone's bridge was much more simply obtained in § 97, page 167, but the method there employed for arriving at the connection that existed between the resistances when no current was passing through the galvanometer, gave us no indication as to what the current would be if this connection between the resistances were not fulfilled. If $q\,r$ equals $p\,s$ there will be no current through the galvanometer, whatever be its resistance, or however it be constructed; but as our only method of insuring that qr shall be equal to ps, is by varying one or more of the resistances until no visible deflection is observed on the galvanometer, it is important to construct the galvanometer so that the needle will deflect even when there is a very small difference between $q\,r$ and $p\,s$. The proper wire to wind on the galvanometer bobbins may be calculated from the formula given in § 98, page 171; and that formula, as will be seen in the next section, can be obtained by multiplying the $\sqrt{g}$ (which we know from § 217, page 418, is proportional to the sensibility of the galvanometer) by the value of G given above when $q\,r - p\,s$ has a fixed small value, and seeing what is the relationship between g and p, q, r, and s that makes $G\sqrt{g}$ a maximum.

237. Best Resistance for the Galvanometer with a Wheatstone's Bridge.—If G be the current passing through the galvanometer of a Wheatstone's bridge, and g be its resistance, the magnetic effect, which is proportional to $G\sqrt{g}$ is, from the last section, proportional to

$$\frac{E\,(q\,r - p\,s)\,\sqrt{g}}{\begin{array}{c} b\,(p+q)\,(r+s) + r\,(p+q)\,(q+s) - q\,(q\,r - p\,s) \\ + \left\{ b\,(p+q+r+s) + (p+r)\,(q+s) \right\} \end{array}}.$$

Now this expression is of the form

$$\frac{a\,x}{b + c\,x^2},$$

where a, b, and c are constants, and x is the variable, and such an expression we saw in § 136, page 245, is a maximum when

$$x^2 = \frac{b}{c}.$$

Therefore it follows that $G\sqrt{g}$ will be a maximum when

$$g = \frac{b(p+q)(r+s) + r(p+q)(q+s) - q(qr - ps)}{b(p+q+r+s) + (p+r)(q+s)}.$$

But we want to find the best value to give to g when balance is nearly established, that is, when $q\,r$ is nearly equal to $p\,s$, since that is when it is most important to have the galvanometer sensitive, hence we may assume that $q\,r$ equals $p\,s$ in the preceding expression for g. Under these circumstances we find that

$$g = \frac{b\,q(r+s)^2 + q\,r(r+s)(q+s)}{b(r+s)(q+s) + r(q+s)^2}$$

$$= q \cdot \frac{r+s}{q+s} \cdot \frac{b(r+s) + r(q+s)}{b(r+s) + r(q+s)}.$$

And this when $q\,r$ equals $p\,s$ is the same as

$$\frac{(p+q)(r+s)}{p+q+r+s},$$

which is therefore the best resistance to give to the galvanometer.

238. Best Arrangement of the Battery and Galvanometer with a Wheatstone's Bridge.—We have seen, in § 97, page 167, that when balance is obtained the battery and galvanometer may be interchanged without disturbing the balance. But when balance has not been obtained a greater current will pass through the galvanometer when it and the battery are arranged one way than will pass when the galvanometer and the battery are interchanged.

In other words, one arrangement is more sensitive than the other, and the object of the following is to ascertain which is the more sensitive arrangement.

As we are dealing with a definite galvanometer of fixed resistance, we are merely concerned with the current, and need not consider the magnetic effect. Let G_1 be the current passing through the galvanometer when it and the battery are placed as shown in Fig. 176, page 463, and let G_2 be the current when the galvanometer and battery are interchanged, then

$$G_2 = \frac{E(qr - ps)}{\begin{array}{c} b\{g(p+q+r+s) + (p+r)(q+s)\} \\ +g(p+q)(r+s) + p(q+s)(r+s) - s(ps - qr) \end{array}}$$

and the value of G_1 is given in § 236, page 465.

Hence

$$G_1 - G_2 = \frac{E(qr - ps)}{D_1 \times D_2}(g - b)\left\{\begin{array}{c} (p+q)(r+s) \\ -(p+r)(q+s) \end{array}\right\}$$

where D_1 and D_2 stand respectively for the denominators of G_1 and G_2. Simplifying, we have

$$G_1 - G_2 = \frac{E(qr - ps)}{D_1 \times D_2}(g - b)(p - s)(r - q).$$

1st.—*Let g be greater than b.* Then $G_1 - G_2$ will be positive, that is, the first arrangement will be more sensitive than the second, when p and r are respectively both greater or both less than s and q. Therefore *the galvanometer should connect the junction of the two greater resistances with the junction of the two less.*

2nd. *Let b be greater than g.* Then $G_1 - G_2$ will be positive, or the first arrangement will be more sensitive than the second, when p is greater than s, and r is less than q, or when p is less than s, and r is greater than q. Therefore *the battery should connect the junction of the two greater resistances with the junction of the two less.*

239. Measuring a Resistance containing an E. M. F. —If in one of the branches 3 4 of the Wheatstone's bridge (Fig. 177) there be an opposing E. M. F. of e volts, Kirch-

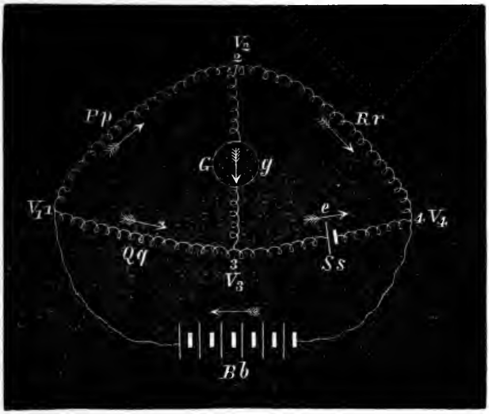

Fig. 177.

hoff's second law tells us that equation (5) (§ 235, page 465) becomes

$$Gg + Ss - Rr = -e,$$

and equation (6) becomes

$$Qq + Ss + Bb = E - e;$$

the other four equations remaining as before. Using these equations we now find that

$$G = \frac{E(qr - ps) - e\{b(p+q) + q(p+r)\}}{b\{g(p+q+r+s) + (p+q)(r+s)\} + g(p+r)(q+s) + r(p+q)(q+s) - q(qr-ps)}.$$

This current is obviously the difference of two currents, the one the current that would exist if e were nought—

that is, if there were no E. M. F. in the branch 3 4 of the bridge—the other the current that would exist if E were nought—that is, if the testing battery had no E. M. F. This is expressed by saying that *each E. M. F. acts independently*, a result that is *universally* true.

$$\text{If} \qquad q\,r = p\,s,$$

the expression given above for G reduces to simply

$$-\frac{p\,e}{g\,(p + r) + r\,(p + q)},$$

the negative sign meaning that the current through the galvanometer is now in the opposite direction to that shown in the figure. This current is independent of E and of b—that is to say, if the resistances be so adjusted that $q\,r$ equals $p\,s$, *no change will be made in the current through the galvanometer by altering the value of E, or of b, or of both.* This leads us to a very simple test for measuring a resistance s containing an E. M. F., and which is :—*Adjust the resistances p, q, and r until on making and breaking the circuit containing the testing battery, no change is produced in the galvanometer deflection.* Or, the testing battery may be dispensed with altogether, and a wire of any convenient resistance substituted for it, the E. M. F. in the branch 3 4 being the only E. M. F. employed. In that case *the resistances p, q, and r must be adjusted until, on connecting and disconnecting this wire, no change is produced in the galvanometer deflection.* Then

$$s = \frac{q\,r}{p}.$$

This latter is known as "*Mance's test*" for measuring the resistance of a conductor containing an E. M. F. such as a battery, a long telegraph line in which an E. M. F. is produced by atmospheric causes, or by there being a P. D. between the ground at the two ends of the line, &c.

Although connecting and disconnecting the wire that is used to join the points 1 and 4 produces no change in the current passing through the galvanometer when s equals $\frac{q\,r}{p}$, the current sent through the circuit by e is increased on connecting the auxiliary wire used to join the points 1 and 4. Hence, this test can only be employed when e, the E. M. F. in the branch 3 4, is not altered by varying the current sent by this E. M. F. through the circuit.

Example 120.—Prove directly the formula employed in *Mance's test* for measuring the resistance of a conductor containing an E. M. F.

From Fig 178, which shows the distribution of cur-

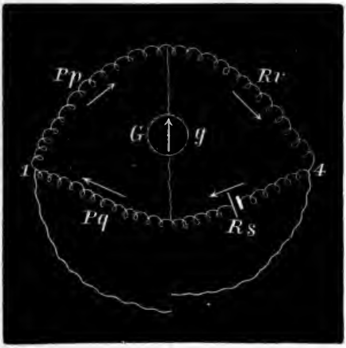

Fig. 178.

rents when the wire used to join the points 1 and 4 is disconnected, we have

$$\mathrm{R} = \mathrm{P} + \mathrm{G} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mathrm{P}\,(p + q) - \mathrm{G}\,g = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\mathrm{R}\,(r + s) + \mathrm{G}\,g = e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When this wire is joined, all the currents will be altered except G. Let them now be P′, Q′, R′, S′ (Fig. 179), then

$$S' = Q' + G \quad . \quad . \quad . \quad . \quad (4)$$

$$R' = P' + G \quad . \quad . \quad . \quad . \quad (5)$$

$$P'p + Q'q - Gg = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$R'r + S's + Gg = e \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

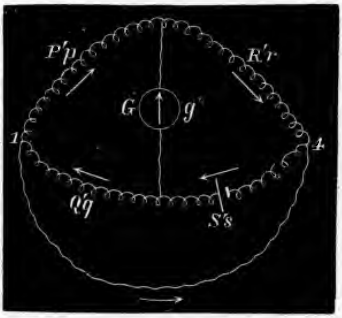

Fig. 179.

From equations (2) and (6) we have

$$(P - P')p = (Q' - P)q \quad . \quad . \quad . \quad (8)$$

and from equations (3) and (7)

$$(R - R')r = (S' - R)s \quad . \quad . \quad . \quad (9)$$

From equations (1) and (5) it follows that

$$P - P' = R - R',$$

and from equations (1) and (4) that

$$Q' - P = S' - R,$$

therefore substituting these values in equations (8) and (9) we have

$$\frac{p}{r}=\frac{q}{s}.$$

Example 121.—A battery having an E. M. F. of $3\frac{1}{2}$ volts and a resistance of $2\frac{1}{4}$ ohms, is employed in sending a current through a circuit consisting of a resistance of 1,234 ohms in series with a galvanometer of 52 ohms'

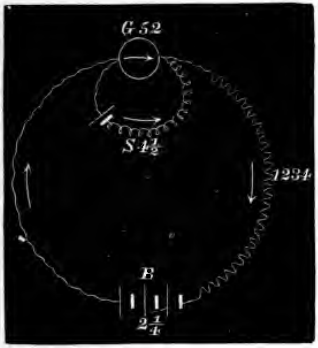

Fig. 180.

resistance, shunted with a shunt of $4\frac{1}{2}$ ohms' resistance, containing an opposing E. M. F of 1 volt. What is the current flowing through the galvanometer?

The arrangement of the circuit is shown in Fig. 180, and if B, G, and S be the currents in amperes flowing respectively through the battery, the galvanometer, and the shunt, we have by Kirchhoff's first and second laws,

$$\mathrm{B} = \mathrm{S} + \mathrm{G},$$

$$(2\tfrac{1}{4} + 1{,}234)\,\mathrm{B} + 52\,\mathrm{G} = 3\tfrac{1}{2},$$

$$52\,\mathrm{G} - 4\tfrac{1}{2}\,\mathrm{S} = 1.$$

Eliminating B and S from these three equations, we find

$$G = 0{\cdot}01786.$$

Answer.—0·01786 amperes.

Example 122.—What E. M. F. must be inserted in the shunt in the last question so that no current shall pass through the shunt?

Let e be this E. M. F. in volts, then we must find the value of e that makes S equal to nought in the following equations:—

$$B = S + G,$$

$$1{,}236\tfrac{1}{4}\,B + 52\,G = 3\tfrac{1}{2},$$

$$52\,G - 4\tfrac{1}{2}\,S = e.$$

Putting S equal to nought we have

$$1{,}288\tfrac{1}{4}\,G = 3\tfrac{1}{2},$$

$$52\,G = e,$$

$$\therefore\ e = \frac{52 \times 3\frac{1}{2}}{1{,}288\frac{1}{4}} \text{ volts.}$$

Answer.—0·1413 volts.

This question may be solved differently thus:—If no current passes through the shunt, the E. M. F. must be equal and opposite to the P. D. that would be produced between the terminals of the galvanometer if there were no shunt circuit at all, and this we know, from § 115, page 204, is equal to the E. M. F. of the battery multiplied by the ratio of the resistance of the galvanometer to that of the whole of the circuit, or

$$\frac{52}{1{,}288\frac{1}{4}} \times 3\tfrac{1}{2} \text{ volts,}$$

the same expression that is given above for e.

Example 123.—Does the presence of the shunt in example 121 increase or diminish the current that would pass through the galvanometer if there were no shunt circuit, and by what amount is the galvanometer current varied?

If there were no shunt the current through the galvanometer would be

$$\frac{3\frac{1}{2}}{1,288\frac{1}{4}}, \text{ or } 0{\cdot}002717 \text{ amperes.}$$

We see, therefore, that the shunt in this particular case, in consequence of the E. M. F. in it, actually increases the galvanometer current by

$$0{\cdot}01786 - 0{\cdot}002717, \text{ or 15-thousandths of an ampere.}$$

Specimens of Instructions for Experiments.

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PHYSICAL DEPARTMENT.

To compare the amount of CHEMICAL DECOMPOSITION produced per second by a current with the corresponding DEFLECTION of a TANGENT GALVANOMETER.

PRELIMINARY.—The current passing through the voltameter and galvanometer can be varied by altering the resistance in circuit. The value of the resistance need not be known.

When the clip is firmly fixed on the small piece of indiarubber tube, the gas evolved by passing a current through the voltameter cannot escape, and so the pressure inside becomes greater than the atmospheric pressure, and forces the liquid up the glass tube. The rate at which the liquid rises in the tube is a measure of the amount of gas evolved per second. Releasing the clip allows the gas to escape. The volume of the tube between the two marks 0 and 7 is 2·284 cubic centimetres.

EXPERIMENTS.—(1.) Adjust the needle of the galvanometer to zero by slightly turning the instrument.

(2.) Send a current through the apparatus by pressing the key, and open the clip so that the gas escapes. Keep the key pressed for a few minutes, until the liquid becomes thoroughly saturated with gas. Now close the

clip, and note the interval of time it takes for the liquid to rise from the lowest to the highest mark on the tube; also note the steady deflection of the galvanometer.*

(3.) Vary the current by altering the resistance in circuit, and repeat the observation mentioned in (2).

(4.) Repeat (3) with as many different strengths of currents as possible.

(5.) Tabulate your results in a convenient form.

(6.) Draw a curve having for abscissæ the quantity of gas evolved per second, and for ordinates the tangents of the corresponding deflections of the galvanometer.

DEDUCTIONS.—Write out clearly all the inferences which can be drawn from this experiment, assuming that the strengths of currents are proportional to the amount of chemical decomposition which they produce per second.

Determine the constant a of the galvanometer such that

$$A = a \tan. d,$$

where A is the current in amperes and d the deflection it produces, having given that

1 ampere liberates 0·1738 c.c. of mixed gas per second, when measured at 0° C. and 760 m.m. pressure.

State clearly the corrections which would have to be applied in making accurate determinations of current strength by this method.

* It is desirable to make two or three determinations with each particular current, and take the mean.

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PHYSICAL DEPARTMENT.

EXPERIMENTS on SHUNTS.

PRELIMINARY.—When the current to be measured in any circuit is too large for the galvanometer available to measure it, only a known fraction of the current is passed through the galvanometer, the remainder being passed from one terminal of the galvanometer to the other through a "*by-pass*" or "*shunt*" circuit. As, however, the introduction of this shunt circuit lessens the resistance between the terminals of the galvanometer, and therefore the *total* resistance used in the experiment, the *main* current is increased. Thus it may happen that the effect of shunting a galvanometer is to scarcely diminish the current passing through it.

The following experiments have been devised to make the student practically acquainted with the effect of shunting a galvanometer, and the manner in which the effect of a given shunt depends on the resistance in the other parts of the circuit.

The resistance of the galvanometer circuit unshunted is about 200 ohms.

EXPERIMENTS.—(1.) Using one cell of the battery, and with no resistance in the main circuit excepting that of galvanometer, battery, and connecting wires, send a current through the unshunted galvanometer and note the deflection d produced.

(2.) Place various resistances from the highest available down to 0 in the shunt circuit, and note all the corresponding deflections.

(3.) Tabulate your results in some convenient form.

(4.) Plot a curve having for abscissæ the resistances in the shunt circuit, and for ordinates the corresponding currents passing through the galvanometer.

(5.) Join up two cells of the battery, and introduce such a resistance into the main circuit as will give the same deflection d as was obtained in (1) when the galvanometer was unshunted.

(6.) Repeat (2), (3), (4), drawing the curve on the same sheet of paper.

(7.) Repeat (5) and (6), using four and six cells respectively.

DEDUCTIONS.—Write out a clear account of the inferences which you can draw from these experiments.

Also determine algebraically the general equation to, and character of, the curves obtained in these experiments, and show how the results obtained experimentally could be deduced from this equation. Prove that the curves have a common asymptote, and find the limits between which the other asymptotes lie.

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PHYSICAL DEPARTMENT.

To CALIBRATE an AMMETER by the CALORIMETRIC METHOD.

PRELIMINARY.—The calorimeter provided consists of a thin copper vessel supported within an air space, and screened from external radiation by a large water jacket. A coil of German silver wire is inserted in the calorimeter, and surrounds the bulb of a delicate thermometer. This thermometer serves to show the rise of temperature of the water and calorimeter caused by passing a current through the wire. Another thermometer indicates the temperature of the large water jacket in which it is immersed.

EXPERIMENTS.—(1.) Carefully dry and weigh the small copper calorimeter, the *approximate* weight of which is 24·8 grammes.

(2.) Partly fill the calorimeter with distilled water by means of the pipette provided, and determine the weight of the water added.

(3.) Replace the calorimeter within the water jacket and connect the wires to the ends of the coil. Adjust the pointer of the ammeter to zero (if necessary) by turning the small milled head at the top.

(4.) Complete the circuit, and adjust the carbon resistance till a suitable deflection is obtained on the ammeter, say 0·8, which must be maintained constant. Keep the water well agitated by means of the stirrer, and take "*time readings*" (about every half-minute) of the temperatures of the inner and outer vessels, until the inner thermometer has risen several degrees. Break the circuit.

(5.) Tabulate your results in a convenient form.

(6.) Plot a curve having times for abscissæ and temperatures of the calorimeter for ordinates.

(7.) Repeat (4), (5), (6), using successively currents which produce deflections of about 1·1, 1·4, 1·7, and 2·0 on the ammeter.

(8.) When all the heating observations have been taken, break the circuit, and allow the calorimeter to cool to nearly its initial temperature, and take time readings of its temperature, keeping the water well stirred all the while.

(9.) Plot a "*cooling curve*" from the observations obtained in (8).

(10.) Correct the heating curves obtained in (6) and (7) by the cooling curve (9), and determine the corrected rise of temperature in a given time (say five minutes).

(11.) Calculate the strength of current passing in each of the above experiments from the formula

$$A = \sqrt{\frac{(W + w)T}{0{\cdot}24 \times rt}},$$

where A stands for the current in amperes,
,, r ,, ,, resistance of the coil in ohms, which is 1·0306 at 15°·6 C.
,, W ,, ,, weight of the water in grammes,
,, w ,, ,, water-equivalent of the calorimeter, thermometer, &c., which equals 2·778 grammes,
,, T ,, ,, corrected rise of temperature in t seconds,
,, t ,, ,, time in seconds,

and compare the values so obtained with the graduations of the ammeter.

DEDUCTIONS.—State clearly how the heating curves are corrected from the cooling curve so as to show what would have been the true rise of temperature if no cooling had taken place during the experiment.

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PHYSICAL DEPARTMENT.

To CALIBRATE an AMMETER by means of a SILVER VOLTAMETER.

Preliminary.—The voltameter consists of a platinum dish containing a 25 per cent. solution of silver nitrate, and in which a silver plate is immersed. An adjustable carbon resistance is provided, by means of which the current passing through the voltameter can be maintained constant during each experiment, and can be varied in the different experiments.

Experiments.—(1.) Carefully clean, dry, and weigh the platinum dish, the *approximate* weight of which is 78 grammes.

(2.) Pour the solution of silver nitrate into the dish and place it on the three brass pins provided for its reception, and which are electrically connected with the left-hand binding screw on the board. Immerse the silver plate in the solution, and clamp it in such a position that its edges are equally distant from the sides and bottom of the dish.

(3.) Turn the small milled head at the top of the ammeter so that the pointer of the ammeter comes opposite the zero on the scale, if not there already. Place the copper connecting wire in the mercury cups marked A and C (which cuts out the voltameter), and adjust the carbon resistance until a convenient current flows round the ammeter. Remove the connecting wire.

(4.) Quickly insert the connecting wire in the mercury cups marked A and B, carefully noting the instant at which the circuit was completed. Allow the current

to pass for a convenient time (10 to 30 minutes, according to the strength of current used), and keep the current constant by the adjustable resistance. Note the temperature of the room during the experiment, and, at the end of the interval decided on, quickly break the circuit.

(5.) Empty the solution from the dish into its bottle and carefully wash the deposited silver with distilled water. Then fill the dish with distilled water and allow it to stand 10 to 15 minutes. Again wash with water, alcohol, and ether, dry over the spirit-lamp, and cool in the desiccator.

(6.) Carefully determine the increase of weight due to the silver deposited on the dish.

(7.) Calculate the strength of current used in the experiment, assuming that one ampere deposits 1·11815 milligrammes of silver per second.

(8.) Repeat the experiment with several different strengths of current.

(9.) Tabulate your results in some convenient form and write them with your name on the card, on which you will find recorded the results of previous experiments.

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